

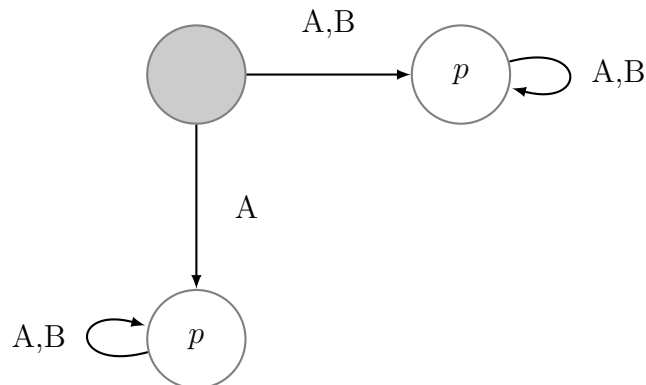
## Exercises Social Software, Lecture 4, Solutions

**Exercise 1** *Is it possible for  $p$  to be common knowledge, while Jan does not know  $p$ ?*

Answer: No, this is impossible, for the indistinguishability relation for Jan is *included* in the common knowledge relation.

**Exercise 2** *Is it possible for  $p$  to be common belief, while  $p$  is not true?*

Answer: Yes, this is very well possible. In fact, it happens a lot. See for example the next picture. The accessibility relations for belief need not be reflexive (only transitive and euclidean), so in the top left world we have  $\neg p$  but  $p$  is still a common belief there because  $p$  holds in all other worlds accessible for the agents in one or more steps.



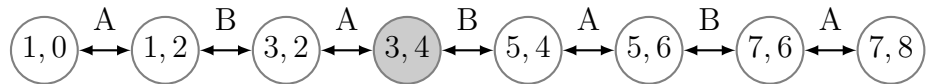
For the next exercises, assume the following scenario is common knowledge among Alice and Bob:

- Alice and Bob are each going to be told a natural number; these numbers will be one apart.
- The numbers are then whispered into Alice's and Bob's ears.

Suppose Alice privately hears 3 and Bob privately hears 4.

**Exercise 3** *Draw a possible worlds model for the situation.*

Answer: In the model below, each world has been represented by an ordered pair, where the first item represents Alice's number and the second one Bob's. There is an infinite sequence continuing to the right in the obvious way.



**Exercise 4** *Do both Alice and Bob individually know that the numbers are smaller than 6?*

Answer: Yes, because in all three worlds accessible in one step from the real world (namely (3,2), (3,4) itself and (5,4)), both numbers are smaller than 6.

**Exercise 5** *Do Alice and Bob have common knowledge that the numbers are smaller than 6?*

Answer: No, because there is a world accessible by the transitive closure of the union of the accessibility relations (for example world (7,8)) where the numbers are both larger than 6.

**Exercise 6** *Do Alice and Bob have common knowledge that the numbers are smaller than 1,000,000?*

Answer: No, in a similar fashion, because there is a world accessible by the transitive closure of the union of the accessibility relations (for example, in the world where Alice has 1,000,001 and Bob has 1,000,002, both numbers are larger than 1,000,000).