Book review

Logic: a Brief Course by Daniele Mundici, xi + 124 pages, Springer, 2012. Paperback, ISBN 978-88-470-2360-4.

This is a short introduction to mathematical logic that covers basic material in 17 chapters that total less than 130 pages. The author takes a different route than the one commonly taken in standard introductory texts which results in an attractive option for a first course on logic for computer science students.

The first non-standard decision is taken at the very beginning. Motivated by an example of the graph colouring problem discussed in Chapter 1, the book starts in Chapter 2 with the introduction of the first class of propositional formulas — those in conjunctive normal form (CNF). Full propositional logic appears only in Chapter 7. This decision has some pedagogical advantage in that the not completely trivial proof of the Unique Reading Lemma does not need to be imposed upon the reader at the very beginning, when she/he is barely introduced to a new formalism.

Chapters 2–6 focus on CNF formulas, gradually introducing their syntax and semantics (in Chapter 2), followed in Chapter 3 by the representation in the form of finite sets, the introduction of the (propositional) resolution method, and, somewhat surprisingly, the Davis-Putnam procedure (DPP). This makes it possible to completely drop the proof theory and focus instead on the procedural way of drawing conclusions, by means of refutations.

In fact, there are no proof rules at all in this book, this being the other non-standard decision. Those who find proof theory at the core of mathematical logic might not like this decision, but those who, like the logic programming community, view resolution as the realisation of a deduction process might instead favour it. Consequently, proofs are introduced as directed graphs whose vertices are clauses and whose edges depict the formation of the resolvents. (In turn, axioms appear only in Chapter 15 that deals with the equality axioms.)

The customary proof-theoretic notation

$$\frac{\phi}{\psi}$$

is used only in exercises, to introduce problems asking for determining whether ψ is a logical consequence of ϕ or can be deduced from ϕ by means of resolution or the DPP.

The exposition of the formulas in the CNF form deals with (in Chapter 4) Completeness Theorem, which in this form is due to Alan Robinson, a short account of Krom and Horn clauses (in Chapter 5) and the Compactness Theorem (in Chapter 6). Because of the restriction to formulas in CNF form the proof of the latter theorem is self-contained and does not rely on any completeness result (or the ultrafilter construction).

Full propositional logic is finally considered in Chapters 7-9 that deal respectively with syntax, semantics and reductions to CNF and DNF forms.

The second part of the book deals with predicate (i.e., first-order) logic. In Chapter 11 the alphabet of predicate logic is introduced and quantifiers are discussed. Next, in Chapter 12 terms and clauses built out of atomic formulas are defined. The chapter ends with a definition of Herbrand universe, followed by an introduction of the ground resolution, defined as an inessential variant of resolution for propositional clauses.

The next two Chapters, 13 and 14, are concerned with Tarski semantics and Gödel's Completeness Theorem for clausal logic. Then, in Chapter 15, models with equality are introduced and related to the equality axioms.

It is remarkable that the formulas of predicate logic are introduced only on page 95, in Chapter 16. Such a long run-up allows the reader to better appreciate the expressive power of full predicate logic. One of the aims of this chapter is to show how to transform by means of an algorithmic procedure a first-order formula into a finite set of propositional clauses, successively using the prenex normal form, Skolemisation, and rewriting of the clauses into a set-based notation. The chapter ends with a presentation of Gödel's Completeness Theorem for the predicate logic with equality, where provability means refutability by means of the Davis-Putnam procedure applied to the corresponding set of ground clauses. Further, Gödel's Compactness theorem and Löwenheim-Skolem Theorem are proved. Also, it is explained how the last theorem implies the existence of non-standard models of the reals with infitesimals.

The books ends with a short chapter that puts the axiomatic method into a broader perspective, by reflecting on completeness, incompleteness and expressivity, and on the limitations of predicate logic.

The book is clearly geared for a one semester course. Given these limitations any material that did not make into it would have to be traded with some existing chapters. In the case of a logic course aimed at the introduction of logic programming, a natural possibility would be to trade Chapter 15 on the equality axioms and the last two sections of Chapter 16, on Skolemisation, completeness and compactness, with a chapter on unification (introduced by means of the Martelli-Montanari algorithm) and another one on SLD-resolution and elements of Prolog.

The point is that resolution is carried out only on ground instantiations, and unification is not studied in the book. Consequently, the *computing process*, as realised in logic programming by means of the SLD-resolution, is not explained. However, all the other ingredients, including Horn clauses are already present.

The book is interspersed with several small references to various scholars involved in the development of logic, which provides for welcome interruptions in the formal exposition. Already on page 5 one is introduced to the idea of a *calculus ratiocinator* of Leibniz. Other, passing, references briefly mention Pythagoras, Saint Augustine, Frege, Dedekind, Peano, Hilbert, Poincaré, Gödel and Lindström. Such comments allow one to better appreciate the origins of propositional and predicate logic and of their limitations, and encourage the reader to reflect on how we ended up with the formalisms that now seem so obvious.

An important aspect of the book is a veritable multitude of exercises. Notably Chapter 16 has no less than fourteen pages of exercises. They often focus on formalisation of various natural language statements using clauses of propositional or predicate logic and on their deductions. The only slightly puzzling aspect of the book seems to be that quantifiers were introduced in Chapter 12 while they are actually dealt with only in Chapter 16.

All in all, it is a very nice booklet that in view of this reviewer is an attractive choice for an introductory logic course for first year computer science students. The wealth of exercises focusing on the relation between formal and natural language also makes it possible to use substantial fragments of this book in introductory logic courses in Philosophy or Linguistic departments, in which one could emphasise the relation between logical formalisms and natural language at the cost of some technical results.

The book originally appeared in Italian and was translated by this reviewer.

Krzysztof R. Apt
CWI and University of Amsterdam