Inefficiency of Games with Social Context*

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Abstract. The study of other-regarding player behavior such as altruism and spite in games has recently received quite some attention in the algorithmic game theory literature. Already for very simple models, it has been shown that altruistic behavior can actually be harmful for society in the sense that the price of anarchy may *increase* as the players become more altruistic. In this paper, we study the severity of this phenomenon for more realistic settings in which there is a complex underlying social structure, causing the players to direct their altruistic and spiteful behavior in a refined player-specific sense (depending, for example, on friendships that exist among the players). Our findings show that the increase in the price of anarchy is modest for congestion games and minsum scheduling games, whereas it is drastic for generalized second price auctions.

1 Introduction

Many practical situations involve a group of strategic decision makers who attempt to achieve their own self-interested goals. It is well known that strategic decision making may result in outcomes that are suboptimal for the society as a whole. The need to gain an accurate understanding of the extent of suboptimality caused by selfish behavior has led to the study of the *inefficiency of equilibria* in algorithmic game theory. In this context, a common inefficiency measure is the *price of anarchy* [21], which relates the worst-case cost of a Nash equilibrium to the one of an optimal outcome.

More recently, quite some attention has been given to more general settings in which the players do not necessarily behave entirely selfishly, but may alternatively exhibit *spiteful* or *altruistic* behavior; see, for instance, [2, 4, 5, 7–9, 14, 17–19]. Studying such alternative behaviors in games is motivated by the observation that altruism and spite are phenomena that frequently occur in real life (see, for example, [15]). Consequently, it is desirable to incorporate such alternative behavior in game-theoretical analyses.

Previous work on the price of anarchy for spiteful and altruistic games has focused on simple models of spite and altruism, where a spite/altruism level α_i is associated to each player i denoting the extent to which his perceived cost is

* This research was partially supported by the EU FET project MULTIPLEX 317532.

influenced by any nonspecific other player. Already for these simple models it has been observed in a series of papers [5, 7, 8] that altruistic behavior can actually be harmful in the sense that the price of anarchy may *increase* as players become more altruistic. This observation served as a starting point for the investigations conducted in this paper. The main question that we address here is: How severe can this effect be if one considers more refined models of altruism that capture complex social relationships between the players?

Our Contributions. In the present paper, we study a more general player-specific model of spite and altruism. Our model can be viewed as extending a given strategic game by imposing a *social-network structure* on top of the players, which specifies for each pair of players (i, j) an altruism/spite level α_{ij} signifying how much player i cares about player j; these relations are not necessarily symmetric. This allows us to model more realistically settings in which the behavior of the players depends on a complex underlying social structure, expressing friendships and animosities among the players. Our altruistic games fall into the framework of *social context games* proposed in [1].

For this general model of games with altruism and spite, we are interested in studying the price of anarchy. The *smoothness framework*, originally introduced by Roughgarden [22], has become a standard method for proving upper bounds on the price of anarchy. Basically, this framework shows that such bounds can be derived by establishing a certain smoothness condition. An additional strength of this approach is that the smoothness condition allows to derive upper bounds on the price of anarchy for various solution concepts, ranging from pure Nash equilibria to coarse correlated equilibria; the latter being naturally related to outcomes resulting from natural learning algorithms (see, for example, Young [26]). Here, we extend the smoothness framework such that it can be used conveniently it our setting.

Using this extension, we prove upper bounds on the price of anarchy for altruistic versions of three classes of well-studied games: congestion games, minsum scheduling games, and generalized second price auctions. We show that for unrestricted altruism levels the price of anarchy is unbounded. In particular, this happens if there is a player i who does not care about himself or he cares more about some friend than about himself, that is, $\alpha_{ij} > \alpha_{ii}$. We therefore derive our upper bounds under the mild assumption that each player cares at least a little about himself and he cares about any other player at most as much as he cares about himself; we refer to this as *restricted altruistic social context*. Under this assumption, we derive the following upper bounds on the coarse price of anarchy:

- A bound of 7 for altruistic linear congestion games, and a bound of $\varphi^3 \approx 4.236$ for the special case of singleton linear congestion games, where $\varphi = (1+\sqrt{5})/2$ denotes the golden ratio.
- A bound of $4 + 2\sqrt{3} \approx 7.4641$ and $12 + 8\sqrt{2} \approx 23.3137$ for altruistic minsum machine scheduling games for related and unrelated machines, respectively.
- A bound of $2(n + 1)$ for altruistic generalized second price auctions, where n is the number of players.

Our results therefore show that for congestion games and minsum scheduling games the price of anarchy cannot drastically increase. Specifically, it remains constant, independently of how complex the underlying altruistic social structure is. On the other hand, for generalized second price auctions the price of anarchy may degrade quite drastically: we prove an upper bound of $O(n)$, as opposed to a small constant which is known for the purely selfish setting [6].

We derive most upper bounds using a simple proof template: we decompose the altruistic game into a selfish part and an altruistic part and prove smoothness for each part separately. We can import known smoothness results for the selfish part and only need to focus on the altruistic part.

Our upper bound proof for singleton congestion games uses a novel proof approach: We use a refined *amortized* argument by distributing some additional "budget" unevenly among the facilities. We believe that this approach might be of independent interest.

Related Work. There are several papers that propose models of altruism and spite [4, 5, 7–9, 14, 17–19]. All these models are special cases of the one studied here. Among these articles, the inefficiency of equilibria in the presence of altruistic/spiteful behavior was studied for various games in [5, 7–9, 14]. After its introduction in [22], the smoothness framework has been adapted in various directions [23–25], including an extension to a particular model of altruism in [8], which constitutes a special case of the altruistic games considered here.

Biló et al. [2] also studied social context congestion games, in the case where the perceived cost of a player is the minimum, maximum, or sum of the immediate cost of his neighbors. [2] establishes, among other results, an upper bound of 17/3 on the pure price of anarchy of linear congestion games for a special case of the setting we study here.

Related but different from our setting, is the concept of *graphical congestion games* [3, 16]. Here the cost and the strategy set of a player depends only on a subset of the players.

2 Preliminaries

Altruistic Extensions of Games. We study the effect of altruistic behavior in strategic games. To model the complex altruistic relationships between the players, we equip the underlying game with an *altruistic social context*. More precisely, let $\Gamma = (N, \{\Sigma_i\}_{i \in N}, \{c_i\}_{i \in N})$ be a strategic game (termed *base game*), where $N = \{1, \dots, n\}$ is the set of players, Σ_i is the strategy set of player i, and $c_i : \Sigma \to \mathbb{R}$ is the direct cost function of player i that maps every strategy profile $s \in \Sigma = \Sigma_1 \times \cdots \times \Sigma_n$ to a real value. Unless stated otherwise, we assume that Γ is a cost minimization game, that is, every player i wants to minimize his individual cost function c_i. Further, we assume that an *altruistic social context* is given by an $n \times n$ matrix $\alpha \in \mathbb{R}^{n \times n}$.

Given a base game Γ and an altruistic social context α, the α-*altruistic extension* of Γ is defined as the strategic game $\Gamma^\alpha = (N, \{\Sigma_i\}_{i \in N}, \{c_i^\alpha\}_{i \in N})$, where for all $i \in N$ and $s \in \Sigma$, the *perceived cost* $c_i^\alpha(s)$ of player i is given by

$$c_i^\alpha(s) = \sum_{j=1}^{n} \alpha_{ij} c_j(s). \tag{1}$$

Thus, the perceived cost of player i in the α-altruistic extension is the α_{ij}-weighted sum of the individual direct costs of all players in the base game. A positive (negative) α_{ij} value signifies that player i cares positively (negatively) about the direct cost of player j, which can be interpreted as an altruistic (spiteful) attitude of i towards j. Note that α_{ii} specifies how player i cares about himself; we also call α_{ii} the *self-perception level*. For simplicity, we will often refer to the resulting game Γ^α as the *α-altruistic game*, without explicitly mentioning the base game Γ and the altruistic social context α.

The above viewpoint has a natural interpretation in terms of *social networks*: Suppose the players in N are identified with the nodes of a complete directed graph $G = (N, A)$. The weight of an edge $(i, j) \in A$ is equal to α_{ij}, specifying the extent to which player i cares about the cost of player j.

The main focus of this paper is on altruistic behavior. We distinguish between *unrestricted* and *restricted* altruistic social contexts α. In the *unrestricted* case we assume that $\alpha_{ij} \geq 0$ for every $i, j \in N$; in particular, the self-perception level of a player can be zero. In this case, one can prove trivial lower bounds for the price of anarchy, just by setting $\alpha_{ij} = 0$, for all i, j. For this reason we consider also the more interesting restricted case. In the *restricted* case, every player has a positive self-perception level and cares about himself at least as much as about any other player, namely, $\alpha_{ii} > 0$ and $\alpha_{ii} \geq \alpha_{ij} \geq 0$ for every $i, j \in N$, $i \neq j$. In the latter case, we can normalize α without loss of generality such that $\alpha_{ii} = 1$ for every player i.[3]

Coarse Equilibria and the Price of Anarchy. We are interested in the efficiency loss caused by altruistic behavior. Let $C : \Sigma \to \mathbb{R}$ be a *social cost* function that maps strategy profiles to real numbers. Most of the time in this paper, the social cost will refer to the sum of the direct costs of all players, namely, $C(s) = \sum_{i=1}^{n} c_i(s)$. The motivation therefore is that we are interested in the efficiency of the outcome resulting from altruistic behavior, which is modeled through the altered perceived cost functions.

We focus on the inefficiency of *coarse equilibria*, which are defined as follows: Let σ be a probability distribution over Σ. Let σ_{-i} denote the projection of σ onto $\Sigma_{-i} = \Sigma_1 \times \cdots \times \Sigma_{i-1} \times \Sigma_{i+1} \times \cdots \times \Sigma_n$. Then σ is a *coarse equilibrium* of the altruistic game Γ^α if, for every player i and every strategy $s_i^* \in \Sigma_i$, it holds that $\mathbf{E}_{s\sim\sigma}[c_i^\alpha(s)] \leq \mathbf{E}_{s_{-i}\sim\sigma_{-i}}[c_i^\alpha(s_i^*, s_{-i})]$. We use $CE(\Gamma^\alpha)$ to denote the set of coarse equilibria of Γ^α. Coarse equilibria include several other solution concepts, such as correlated equilibria, mixed Nash equilibria, and pure Nash equilibria.

We study the *price of anarchy* [21] of coarse equilibria of altruistic games. For an altruistic game Γ^α, define $POA(\Gamma^\alpha) = \sup_{s \in CE(\Gamma^\alpha)} C(s)/C(s^*)$, where

[3] To see this, note that, by dividing all α_{ij} by $\alpha_{ii} > 0$, the set of equilibria and the social cost of any outcome remain the same.

s^* is a strategy profile that minimizes C. The *coarse price of anarchy* of a class of altruistic games $\mathcal{G}$ is defined as $POA(\mathcal{G}) = \sup_{\Gamma^\alpha \in \mathcal{G}} POA(\Gamma^\alpha)$.

Because of space restrictions, some material has been omitted from this extended abstract and will appear in a full version of this work.

3 Smoothness and a Proof Template

Smoothness. Roughgarden [22] introduced a general smoothness framework to derive bounds on the coarse price of anarchy. Next we extend this framework to α-altruistic games with arbitrary social cost functions.

Definition 1. *Let Γ^α be an α-altruistic extension of a cost minimization game with $\alpha \in \mathbb{R}^{n\times n}$ and social cost function C. Further, let s^* be a strategy profile that minimizes C. Γ^α is* (λ, μ)-smooth *if there exists a strategy profile $\bar{s} \in \Sigma$ such that for every strategy profile $s \in \Sigma$ it holds that*

$$\sum_{i=1}^{n} \sum_{j=1}^{n} \alpha_{ij}(c_j(\bar{s}_i, s_{-i}) - c_j(s)) \leq \lambda C(s^*) + (\mu - 1)C(s). \tag{2}$$

The following theorem shows that (λ, μ)-smoothness implies a bound on the coarse price of anarchy of α-altruistic games.

Theorem 1. *Let Γ^α be an α-altruistic extension of a cost minimization game with $\alpha \in \mathbb{R}^{n\times n}$ and social cost function C. If Γ^α is (λ, μ)-smooth with $\mu < 1$, then the coarse price of anarchy of Γ^α is at most $\lambda/(1 - \mu)$.*

The above smoothness definition allows us to import some additional results from [22] (e.g., on the efficiency of natural learning algorithms). The proof of Theorem 1 and further discussion wll appear in a full version of this work.[4]

Proof Template. Most of our smoothness results are based on the following decomposition idea. Recall that for restricted altruistic social contexts we have $\alpha_{ii} = 1$. Suppose that the underlying base game is known to be (λ_1, μ_1)-smooth (in the purely selfish setting), that is, there is some $\bar{s} \in \Sigma$ such that

$$\sum_{i=1}^{n} c_i(\bar{s}_i, s_{-i}) \leq \lambda_1 C(s^*) + \mu_1 C(s), \tag{3}$$

and that $C(s) \leq \sum_i c_i(s)$. Then, to establish $(\lambda, \mu) = (\lambda_1 + \lambda_2, \mu_1 + \mu_2)$-smoothness for the altruistic game Γ^α, it suffices to prove that for $\bar{s}$ it holds

$$\sum_{i=1}^{n} \sum_{j\neq i} \alpha_{ij}(c_j(\bar{s}_i, s_{-i}) - c_j(s)) \leq \lambda_2 C(s^*) + \mu_2 C(s). \tag{4}$$

[4] In the purely selfish setting (i.e., when $\alpha_{ii} = 1$ and $\alpha_{ij} = 0$ for every $i, j \in N$, $i \neq j$) our smoothness definition is slightly more general than the one in [22] where (2) is required to hold for any arbitrary strategy profile s^* and with $\bar{s} = s^*$. Also, in [22] the analogue of Theorem 1 is shown under the additional assumption that C is *sum-bounded*, that is, $C(s) \leq \sum_i c_i(s)$. Here, we get rid of this assumption.

4 Congestion Games

In a congestion game $\Gamma = (N, E, \{d_e\}_{e\in E}, \{\Sigma_i\}_{i\in N})$ we are given a set of players $N = \{1, \dots, n\}$, a set of *facilities* E with a *delay function* $d_e : \mathbb{N} \to \mathbb{R}$ for every facility $e \in E$, and a strategy set $\Sigma_i \subseteq 2^E$ for every player $i \in N$. For a strategy profile $s \in \Sigma = \Sigma_1 \times \cdots \times \Sigma_n$, define $x_e(s)$ as the number of players using facility $e \in E$, that is, $x_e(s) = |\{i \in N : e \in s_i\}|$. The direct cost of player i is defined as $c_i(s) = \sum_{e\in s_i} d_e(x_e(s))$ and the social cost function is given by $C(s) = \sum_{i=1}^n c_i(s)$. In a *linear* congestion game, the delay function of every facility $e \in E$ is of the form $d_e(x) = a_e x + b_e$, where $a_e, b_e \in \mathbb{Q}_{\geq 0}$ are nonnegative rational numbers.

4.1 General Linear Congestion Games

Theorem 2. *Every α-altruistic extension of a linear congestion game with restricted altruistic social context α is $(\frac{7}{3}, \frac{2}{3})$-smooth. Therefore, the coarse price of anarchy is at most 7 for these games.*

We need the following simple lemma for the proof of Theorem 2. Its proof will appear in a full version of this paper.

Lemma 1. *For every two integers $x, y \in \mathbb{N}$, $xy \leq \frac{2}{3}y^2 + \frac{1}{3}x^2$.*

Proof (Theorem 2). Let s be an arbitrary strategy profile and let s^* be a strategy profile that minimizes C. We can assume without loss of generality that $d_e(x) = x$ for all $e \in E$.

The base game is known to be $(\lambda_1, \mu_1) = (\frac{5}{3}, \frac{1}{3})$-smooth for $\bar{s} = s^*$ [10,11,22]. Using our proof template, it is sufficient to show that (4) holds with $(\lambda_2, \mu_2) = (\frac{2}{3}, \frac{1}{3})$.

Let x_e and x_e^* refer to $x_e(s)$ and $x_e(s^*)$, respectively. Fix some player $i \in N$ and let $x'_e = x_e(s_i^*, s_{-i})$. Note that $x'_e = x_e + 1$ for $e \in s_i^* \setminus s_i$, $x'_e = x_e - 1$ for $e \in s_i \setminus s_i^*$ and $x'_e = x_e$ otherwise. Using these relations, we obtain

$$\begin{aligned}\sum_{j\neq i} \alpha_{ij}(c_j(s_i^*, s_{-i}) - c_j(s)) &= \sum_{j\neq i} \left(\sum_{e\in s_j\cap(s_i^*\setminus s_i)} \alpha_{ij} - \sum_{e\in s_j\cap(s_i\setminus s_i^*)} \alpha_{ij} \right) \\ &= \sum_{e\in s_i^*\setminus s_i} \sum_{j\neq i: e\in s_j} \alpha_{ij} - \sum_{e\in s_i\setminus s_i^*} \sum_{j\neq i: e\in s_j} \alpha_{ij}.\end{aligned}$$

Summing over all players and exploiting that in the restricted case $0 \leq \alpha_{ij} \leq 1$ for every $i, j \in N$, $i \neq j$, we can bound

$$\sum_{i=1}^n \left(\sum_{e\in s_i^*\setminus s_i} \sum_{j\neq i: e\in s_j} \alpha_{ij} - \sum_{e\in s_i\setminus s_i^*} \sum_{j\neq i: e\in s_j} \alpha_{ij} \right) \leq \sum_{i=1}^n \sum_{e\in s_i^*} \sum_{j: e\in s_j} 1 = \sum_{e\in E} x_e x_e^*.$$

Using Lemma 1, we conclude that $\sum_{e\in E} x_e x_e^* \leq \frac{2}{3}C(s^*) + \frac{1}{3}C(s)$ as desired. □

4.2 Singleton Congestion Games

We derive a better smoothness result for *singleton congestion games with identical delay functions*, that is, when $\Sigma_i \subseteq E$ for every $i \in N$, so that for each strategy $s \in \Sigma_i$ we have that $|s| = 1$.

Theorem 3. *Every α-altruistic extension of a singleton linear congestion game with identical delay functions on all facilities under restricted altruistic social context α is $(1+\varphi, 1/\varphi^2$-smooth, where $\varphi = (1+\sqrt{5})/2$ is the golden ratio. Therefore, the coarse price of anarchy is at most $\varphi^3 \approx 4.236$ for these games.*

To prove this theorem, we use a novel proof approach. In most existing proofs one first massages the smoothness condition to derive an equivalent condition summing over all facilities (instead of players), and then establishes smoothness by reasoning for each facility separately. If we follow this approach here, we again obtain an upper bound of 7. Instead, we use an *amortized* argument here to derive our improved bound.

A careful analysis (details omitted because of paucity of space) can show that the smoothness definition (2) for singleton linear congestion games with $\bar{s} = s^*$ is equivalent to

$$\sum_{i=1}^{n} \sum_{j \neq i} \left(\lambda \left| s_i^* \cap s_j^* \right| + (\mu + \alpha_{ij}) \left| s_i \cap s_j \right| - (1 + \alpha_{ij}) \left| s_i^* \cap s_j \right| \right) + (\lambda + \mu - 1)n \geq 0. \tag{5}$$

We translate the proof of this inequality to a coloring problem on a suitably defined graph. We construct an *extended social network* as follows: For every player $i \in N$ we introduce two nodes i and i^* representing player i under s and s^*, respectively. We call the former type of nodes *s-nodes* and the latter type of nodes *s^*-nodes*. For every two players $i, j \in N$ with $i \neq j$ we introduce four edges: (i, j) with weight $2\mu + \alpha_{ij} + \alpha_{ji}$, (i^*, j^*) with weight 2λ, (i^*, j) with weight $-(1 + \alpha_{ij})$, and (i, j^*) with weight $-(1 + \alpha_{ji})$. We identify the set of facilities E with a set of m *colors*, such that $E = [m]$. The colors assigned to i and i^* are s_i and s_i^*, respectively. Call an edge $e = (u, v)$ in the extended network *c-monochromatic* if both u and v have color c. In addition, we distribute a total budget of $(\lambda + \mu - 1)n$ among the $2n$ nodes of the extended network.

With the viewpoint of the previous paragraph, the left-hand side of (5) is equal to the total weight of all c-monochromatic edges (summed over all colors c) plus the total budget of all nodes. The idea now is to argue that we can fix λ and μ such that for each color $c \in [m]$ the total weight of all c-monochromatic edges plus the respective node budget is at least 0. The crucial insight to derive our improved bound is that the budget is split unevenly among the nodes: we assign a budget of $(\lambda + \mu - 1)$ to every s-node and 0 to every s^*-node.

Fix some color $c \in [m]$ and consider the subgraph of the extended network induced by the nodes having color c. Partition the nodes into the set S_c of s-nodes and the set S_c^* of s^*-nodes. Imagine we draw this subgraph with all nodes in S_c put on the left-hand side and all nodes in S_c^* put on the right-hand side.

The edges from S_c to S_c^* are called *crossing edges*. The edges that stay within S_c or S_c^* are called *internal* edges. Let $x = x_c = |S_c|$ and $y = y_c = |S_c^*|$. Note that the internal edges in S_c constitute a complete graph on x nodes. Similarly, the internal edges in S_c^* constitute a complete graph on y nodes. Note that the crossing edges constitute a $K_{x,y}$ with a few edges missing, namely the pairs (i, i^*) representing the same player i (which are nonexistent by construction). Let $z = z_c$ be the number of such pairs.

In the worst case, $\alpha_{ij} = 0$ for all internal edges and $\alpha_{ij} = 1$ for all crossing edges. The total contribution to the left-hand side of (5) that we can account for color c is then

$$
\begin{aligned}
&2\mu \cdot \tfrac{1}{2}x(x-1) + 2\lambda \cdot \tfrac{1}{2}y(y-1) - 2 \cdot (xy - z) + (\lambda + \mu - 1) \cdot x \\
&\quad = \mu x^2 + \lambda y^2 - 2xy + (\lambda - 1)x - \lambda y + 2z. \qquad (6)
\end{aligned}
$$

We need the following lemma, whose proof will appear in a full version of this work. It is actually tight, implying that under the smoothness framework we cannot show a better bound. It is a small variation of Lemma 1 in [12].

Lemma 2. *Let $\varphi = \frac{1+\sqrt{5}}{2}$ be the golden ratio. For every two integers $x, y \in \mathbb{N}$, $2xy - \varphi x + \varphi^2 y \leq \frac{1}{\varphi^2}x^2 + \varphi^2 y^2$.*

Fix $\lambda = 1 + \varphi$ and $\mu = 1/\varphi^2$. Then (6) is nonnegative by Lemma 2. Summing over all colors $c \in [m]$ proves (5). Given our choices of $\lambda = 1 + \varphi$ and $\mu = 1/\varphi^2$ we obtain a bound on the coarse price of anarchy of $\varphi^3 \approx 4.236$. □

5 Minsum Machine Scheduling

In a scheduling game, we deal with a set of machines $[m]$, and a set of jobs $[n]$ that are to be scheduled on the machines. For each job $i \in [n]$ and machine $k \in [m]$, we are given a *processing time* $p_{i,k} \in \mathbb{R}_{\geq 0}$, which is the time it takes to run job i on machine k.

There are many ways in which a machine may execute the set of jobs it gets assigned. We restrict ourselves here to a popular policy where the jobs on a machine are executed one-by-one, in order of increasing processing time. Ties are broken deterministically, and we write $i \prec_k j$ if $p_{i,k} < p_{j,k}$ or $p_{i,k} = p_{j,k}$ and the tie breaking rule schedules job i before job j on machine k. A *schedule* is a vector $s = (s_1, \ldots, s_n)$, where for $i \in [n]$, s_i is the machine on which job i is to be ran. We define the value $N(i, k, s)$ to be the number of jobs j on machine k under strategy profile s for which it holds that $i \prec_k j$. Given s, the *completion time* of a job i under s is $p_{i,s_i} + \sum_{j: j \prec_{s_i} i, s_j = s_i} p_{j,s_j}$. The jobs take the role of the players: the strategy set of a player is $[m]$, so the strategy profiles are schedules. The cost $c_j(s)$ of a job $j \in [n]$ under strategy profile s is the completion time of j under s.

We define the social cost function for this game to be the sum of the completion times of the jobs. The social cost can be written as
$C(s) = \sum_{k=1}^{m} \sum_{i: s_i = k} (N(i, k, s) + 1) p_{i,k}$.

If the processing times are not restricted, we speak of *unrelated machine scheduling games*. We speak of *related machine scheduling games* if the processing times are defined as follows: For each machine $k \in [m]$, there is a *speed* $t_k \in \mathbb{R}_{>0}$ and for each job $j \in [n]$ there is a *length* $p_j \in \mathbb{R}_{\geq 0}$ such that $p_{i,k} = p_j/t_k$ for all $i \in [n]$, $k \in [m]$.

Next, we prove constant upper bounds on the price of anarchy for restricted altruistic social contexts.

Theorem 4. *Every α-altruistic extension of a machine scheduling game with restricted altruistic social context α is $(2+x, 1/x)$-smooth for related machines and $(2+x, 1/2+1/x)$-smooth for unrelated machines for every $x \in \mathbb{R}_{>0}$. Therefore, the coarse price of anarchy is at most $4 + 2\sqrt{3} \approx 7.4641$ (choosing $x = 1 + \sqrt{3}$) and $12 + 8\sqrt{2} \approx 23.3137$ (choosing $x = 2 + 2\sqrt{2}$) for these games, respectively.*

Proof. We only give the main steps of the proof here. All missing details will appear in a full version of this extended abstract. In [20] it is proved that the base game for the case of related machines is $(2, 0)$-smooth, and from [13], it follows that the base game for the case of unrelated machines is $(2, 1/2)$-smooth. Let s^* be an optimal schedule and let s be any schedule. We show that for all $x > 0$

$$\sum_{i=1}^{n} \sum_{j \neq i} \alpha_{ij}(c_j(s_i^*, s_{-i}) - c_j(s)) \leq xC(s^*) + \frac{C(s)}{x}.$$

Let $P_1 = \{(i, j) : s_i^* = s_j, s_i^* \neq s_i, i \prec_{s_i^*} j\}$. Informally, P_1 is the set of pairs of jobs (i, j) such that i's strategy change from s_i to s_i^* makes j become scheduled later. After some derivations, we obtain

$$\sum_{i=1}^{n} \sum_{j \neq i} \alpha_{ij}(c_j(s_i^*, s_{-i}) - c_j(s)) \leq \sum_{(i,j) \in P_1} p_{i,s_i^*}.$$

Turning the last expression into a summation over the machines, and again after a series of calculations, we obtain

$$\begin{aligned}\sum_{(i,j) \in P_1} p_{i,s_i^*} \leq &\sum_{k=1}^{m} \sum_{i: s_i^* = k} (x(N(i, k, s^*) + 1) - 1)p_{i,k} \\ &+ \sum_{k=1}^{m} \sum_{\substack{i: s_i^* = k, s_i \neq k, \\ N(i,k,s) > xN(i,k,s^*) + x - 1}} \lceil N(i, k, s) - xN(i, k, s^*) - x + 1 \rceil p_{i,k}.\end{aligned}$$

Consider a job i and machine k such that it holds that $s_i^* = k$, $s_i \neq k$, and $N(i, k, s) > xN(i, k, s^*) + x - 1$. Let $S(i, k)$ be the set of $\lceil N(i, k, s) - xN(i, k, s^*) - x \rceil$ smallest jobs $j \succ_k i$ such that $s_j = k$. Note that $S(i, k)$ is well defined in the sense that this number of jobs exists because $N(i, k, s) > xN(i, k, s^*) + x - 1$ implies $\lceil N(i, k, s) - xN(i, k, s^*) - x \rceil \geq 0$, and because there exist $N(i, k, s) \geq |S(i, k)|$ jobs $j \succ_k i$ with $s_j = k$. Note that for every job $j \in S(i, k)$ it holds that

$N(j,k,s) \geq N(i,k,s) - |S(i,k)| > xN(i,k,s^*) + x - 1$. We use this to upper bound the above and eventually obtain:

$$\sum_{(i,j)\in P_1} p_{i,s_i^*} \leq xC(s^*) + \sum_{k=1}^{m} \sum_{j:s_j=k} \sum_{\substack{i:s_i^*=k, s_i\neq k, i\prec_k j,\\ N(j,k,s)>xN(i,k,s^*)+x-1}} p_{j,k}. \qquad (7)$$

The next step in the derivation is made by observing that for each job j and each machine k such that $s_j = k$, there are at most $\lceil (N(j,k,s) - x + 1)/x \rceil$ jobs $i \prec_k j$ such that $s_i^* = k$, $s_i \neq k$ and $N(j,k,s) > xN(i,k,s^*) + x - 1$. To see this, assume for contradiction that there are *more* than $\lceil (N(j,k,s) - x + 1)/x \rceil$ jobs $i \prec_k j$ such that $s_i^* = k$, $s_i \neq k$ and $N(j,k,s) > xN(i,k,s^*) + x - 1$. Let i be the $(\lceil (N(j,k,s) - x + 1)/x \rceil + 1)$-th largest job for which these three properties hold. Then, there are at least $(\lceil (N(j,k,s) - x + 1)/x \rceil + 1)$ jobs scheduled on machine k that have these properties and that are scheduled after i on machine k under strategy s^*. Therefore, we have that $xN(i,k,s^*) + x - 1 \geq x\lceil (N(j,k,s) - x + 1)/x \rceil + x - 1 \geq N(j,k,s)$, which is a contradiction. Exploiting this observation, we derive that the right-hand side of (7) is at most $xC(s^*) + \frac{C(s)}{x}$, which concludes the proof. □

6 Generalized Second Price Auctions

We study auctions where a set $N = [n]$ of n bidders compete for k slots. Each bidder $i \in N$ has a valuation $v_i \in \mathbb{R}_{\geq 0}$ and specifies a bid $b_i \in \mathbb{R}_{\geq 0}$. Each slot $j \in [k]$ has a *click-through rate* $\gamma_j \in \mathbb{R}_{\geq 0}$. Without loss of generality, we assume that the slots are sorted according to their click-through rates such that $\gamma_1 \geq \dots \geq \gamma_k$ and that $k = n$.[5]

We consider the *generalized second price auction (GSP)* as the underlying mechanism. Given a bidding profile $b = (b_1, \dots, b_n)$, GSP orders the bidders by nonincreasing bids and assigns them in this order to the slots. Each bidder pays the next highest bid for his slot. More precisely, let $b_1 \geq \dots \geq b_n$ be the ordered list of bids. We assume without loss of generality that if $b_i = b_j$ for two bidders $i > j$ then i precedes j in the order. Then bidder i is assigned to slot i and has to pay b_{i+1}, where we define $b_{n+1} = 0$. The utility of player i for bidding profile b is defined as $u_i(b) = \gamma_i(v_i - b_{i+1})$. The *social welfare* for a bidding profile b is defined as $\Pi(b) = \sum_{i=1}^{n} \gamma_i v_i$.

A standard assumption we make in this setting is that bidders do not *overbid* their valuations, that is, $\Sigma_i = [0, v_i]$. This assumption is made for reasons related to individual rationality.

We prove that the coarse price of anarchy of α-altruistic GSP auctions is $O(n)$ if the altruistic social context is restricted. Note that we consider a profit maximization game here. Definition 1, Theorem 1, and our proof template extend naturally to profit maximization games. The details will appear in a full version of this paper. We are able to prove the following theorem.

[5] If $k < n$ we can add $n - k$ dummy slots with click-through rate 0; if $k > n$ we can remove the $k - n$ last slots.

Theorem 5. *Every α-altruistic extension of a generalized second price auction with restricted altruistic social context α is $(\frac{1}{2}, n)$-smooth. Therefore, the coarse price of anarchy is at most $2n+1$ for these games.*

Proof. Let b^* and b be two bidding profiles. By renaming, we assume that for all j, bidder j gets assigned to slot j under bidding profile b.

The base game is known to be $(\lambda_1, \mu_1) = (\frac{1}{2}, 1)$-smooth [23]. That is, for every two bidding profiles b, b^*, it holds that $\sum_{i \in N} u_i(b_i^*, b_{-i}) \geq \frac{1}{2}\Pi(b^*) - \Pi(b)$.

It remains to bound

$$\sum_{i=1}^{n} \sum_{j \neq i} \alpha_{ij}(u_j(b_i^*, b_{-i}) - u_j(b)) \geq \sum_{i=1}^{n} \sum_{j \neq i} \alpha_{ij}(-u_j(b)) \geq \sum_{i=1}^{n} \sum_{j \neq i} \alpha_{ij}(-\gamma_j v_j)$$
$$\geq \sum_{i=1}^{n} \sum_{j \neq i} -\gamma_j v_j \geq -(n-1)\Pi(b).$$

Combining these inequalities proves $(\lambda, \mu) = (\frac{1}{2}, n)$-smoothness. □

As in the case of congestion games, the analysis is essentially tight. Details will appear in an extended version of this work.

Concluding Remarks. The main focus of this paper was put on deriving upper bounds on the price of anarchy that are *independent* of the underlying social network structure. An interesting open question is whether one can derive refined bounds by exploiting *structural properties* of the underlying social network.

Our model of altruistic games and the smoothness definition introduced in Section 2 allows us to incorporate spiteful player behavior. We leave it as an interesting open direction for future research to pursue such analyses for spiteful behavior.

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