NLP, Philosophy, and Logic

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Abstract

In this tutorial, the meaning of natural language is analysed along the lines proposed by Gottlob Frege and Richard Montague. In building meaning representations, we assume that the meaning of a complex expression derives from the meanings of its components. Typed logic is a convenient tool to make this process of composition explicit. Typed logic allows for the building of semantic representations for formal languages and fragments of natural language in a compositional way. The tutorial ends with the discussion of an example fragment, implemented in the functional programming language Haskell Haskell Team; Jones [2003].

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- When listening to Nature, we use propositions rather than sentences. When the temperature is 32°F we feel cold, and when the temperature is 0°C we also feel cold.
- But depending on our mother tongue, we will utter different sentences.

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- Now suppose we only have a thermometer, and we have instructed a person with the following sentence: “Turn on the heat when the temperature falls below 18°C.” If the thermometer has a Fahrenheit scale, will she turn on the heat if it shows 50°F?

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- Many different proposals for how this works ...
- Fortunately, philosophical problems can be put 'on hold' ...

The Use of Logic: Checking for Consistency

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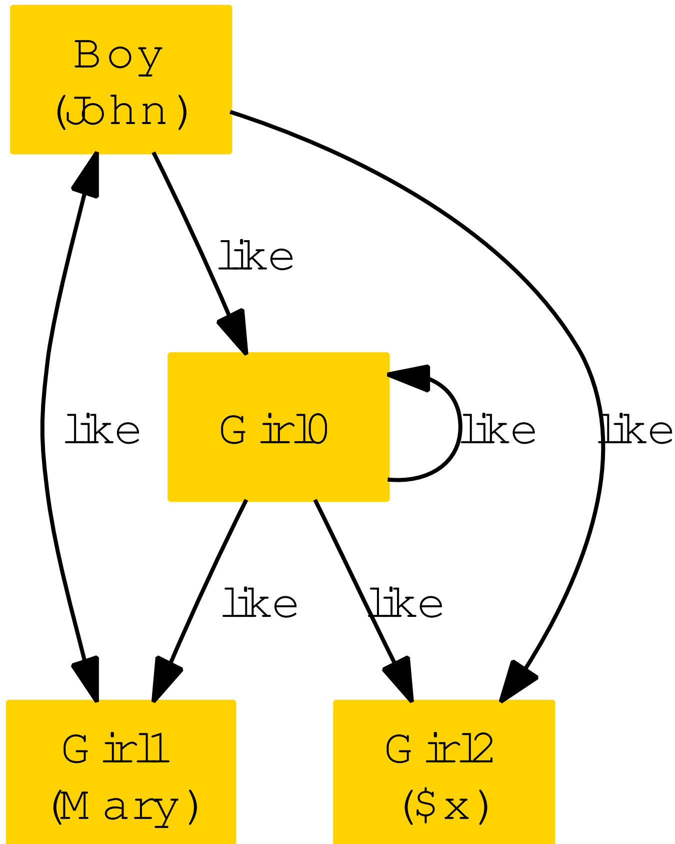
- How do we check whether this piece of text is consistent?
- One possibility: translate the sentences into predicate logic and use an analyzer for predicate logic, e.g. Alloy MIT Software Design Group; Jackson [2006].

First Order Analysis

```
abstract sig Person { like: set Person }
sig Boy extends Person {}
one sig John in Boy {}
sig Girl extends Person {}
one sig Mary in Girl {}

fact likesAndDislikes {
   all x: Girl | x in like[John]
   some x: Girl | all y: Person | y != John => y in like[x]
   all x: Person | x in like[Mary] <=> x = John
   no x: Boy - John | all y: Girl | y in like[x]
}
run {} for exactly 4 Person
```

Example Model



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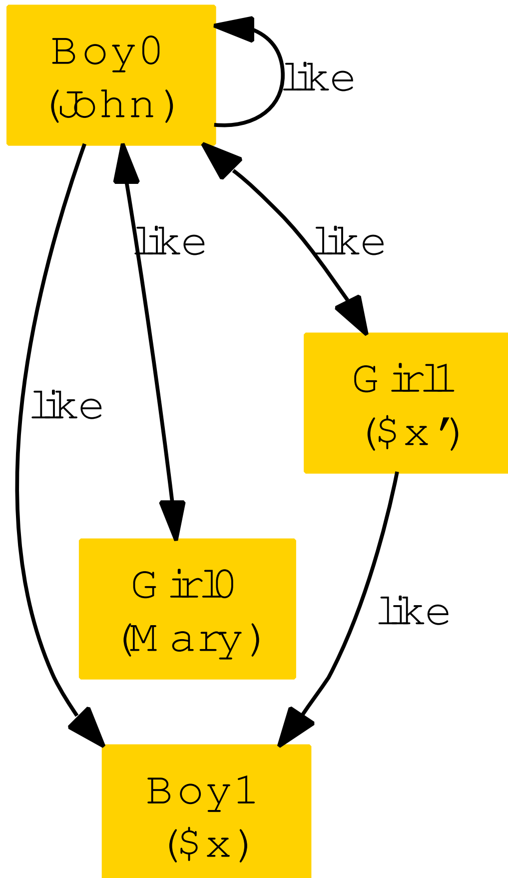
- And . . . , we get a counterexample.
- This shows that the text does not entail that there is only one boy.

CounterExample Model



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- Is it also possible to make knowledge of the meaning of a fragment of natural language fully explicit?

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- The meanings of complex expressions should be derivable in a systematic fashion from the meanings of the smallest building blocks occurring in those expressions.
- The meaning of these smallest building blocks is taken as given.

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- This mystery is not explained away by logical analysis.
- Logic is a manifestation of the Power of Pure Intelligence rather than an explication of it.
- Logic gives us patterns and connections.
- Logic is simple. This simplicity comes at a price. It is achieved by abstracting away from what is difficult.

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- The intuition that this is always possible can be stated somewhat more precisely; it is called the Principle of Compositionality:

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  > The meaning of an expression is a function of the meanings of its immediate syntactic components plus their syntactic mode of composition.

- The principle of compositionality is implicit in Gottlob Frege's writings on philosophy of language Frege [1892]; it has been made fully explicit in Richard Montague's approach to natural language semantics.

Misleading Form and Logical Form

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- From Alice walked on the road it follows that someone walked.
- From Nobody walked on the road it does not follow that someone walked.

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- Frege remarks that a quantified expression like every unmarried man or nobody does not give rise to a concept by itself (eine selbstandige Vorstellung), but can only be interpreted in the context of the translation of the whole sentence.

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- Strictly speaking the expression A does not reveal that it is supposed to combine with an individual term to form a formula (an expression denoting a truth value).
- One way to make this explicit is by means of lambda notation. The function expression of this example is then written as $(\lambda x.Ax)$. It is also possible to be even more explicit, and write

  $\lambda x.(Ax) :: e \rightarrow t$

  to indicate the type of the expression, or even:

  $(\lambda x_e.A_{e\rightarrow t}x)_{e\rightarrow t}$.

  The subscripts reveal the types.

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- In general: $T_1 \rightarrow T_2$ is the type of expressions denoting functions from denotations of T_1 expressions to denotations of T_2 expressions.

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- For complex types we use recursion.
- This gives:

$$D_e = D, D_t = \{0,1\}, D_{A\to B} = {D_B}^{D_A}.$$

Here ${D_B}^{D_A}$ denotes the set of all functions from D_A to D_B.

Exercise 1 *Let a set $D_e = \{a,b,c\}$ be given. Draw a picture of an element of $D_{e\to t}$.*

Characteristic Functions

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- As another example, consider $D_{(e \to t) \to t}$. According to the type definition this is the domain of functions ${D_t}^{D_{e \to t}}$, i.e., the functions in $\{0, 1\}^{D_{e \to t}}$. These functions characterize sets of subsets of the domain of individuals D_e.

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$$D_{e \to e \to t} = {D_{e \to t}}^{D_e} = (D_t^{D_e})^{D^e} = (\{0,1\}^{\{a,b,c\}})^{\{a,b,c\}}.$$

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- A picture of a single element of $D_{e \to e \to t}$.

$$
\begin{aligned}
a &\mapsto \begin{pmatrix} a \mapsto 1 \\ b \mapsto 0 \\ c \mapsto 0 \end{pmatrix} \\
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- A function in $D_{e \to e \to t}$ maps every element d of D_e to (the characteristic function of) the set of those elements of D_e to which d has the R-relation, i.e., to the set $\{x \in D_e \mid (d, x) \in R\}$.

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- As another example, note that $D_{t \to t}$ has precisely four members, namely:

identity	negation	constant 1	constant 0
$1 \mapsto 1$ $0 \mapsto 0$	$1 \mapsto 0$ $0 \mapsto 1$	$1 \mapsto 1$ $0 \mapsto 1$	$0 \mapsto 0$ $0 \mapsto 0$

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$$\begin{aligned} 1 &\mapsto \begin{pmatrix} 1 \mapsto 1 \\ 0 \mapsto 1 \end{pmatrix} \\ 0 &\mapsto \begin{pmatrix} 1 \mapsto 1 \\ 0 \mapsto 0 \end{pmatrix} \end{aligned}$$

- Note that we can view this as a 'two step' version of the semantic operation of taking a disjunction.

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Exercise 3 *Specify the conjunction function in* $D_{t \to t \to t}$.

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- If Px is an expression of type t and x is of type e, then $(\lambda x.Px)$ is an expression of type $e \to t$. In fact, $(\lambda x.Px)$ is nothing but a more explicit notation for the so-called set abstraction notation $\{x \mid Px\}$.

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- If P is an expression of type $e \to t$ and x is of type e, then (Px) denotes the application of P to x; it is an expression of type t.

Abstraction and Application

- Now that we know in principle what the type domains D_T look like, for every type T in the type hierarchy, it should be clear that the process of abstraction (creating new functions) brings one higher up in the hierarchy, while the operation of application (applying a function to an argument) brings one down in the hierarchy.

- If Px is an expression of type t and x is of type e, then $(\lambda x.Px)$ is an expression of type $e \rightarrow t$. In fact, $(\lambda x.Px)$ is nothing but a more explicit notation for the so-called set abstraction notation $\{x \mid Px\}$.

- If P is an expression of type $e \rightarrow t$ and x is of type e, then (Px) denotes the application of P to x; it is an expression of type t.

- Compositional functional viewpoint: write everything as function application.

Example

The natural language sentence Jan admires the Dalai Lama will get represented as $((Aj)d)$, where j is an argument of the functional expression A, and d in turn is an argument of the functional expression (Aj).

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- The property of admiring oneself is expressed by $(\lambda x.((Ax)x))$.
- The property of being a property of Jan is given by $\lambda X.(Xj)$.
- The property of being a relation between Jan and the Dalai Lama is given by $\lambda Y.((Yj)d)$.

- If X in $\lambda X.(Xj)$ is of type $e \longrightarrow t$, then the expression $\lambda X.(Xj)$ has type $(e \longrightarrow t) \longrightarrow t$.

- If X in $\lambda X.(Xj)$ is of type $e \rightarrow t$, then the expression $\lambda X.(Xj)$ has type $(e \rightarrow t) \rightarrow t$.
- Y in $\lambda Y.((Yj)d)$ should have type $e \rightarrow e \rightarrow t$, so expression $\lambda Y.((Yj)d)$ itself has type $(e \rightarrow e \rightarrow t) \rightarrow t$.

Type Logic as Solution to the Misleading Form Problem

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- While fully reduced logical translations of natural language sentences may be misleading in some sense, the fully unreduced original expressions are not.
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- The translation of nobody is of the same type:
  $(\lambda P.\neg\exists x((\text{person } x) \wedge (Px)))$.

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- The translation of nobody is of the same type:
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- Before reduction, the translations of John walked and nobody walked look very similar. These similarities disappear only after both translations have been reduced to their simplest forms.

Meaning in Natural Language

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- X is always used for the meaning of the whole.
- X_n refers to the meaning representation of the n-th daughther.

S	$\longrightarrow$	**NP VP**	X	$\longrightarrow$	$(X_1 X_2)$
NP	$\longrightarrow$	*Mary*	X	$\longrightarrow$	$(\lambda P.(Pm))$
NP	$\longrightarrow$	*Bill*	X	$\longrightarrow$	$(\lambda P.(Pb))$
NP	$\longrightarrow$	**DET CN**	X	$\longrightarrow$	$(X_1 X_2)$
NP	$\longrightarrow$	**DET RCN**	X	$\longrightarrow$	$(X_1 X_2)$
DET	$\longrightarrow$	*every*	X	$\longrightarrow$	$(\lambda P.(\lambda Q.\forall x((Px) \Rightarrow (Qx)))))$
DET	$\longrightarrow$	*some*	X	$\longrightarrow$	$(\lambda P.(\lambda Q.\exists x((Px) \land (Qx)))))$
DET	$\longrightarrow$	*no*	X	$\longrightarrow$	$(\lambda P.(\lambda Q.\forall x((Px) \Rightarrow \neg(Qx)))))$
DET	$\longrightarrow$	*the*	X	$\longrightarrow$	$(\lambda P.(\lambda Q.\exists x(\forall y((Py) \leftrightarrow x = y)$
CN	$\longrightarrow$	*man*	X	$\longrightarrow$	$(\lambda x.(Mx))$
CN	$\longrightarrow$	*woman*	X	$\longrightarrow$	$(\lambda x.(Wx))$
CN	$\longrightarrow$	*boy*	X	$\longrightarrow$	$(\lambda x.(Bx))$
RCN	$\longrightarrow$	**CN** *that* **VP**	X	$\longrightarrow$	$(\lambda x.((X_1\ x) \land (X_3\ x)))$
RCN	$\longrightarrow$	**CN** *that* **NP TV**	X	$\longrightarrow$	$(\lambda x.((X_1\ x) \land (X_3(\lambda y.((X_4\ x)y)$
VP	$\longrightarrow$	*laughed*	X	$\longrightarrow$	$(\lambda x.(Lx))$
VP	$\longrightarrow$	*smiled*	X	$\longrightarrow$	$(\lambda x.(Sx))$
VP	$\longrightarrow$	**TV NP**	X	$\longrightarrow$	$(\lambda x.(X_2\ (\lambda y.((X_1\ x)y))))$
TV	$\longrightarrow$	*loved*	X	$\longrightarrow$	L

Consider the sentence Bill loved Mary:

$$[_S[_{NP}\textit{Bill}\ ][_{VP}[_{TV}\textit{loved}\ ][_{NP}\textit{Mary}\ ]]]$$

According to the rules above, this gets assigned the following meaning:

$$((\lambda P.(Pb))(\lambda x.(\lambda P.(Pm))(\lambda y.Lxy)))$$

Reducing this gives:

$$\xrightarrow{\beta} ((\lambda P.(Pb))(\lambda x.((\lambda y.Lxy)m)))$$

$$\xrightarrow{\beta} ((\lambda P.(Pb))(\lambda x.(Lxm)))$$

$$\xrightarrow{\beta} (\lambda x.Lxm)b$$

$$\xrightarrow{\beta} Lbm$$

Exercise 4 *Give the compositional translation for 'Bill loved some woman', and reduce it to normal form.*

Datastructures for Syntax

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Datastructures for Syntax

- It is straightforward to give an implementation of compositional semantics of natural language if we use a programming language that is itself based on type theory.
- First we define the data structures for the predicates, the variables, and the formulas, with data declarations for the various syntactic categories.
- The text in the square boxes below is the actual program code.

```
module CM where


import MOTT
import EPLIH
import Domain
import Model
import List
```

```haskell
data Sent = Sent NP VP
     deriving (Eq,Show)

data NP = Ann | Mary | Bill | Johnny | NP1 DET CN
        | NP2 DET RCN
     deriving (Eq,Show)

data DET = Every | Some | No | The | Most
         | Atleast Int
     deriving (Eq,Show)

data CN = Man | Woman | Boy | Person | Thing | House
     deriving (Eq,Show)
```

```
data RCN = CN1 CN VP | CN2 CN NP TV
     deriving (Eq,Show)

data VP = Laughed | Smiled | VP1 TV NP
     deriving (Eq,Show)

data TV = Loved | Respected | Hated | Owned
     deriving (Eq,Show)
```

The suffix `deriving (Eq,Show)` in the data type declarations is the Haskell way to ensure that equality is defined for these data types, and that they can be displayed on the screen.

Semantic Interpretation: Sentences

Next, we define for every syntactic category an interpretation function of the appropriate type, using `Entity` for e and `Bool` for t. The interpretation of sentences has type `Bool`, so the interpretation function `intS` gets type `Sent -> Bool`. Since there is only one rewrite rule for S, the interpretation function `intS` consists of only one equation:

```
intSent :: Sent -> Bool
intSent (Sent np vp) = (intNP np) (intVP vp)
```

Semantic Interpretation: NPs

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- The interpretation function `intNP` consists of four equations, one for every rewrite rule for NP in the grammar fragment.
- The function has type `NP -> (Entity -> Bool) -> Bool`.
- Here `Entity -> Bool` is the Haskell counterpart to $e \longrightarrow t$, which is the type of the VP interpretation that the NP combines with to form a sentence.

```haskell
intNP :: NP -> (Entity -> Bool) -> Bool
intNP Ann = \ p -> p ann
intNP Mary = \ p -> p mary
intNP Bill = \ p -> p bill
intNP Johnny = \ p -> p johnny
intNP (NP1 det cn) = (intDET det) (intCN cn)
intNP (NP2 det rcn) = (intDET det) (intRCN rcn)
```

Note the close connection between `\ p -> p mary` and $\lambda P.(Pm)$ that we get by employing the Haskell counterpart to λ.

Semantic Interpretation: VPs

For the interpretation of verb phrases we invoke the information encoded in our first order model.

```
intVP :: VP -> Entity -> Bool
intVP Laughed = laugh
intVP Smiled = smile
```

The interpretation of complex VPs is a bit more involved. We have to find a way to make reference to the property of 'standing into the TV relation to the subject of the sentence'. We do this in the same way as in the type logic specification of the semantic clause for $[\mathrm{TV\ NP}]_{\mathrm{VP}}$.

```
intVP (VP1 tv np) =
   \ subj -> intNP np (\ obj -> intTV tv (subj,obj))
```

Note that `subj` refers to the sentence subject and `obj` to the sentence direct object.

Semantic Interpretation: TVs

The interpretation of transitive verbs discloses another bit of information about the world. Again, we invoke the information about the world encoded in our first order model.

```
intTV :: TV -> (Entity,Entity) -> Bool
intTV Loved     = love
intTV Respected = respect
intTV Hated     = hate
intTV Owned     = own
```

Semantic Interpretation: Common Nouns

The interpreation of CNs is similar to that of VPs.

```
intCN :: CN -> Entity -> Bool
intCN Man = man
intCN Boy = boy
intCN Woman = woman
intCN Person = person
intCN Thing = thing
intCN House = house
```

Semantic Interpretation: Determiners

The most involved part of the implementation: the definition of the determiner interpretations. First the type. The interpretation of a DET needs two properties (type `Entity -> Bool`): one for the CN and one for the VP, to yield the type of an S interpretation, i.e., `Bool`.

```
intDET :: DET -> (Entity -> Bool)
              -> (Entity -> Bool) -> Bool
```

The interpretation of Some just checks whether the two properties corresponding to CN and VP have anything in common. This is what this check looks like in Haskell:

```
intDET Some p q = any q (filter p entities)
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- `entities` gives the domain of entities in the form of a list.
- `any` is a function taking a property and a list that returns `True` if the sublist of elements satisfying the property is non-empty, `False` otherwise.
- Thus, `any q list` checks whether any element of the list satisfies property `q`.

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- `all q list` checks whether every member of `list` satisfies property `q`.

The interpretation of The consists of two parts:

1. a check that the CN property is unique, i.e., that it is true of precisely one entity in the domain,
2. a check that the CN and the VP property have an element in common, in other words, the Some check on the two properties.

```
intDET The p q = singleton plist && q (head plist)
  where
    plist = filter p entities
    singleton [x] = True
    singleton _   = False
```

The interpretation of No is just the negation of the interpretation of Some:

```
intDET No p q = not (intDET Some p q)
```

The interpretation of Most compares the length of the list of entities satisfying both arguments (the restrictor argument and the body argument) with the length of the list of entities satisfying only the restrictor argument.

```
intDET Most p q =
  length pqlist > length (plist \\ qlist)
  where
   plist  = filter p entities
   qlist  = filter q entities
   pqlist = filter q plist
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Exercise 5 *Implement the interpretation function for* `(Atleast n)`*.*

Semantic Interpretation: Relativised Common Nouns

The interpretation of relativised common nouns of the form That CN VP checks whether an entity has both the CN and the VP property:

```
intRCN :: RCN -> Entity -> Bool
intRCN (CN1 cn vp) =
  \ e -> ((intCN cn e) && (intVP vp e))
```

The interpretation of relativised common nouns of the form That CN NP TV checks whether an entity has both the CN property as the property of being the object of NP TV.

```
intRCN (CN2 cn np tv) =
   \ e -> ((intCN cn e) &&
            (intNP np
               (\ subj -> (intTV tv (subj,e)))))
```

Examples

```
example1 = intSent (Sent (NP1 The Boy) Smiled)
example2 = intSent (Sent (NP1 The Boy) Laughed)
example3 = intSent (Sent (NP1 Some Man) Laughed)
example4 = intSent (Sent (NP1 No Man) Laughed)
example5 = intSent
 (Sent (NP1 Some Man)(VP1 Loved (NP1 Some Woman)))
example6 = intSent
 (Sent (NP2 No (CN1 Man (VP1 Loved Mary))) Laughed)
```

```
CM> example1
True
CM> example2
False
CM> example3
False
CM> example4
True
CM> example5
True
CM> example6
True
```

It is a bit awkward that we have to provide the datastructures of syntax ourselves. The process of constructing syntax datastructures from strings of words is called parsing; this topic will be taken up in various other tutorials.

Translation into Logical Form

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- In Montague [1973] and Montague [1974b] a typed language of logical formulas is used as a stepping stone on the way to semantic specification, but in Montague [1974a] the meaning of of a fragment of English is specified without a detour through logical form.
- The set-up above is along the lines of Montague [1974a].
- We will now demonstrate how translation into logical form can be implemented. We model our logical form language after the language of predicate logic. In particular, we use data types for `Var` and `Term`.

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- In Montague [1973] and Montague [1974b] a typed language of logical formulas is used as a stepping stone on the way to semantic specification, but in Montague [1974a] the meaning of of a fragment of English is specified without a detour through logical form.
- The set-up above is along the lines of Montague [1974a].
- We will now demonstrate how translation into logical form can be implemented. We model our logical form language after the language of predicate logic. In particular, we use data types for `Var` and `Term`.
- Advantage of generating LFs: can also be used for consistency clecking.

Here is a data type for generalized quantifiers.

```
data GQ = ALL | SOME | THE | NO | MOST | ATLEAST Int
          deriving (Show,Eq,Ord)
```

In fact, the only difference between logical forms and formulas of predicate logic is the presence of generalized quantifiers.

```
data LF = Atom1 Name [Term]
        | Eq1 Term Term
        | Neg1 LF
        | Impl1 LF LF
        | Equi1 LF LF
        | Conj1 [LF]
        | Disj1 [LF]
        | Quant GQ Var LF LF
     deriving (Eq,Ord)
```

NOTE: `Term` is defined elsewhere as

```
data Term = Vari Var
          | Struct String [Term] deriving (Eq,Ord)
```

A `show` function for logical forms:

```
instance Show LF where
  show (Atom1 id []) = id
  show (Atom1 id ts) = id ++ concat [ show ts ]
  show (Eq1 t1 t2)   = show t1 ++ "==" ++ show t2
  show (Neg1 form)   = '~': (show form)
  show (Impl1 f1 f2) =
     "(" ++ show f1 ++ "==>" ++ show f2 ++ ")"
  show (Equi1 f1 f2) =
     "(" ++ show f1 ++ "<=>" ++ show f2 ++ ")"
  show (Conj1 []) =  "true"
  show (Conj1 fs) =  "conj" ++ concat [ show fs ]
  show (Disj1 []) =  "false"
  show (Disj1 fs) =  "disj" ++ concat [ show fs ]
  show (Quant gq var f1 f2) =
      show gq ++ " " ++ show var ++
      " (" ++ show f1 ++ "," ++ show f2 ++ ")"
```

Translation to LF: Sentences

The process of translating to logical form is a very easy variation on the interpretation process for syntactic structures. Instead of the type `Bool`, for interpretation in a model, we take `LF`, for a logical form of type t, and instead of the type `Entity` for entities in the model, take `Var`, for variables that are supposed to get mapped to entities.

In the basic cases, we translate proper names into constant terms and lexical CNs, VPs, TVs into atomic formulas.

```
lfSent :: Sent -> LF
lfSent (Sent np vp) = (lfNP np) (lfVP vp)
```

Translation to LF: Noun Phrases

```
lfNP :: NP -> (Term -> LF) -> LF
lfNP Ann  = \ p -> p (Struct "Ann" [])
lfNP Mary = \ p -> p (Struct "Mary" [])
lfNP Bill = \ p -> p (Struct "Bill" [])
lfNP Johnny = \ p -> p (Struct "Johnny" [])
lfNP (NP1 det cn) = (lfDET det) (lfCN cn)
lfNP (NP2 det rcn) = (lfDET det) (lfRCN rcn)
```

Translation to LF: VPs

```
lfVP :: VP -> Term -> LF
lfVP Laughed = \ t -> Atom1 "laugh" [t]
lfVP Smiled  = \ t -> Atom1 "smile" [t]
lfVP (VP1 tv np) =
  \ subj -> lfNP np (\ obj -> lfTV tv (subj,obj))
```

Translation to LF: TVs

```
lfTV :: TV -> (Term,Term) -> LF
lfTV Loved     =
  \ (t1,t2) -> Atom1 "love"    [t1,t2]
lfTV Respected =
  \ (t1,t2) -> Atom1 "respect" [t1,t2]
lfTV Hated     =
  \ (t1,t2) -> Atom1 "hate"    [t1,t2]
lfTV Owned     =
  \ (t1,t2) -> Atom1 "own"     [t1,t2]
```

Translation to LF: CNs

```
lfCN :: CN -> Term -> LF
lfCN Man    = \ t -> Atom1 "man"    [t]
lfCN Boy    = \ t -> Atom1 "boy"    [t]
lfCN Woman  = \ t -> Atom1 "woman"  [t]
lfCN Person = \ t -> Atom1 "person" [t]
lfCN Thing  = \ t -> Atom1 "thing"  [t]
lfCN House  = \ t -> Atom1 "house"  [t]
```

Translation to LF: Determiners

The type of the logical form translation of determiners indicates that the translation takes a determiner phrase and two arguments for objects of type `Term -> LF` (logical forms with term holes in them), and produces a logical form.

```
lfDET :: DET -> (Term -> LF) -> (Term -> LF) -> LF
```

The translation of determiners should be done with some care. It involves the construction of a logical form where a variable gets bound. To ensure proper binding, we have to make sure that the newly introduced variable will not get accidentally bound by a quantifier already present in the logical form. For this, we first list the (free) variables in logical forms.

```
varsInLf :: LF -> [Var]
varsInLf (Atom1 _ ts)  = varsInTerms ts
varsInLf (Eq1 t1 t2)   = varsInTerms [t1,t2]
varsInLf (Neg1 form)   = varsInLf form
varsInLf (Impl1 f1 f2) = varsInLfs [f1,f2]
varsInLf (Equi1 f1 f2) = varsInLfs [f1,f2]
varsInLf (Conj1 fs)    = varsInLfs fs
varsInLf (Disj1 fs)    = varsInLfs fs
varsInLf (Quant gq var f1 f2) =
   varsInLfs [f1,f2] \\ [var]

varsInLfs :: [LF] -> [Var]
varsInLfs = nub . concat . map varsInLf
```

We need the list of variable indices of a list of logical forms in order to compute a fresh variable index. All variables will get introduced by means of this mechanism, so if we start out with variables of the form `Var "x" [0]`, and only introduce new variables of the form `Var "x" [i]`, we can assume that all variables occurring in our logical form will have the shape `Var "x" [i]` for some integer `i`.

```
freshvar :: [LF] -> Var
freshvar lfs = (Var "x" [i+1])
  where
  i = foldr max 0 (xindices (varsInLfs lfs))
  xindices = map (\ (Var "x" [j]) -> j)
```

The definition uses `foldr` for defining the maximum of a list of integers.

The term `zero` is defined elsewhere as a constant. Use this term as a dummy to provisionally fill in the term slot in formulas with term holes in them.

```
lfDET Some p q =
  Quant SOME v (p (Vari v)) (q (Vari v))
   where v = freshvar [p zero,q zero]
lfDET Every p q =
  Quant ALL v (p (Vari v)) (q (Vari v))
   where v = freshvar [p zero,q zero]
lfDET No p q =
  Quant NO v (p (Vari v)) (q (Vari v))
   where v = freshvar [p zero,q zero]
lfDET The p q =
  Quant THE v (p (Vari v)) (q (Vari v))
   where v = freshvar [p zero,q zero]
lfDET Most p q =
  Quant MOST v (p (Vari v)) (q (Vari v))
   where v = freshvar [p zero,q zero]
```

Exercise 6 *Implement the translation function for* `(Atleast n)`.

Translation to LF: Relativized CNs

Use `Conj1` to conjoin the logical form for a common noun and the logical form for a relative clause into a logical form for a complex common noun.

```
lfRCN :: RCN -> Term -> LF
lfRCN (CN1 cn vp) =
   \ t -> Conj1 [lfCN cn t, lfVP vp t]
lfRCN (CN2 cn np tv) =
   \ t -> Conj1 [lfCN cn t,
                 lfNP np (\ subj -> lfTV tv (subj,t))]
```

Examples

```
lf1 = lfSent
 (Sent (NP1 Some Man)
       (VP1 Loved (NP1 Some Woman)))
lf2 = lfSent
  (Sent (NP2 No (CN1 Man (VP1 Loved Mary))) Laughed)
```

```
CM> lf1
SOME x1 (man[x1],SOME x2 (woman[x2],love[x1,x2]))
CM> lf2
NO x1 (conj[man[x1],love[x1,Mary]],laugh[x1])
```

Exercise 7 *Implement an evaluation function for logical forms in appropriate models.*

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- Up-to-date overview of dynamic semantics for natural language, with links to computer science: Eijck and Stokhof [2006].

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