Propositional Dynamic Logic as a Logic of Knowledge Update and Belief Revision

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(based on joint work with Yanjing Wang and Floor Sietsma)

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Abstract

This talk shows how propositional dynamic logic (PDL) can be interpreted as a logic for multi-agent knowledge update and belief revision, or as a logic of preference change, if the basic relations are read as preferences instead of plausibilities.

Our point of departure is the logic of communication and change (LCC) of [9]. Like LCC, our logic uses PDL as a base epistemic language. Unlike LCC, we start out from agent plausibilities, add their converses, and build knowledge and belief operators from these with the PDL constructs. We extend the update mechanism of LCC to an update mechanism that handles belief change as relation substitution, and we show that the update part of this logic is more expressive than either that of LCC or that of epistemic/doxastic PDL with a belief change modality. Next, we show that the properties of knowledge and belief are preserved under any update, unlike in LCC. We prove completeness of the logic and give examples of its use.

If there is time, we will also look at the preference interpretation of the system, and at preference change scenarios that can be modelled with it.

Motivation

Proposals for treating belief revision in the style of dynamic epistemic logic (see Gerbrandy [16], van Ditmarsch [13], van Benthem [6, 10], and Baltag, Moss and coworkers [3, 1, 2], or the textbook treatment in [14]) were made in Van Benthem and Liu [8] and Van Benthem [7], where it is suggested that belief revision should be treated as relation substitution. This is different from the standard action product update from Baltag, Moss and Solecki [3], and it suggests that the proper relation between these two update styles should be investigated.

Main contribution

We propose a new version of action product update that integrates belief revision by means of relation substitution with belief update by means of the action product construction. We show that this allows to express updates that cannot be expressed with action product only or with relation substitution only.

We graft this new update mechanism on a base logic that can express knowledge, strong belief, conditional belief, and plain belief, and we show that the proper relations between these concepts are preserved under any update. We prove that our system is complete.

Related work

Our main source of inspiration is the logic of communication and change (LCC) from Van Benthem, Van Eijck and Kooi [9]. This system has the flaw that updates with non-S5 action models may destroy knowledge or belief; in our redesign this problem is avoided. Our completeness proof is an adaptation from the completeness proof for LCC. The treatment of conditional belief derives from Boutillier [12]. Our work can be seen as a proposal for integrating belief revision by means of relation substitution, as proposed in Van Benthem [7] with belief and knowledge update in the style of Baltag, Moss and Solecki [3].

PDL as a Belief Revision Logic

A preference model $\mathbf{M}$ for set of agents *Ag* and set of basic propositions *Prop* is a tuple (W, P, V) where W is a non-empty set of worlds, P is a function that maps each agent a to a relation P_a (the preference relation for a), and V is a map from W to $\mathcal{P}(\textit{Prop})$ (a map that assigns to each world a *Prop*-valuation). There are no conditions at all on the P_a. Appropriate conditions will be imposed by constructing the operators for belief and knowledge by means of PDL operations.

We fix a PDL style language for talking about preference (or: plausibility). Assume p ranges over a set of basic propositions *Prop* and a over a set of agents *Ag*.

$$\begin{aligned} \phi &::= \top \mid p \mid \neg\phi \mid \phi_1 \wedge \phi_2 \mid [\pi]\phi \\ \pi &::= a \mid a^{\smile} \mid ?\phi \mid \pi_1;\pi_2 \mid \pi_1 \cup \pi_2 \mid \pi^* \end{aligned}$$

This is to be interpreted in the usual PDL manner, with $[\![\pi]\!]^{\mathbf{M}}$ giving the relation that interprets relational expression π in $\mathbf{M} = (W, V, P)$. $[\pi]\phi$ is true in world w of $\mathbf{M}$ if for all v with $(w, v) \in [\![\pi]\!]^{\mathbf{M}}$ it holds that ϕ is true in v. We adopt the usual abbreviations.

The following additional abbreviations allow us to express knowledge, strong belief, conditional belief and plain belief:

knowledge $\sim_a$ abbreviates $(a \cup a^{\smile})^*$.

strong belief $\geq_a$ abbreviates a^*.

conditional belief $[\to_a^{\phi}]\psi$ abbreviates

$$\langle\sim_a\rangle\phi \to \langle\sim_a\rangle(\phi \wedge [\geq_a](\phi \to \psi)).$$

plain belief $[\to_a]\phi$ abbreviates $[\to_a^{\top}]\phi$.
(note: it follows that $[\to_a]\phi$ is equivalent to $\langle\sim_a\rangle[\geq_a]\phi$).

The definition of $\rightarrow_a^\phi$ (conditional belief for a, with condition ϕ) is from Boutillier [12] This definition, also used in Baltag and Smets [5], states that conditional to ϕ, a believes in ψ if either there are no accessible ϕ worlds, or there is an accessible ϕ world in which the belief in $\phi \rightarrow \psi$ is safe. The definition of $\rightarrow_a^\phi$ matches the well-known accessibility relations $\rightarrow_a^P$ for each subset P of the domain, given by:

$$\rightarrow_a^P := \{(x,y) \mid x{\sim_a}y \wedge y \in \mathsf{MIN}_{\leq_a} P\},$$

where $\mathsf{MIN}_{\leq_a} P$, the set of minimal elements of P under $\leq_a$, is defined as

$$\{s \in P : \forall s' \in P(s' \leq_a s \Rightarrow s \leq_a s')\}.$$

This logic is axiomatised by the standard PDL rules and axioms ([22, 21]) plus axioms that define the meanings of the relation names $a^{\smile}$. The PDL rules and axioms are:

Modus ponens	and axioms for propositional logic
Modal generalisation	From $\vdash \phi$ infer $\vdash [\pi]\phi$

Normality	$\vdash [\pi](\phi \to \psi) \to ([\pi]\phi \to [\pi]\psi)$
Test	$\vdash [?\phi]\psi \leftrightarrow (\phi \to \psi)$
Sequence	$\vdash [\pi_1; \pi_2]\phi \leftrightarrow [\pi_1][\pi_2]\phi$
Choice	$\vdash [\pi_1 \cup \pi_2]\phi \leftrightarrow ([\pi_1]\phi \wedge [\pi_2]\phi)$
Mix	$\vdash [\pi^*]\phi \leftrightarrow (\phi \wedge [\pi][\pi^*]\phi)$
Induction	$\vdash (\phi \wedge [\pi^*](\phi \to [\pi]\phi)) \to [\pi^*]\phi$

The relation between the basic programs a and $a^{\smile}$ is expressed by the standard modal axioms for converse:

$$\vdash \phi \to [a]\langle a^{\smile}\rangle\phi \qquad \vdash \phi \to [a^{\smile}]\langle a\rangle\phi$$

Completeness

This yields a very expressive complete and decidable PDL logic for belief revision, to which we can add mechanisms for belief update and for belief change.

Theorem 1 *The above system of belief revision PDL is complete for preference models.*

Knowledge is S5 (equivalence), strong belief is S4 (reflexive and transitive), plain belief is KD45 (serial, transitive and euclidean).

To see that plain belief is euclidean, note that

$$\langle \sim_a \rangle [\geq_a] \phi \rightarrow [\sim_a] \langle \geq_a \rangle \langle \sim_a \rangle [\geq_a] \phi$$

holds.

Action Model Update

Definition of update models $\mathbf{A}$ and of the update product operation $\otimes$ from Baltag, Moss, Solecki [3]. An action model is like an preference model, but with the valuation replaced by a precondition map $\mathbf{pre}$. Updating a static model $\mathbf{M} = (W, P, V)$ with an action model $\mathbf{A} = (E, \mathbf{P}, \mathbf{pre})$ succeeds if the set

$$\{(w, e) \mid w \in W, e \in E, \mathbf{M}, w \models \mathbf{pre}(e)\}$$

is non-empty. The update result is a new static model $\mathbf{M} \otimes \mathbf{A} = (W', P', V')$ with

- $W' = \{(w, e) \mid w \in W, e \in E, \mathbf{M}, w \models \mathbf{pre}(e)\}$,
- P'_a is given by $\{(w, e), (v, f)) \mid (w, v) \in P_a, (e, f) \in \mathbf{P}_a\}$,
- $V'(w, e) = V(w)$.

If the static model has a set of distinguished states W_0 and the action model a set of distinguished events E_0, then the distinguished worlds of $\mathbf{M} \otimes A$ are the (w, e) with $w \in W_0$ and $e \in E_0$.

Figure 1: Static model and update model

Figure 1 gives an example pair of a static model with an update action. The static model, on the left, pictures the result of a hidden coin toss, with three onlookers, Alice, Bob and Carol.

The update model represents a secret test whether the result of the toss is h. The result of the update is that the distinction mark on the $\overline{h}$ world has disappeared, without any of a, b, c being aware of the change.

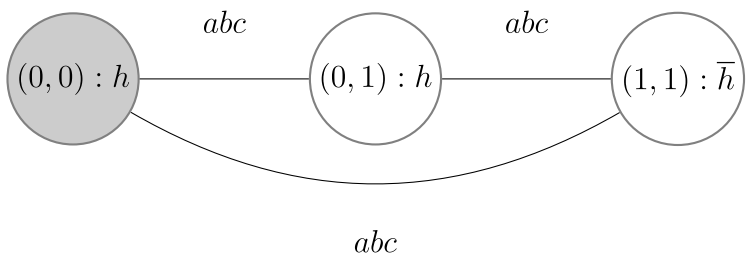


Figure 2: Result of the update in Figure 1.

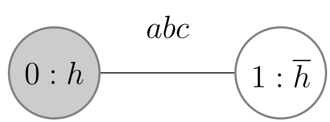


Figure 3: Bisimulation-minimal version of result of the update in Figure 1.

Adding Factual Change

Factual change was already added to update models in LCC. We will now also add belief change. We let an action model be a quintuple

$$A = (E, \mathbf{P}, \mathbf{pre}, \textbf{Sub}, \textbf{SUB})$$

where $E, \mathbf{P}, \mathbf{pre}$ are as before, **Sub** is a function that assigns a propositional binding to each $e \in E$, and **SUB** is a function that assigns a relational binding to each $e \in E$. A propositional binding is a map from proposition letters to formulas, represented by

$$\{p_1 \mapsto \phi_1, \ldots, p_n \mapsto \phi_n\}$$

where the p_k are all different, and where no ϕ_k is equal to p_k. It is assumed that each p that does not occur in a lefthand side of a binding is mapped to itself.

Adding Belief Change

A relational binding is a map from agents to program expressions, represented by

$$\{a_1 \mapsto \pi_1, \ldots, a_n \mapsto \pi_n\}$$

where the a_j are agents, all different, and where the π_j are program expressions from the PDL language. It is assumed that each a that does not occur in the lefthand side of a binding is mapped to itself. The set $\{a \in \mathit{Ag} \mid \rho(a) \neq a\}$ is the domain of ρ. Use ϵ for the identity propositional or relational substitution.

Update Execution (new style)

The **update execution** of static model $\mathbf{M} = (W, R, V)$ with update model $A = (E, \mathbf{P}, \mathbf{pre}, \textsf{\textbf{Sub}}, \textsf{\textbf{SUB}})$ is a tuple: $\mathbf{M} \circledast \mathbf{A} = (W', P', V')$ where:

- $W' = \{(w, e) \mid M, w \models \mathbf{pre}(e)\}$.
- P'_a is given by

  $\{((w_1, e_1), (w_2, e_2)) \mid$
  there is a $\textsf{\textbf{SUB}}(e_1)(a)$ path from (w_1, e_1) to (w_2, e_2) in $\mathbf{M} \otimes \mathbf{A}\}$.

- $V'(p) = \{(w, e) \mid \mathbf{M}, w \models \textsf{\textbf{Sub}}(e)(p)\}$.

Note: the definition of P'_a refers to paths in the old style update product.

Example: Public belief change

Consider the suggestive upgrade $\sharp_a\phi$ discussed in Van Benthem and Liu [8]:

$$\sharp_a\phi =_{\mathsf{def}} \ ?\phi; a; ?\phi \ \cup ?\neg\phi; a; ?\neg\phi \ \cup ?\neg\phi; a; ?\phi.$$

The following update model for public belief change uses this:

$$G = (\{e\}, \mathbf{P}, \mathbf{pre}, \textbf{Sub}, \textbf{SUB})$$

where:

- For all $i \in Ag$, $\mathbf{P}_i = \{(e,e)\}$.
- $\mathbf{pre}(e) = \top$.
- $\textbf{Sub}(e) = \epsilon$.
- $\textbf{SUB}(e) = \{a \mapsto \ \sharp_a\phi, b \mapsto \ \sharp_b\phi\}$.

Example: Non-public belief change

$$G' = (\{e_0, e_1\}, \mathbf{P}, \mathbf{pre}, \textsf{\textbf{Sub}}, \textsf{\textbf{SUB}})$$

where:

- For all $i \in \mathit{Ag}$, if $i \neq b$ then $\mathbf{P}_i = \{(e_0, e_0), (e_1, e_1)\}$, $\mathbf{P}_b = \{(e_0, e_0), (e_1, e_1), (e_0, e_1), (e_1, e_0)\}$
- $\mathbf{pre}(e_0) = \mathbf{pre}(e_1) = \top$.
- $\textsf{\textbf{Sub}}(e_0) = \textsf{\textbf{Sub}}(e_1) = \epsilon$.
- $\textsf{\textbf{SUB}}(e_0) = \{a \mapsto \sharp_a \phi\}$, $\textsf{\textbf{SUB}}(e_1) = \epsilon$.

Assume e_0 is the actual event.

This changes the belief of a while b remains unaware of the change.

Expressivity of Update Mechanism

- Action product update can express updates that cannot be expressed with relational substitution alone.
- Relational substitution can express updates that cannot be expressed with action product update alone.
- Action product update/upgrade new style can express updates that cannot be expressed with either action product or relational substitution.

PDL$^+$: PDL with update/upgrade action product

Let PDL$^+$ be the result of adding modalities of the form $[\mathbf{A}, e]\phi$ to PDL, with the following interpretation clause:

$$\mathbf{M} \models_w [\mathbf{A}, e]\phi \quad \text{iff} \quad \mathbf{M} \models_w \mathbf{pre}(e) \text{ implies } \mathbf{M} \circledast \mathbf{A} \models_{(w,e)} \phi.$$

Completeness

Completeness can be proved by a patch of the LCC completeness proof in [9].

The definition of converse for PDL programs is captured by reduction axioms: add programs of the form $\pi\check{}$ to the language, with the obvious interpretation, and describe their meanings by means of:

$$
\begin{array}{lll}
\vdash & (?\phi)\check{} & \leftrightarrow \; ?\phi \\
\vdash & (\pi_1;\pi_2)\check{} & \leftrightarrow \; \pi_2\check{};\pi_1\check{} \\
\vdash & (\pi_1 \cup \pi_2)\check{} & \leftrightarrow \; \pi_1\check{} \cup \pi_2\check{} \\
\vdash & (\pi^*)\check{} & \leftrightarrow \; (\pi\check{})^*
\end{array}
$$

Redefinition of Program transformation

$$
\begin{array}{rcl}
\underline{T}^{\mathbf{A}}_{ij}(a) & = & \begin{cases} ?pre(e_i); \mathsf{SUB}(e_i)(a) & \text{if } e_i \mapsto_{\mathsf{SUB}(e_i)(a)} e_j \text{ in } \mathbf{A} \\ ?\bot & \text{otherwise} \end{cases} \\
\underline{T}^{\mathbf{A}}_{ij}(a^{\smallsmile}) & = & \begin{cases} ?pre(e_i); (\mathsf{SUB}(e_i)(a))^{\smallsmile} & \text{if } e_i \mapsto_{\mathsf{SUB}(e_i)(a)} e_j \text{ in } \mathbf{A} \\ ?\bot & \text{otherwise} \end{cases} \\
\underline{T}^{\mathbf{A}}_{ij}(?\phi) & = & \begin{cases} ?(pre(e_i) \wedge [\mathbf{A}, e_i]\phi) & \text{if } i = j \\ ?\bot & \text{otherwise} \end{cases} \\
\underline{T}^{\mathbf{A}}_{ij}(\pi_1; \pi_2) & = & \bigcup_{k=0}^{n-1} (\underline{T}^{\mathbf{A}}_{ik}(\pi_1); \underline{T}^{\mathbf{A}}_{kj}(\pi_2)) \\
\underline{T}^{\mathbf{A}}_{ij}(\pi_1 \cup \pi_2) & = & \underline{T}^{\mathbf{A}}_{ij}(\pi_1) \cup \underline{T}^{\mathbf{A}}_{ij}(\pi_2)) \\
\underline{T}^{\mathbf{A}}_{ij}(\pi^*) & = & K^{\mathbf{A}}_{ijn}(\pi)
\end{array}
$$

where it is assumed that the update model $\mathbf{A}$ has n states, and the states are numbered $0, \ldots, n-1$.

Proof System of PDL$^+$

The proof system for PDL$^+$ contains all axioms and rules of LCC except the reduction axiom:

$$[\mathbf{A}, e_i][\pi]\phi \leftrightarrow \bigwedge_{j=0}^{n-1} [T_{ij}^{\mathbf{A}}(\pi)][\mathbf{A}, e_j]\phi.$$

Instead, we have the axioms for converse atoms, the axioms for converse composite programs, and reduction axioms of the form:

$$[\mathbf{A}, e_i][\pi]\phi \leftrightarrow \bigwedge_{j=0}^{n-1} [\underline{T}_{ij}^{\mathbf{A}}(\pi)][\mathbf{A}, e_j]\phi.$$

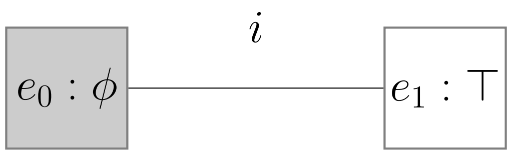
Theorem 2 (Completeness for PDL$^+$) $\models \phi$ *iff* $\vdash \phi$.

Digression: Restricted Announcements

A restricted announcement of ϕ is an announcement of ϕ that is not delivered to one of the agents i. Notation $!\phi^{-i}$. The action model for $!\phi^{-i}$ has event set $\{e_0, e_1\}$, with e_0 the actual event, where e_0 has precondition ϕ and e_1 precondition $\top$, and with the preference relation given by

$$P_i = \{(e_0, e_0), (e_1, e_1), (e_0, e_1), (e_1, e_0)\},$$

and $P_j = \{(e_0, e_0), (e_1, e_1)\}$ for all $j \neq i$.



Protocols for Restricted Announcements

A protocol for restricted announcements, for epistemic situation $\mathbf{M}$, is a set of finite sequences of formula-agent pairs, such that each sequence

$$(\phi_0, i_0), \ldots, (\phi_n, i_n)$$

has the following property:

$$\forall k \in \mathbb{N} : 0 \leq k < n \rightarrow \exists i \in \mathit{Ag} : \mathbf{M} \models_w [!\phi_0^{-i_0}], \ldots, [!\phi_{k-1}^{-i_{k-1}}][\sim_i]\phi_k.$$

Intuitively, at every stage in the sequence of restricted announcements, some agent has to possess the required knowledge to make the next announcement in the sequence.

Restricted Announcements Cannot Achieve Common Knowledge

Theorem 3 *Let C express common knowledge among set of agents Ag. Let $\mathbf{M}$ be an epistemic model with actual world w such that $\mathbf{M} \models_w \neg C\phi$. Then there is no protocol with*

$$\mathbf{M} \models_w [!\phi_0^{-i_0}], \ldots, [!\phi_n^{-i_n}]C\phi.$$

for any sequence $(\phi_0, i_0), \ldots, (\phi_n, i_n)$ in the protocol.

Proof: We show that $\neg C\phi$ is an invariant of any restricted announcement.

Assume $\mathbf{M} \models_w \neg C\phi$. Let $(\mathbf{A}, e)$ be an action model for announcement $!\psi^{-i}$, the announcement of ψ, restricted to *Ag* $- \{i\}$. Then $\mathbf{A}$ has events e and e', with $\mathbf{pre}(e) = \psi$ and $\mathbf{pre}(e') = \top$. If $\mathbf{M} \models_w \neg\psi$ then the update does not succeed, and there is nothing to prove.

Suppose therefore that $\mathbf{M} \models_w \psi$. Since $\mathbf{pre}(e') = \top$, the model $\mathbf{M} \otimes \mathbf{A}$ restricted to domain $D = \{(w, e') \mid w \in W_{\mathbf{M}}\}$ is a copy of the original model $\mathbf{M}$. Thus, it follows from $\mathbf{M} \models_w \neg C\phi$ that

$$\mathbf{M} \otimes A \upharpoonright D \models_{(w,e')} \neg C\phi.$$

Observe that since common knowledge is preserved under model restriction, absence of common knowledge is preserved under model extension. Therefore, it follows from $\mathbf{M} \otimes A \upharpoonright D \models_{(w,e')} \neg C\phi$ that $\mathbf{M} \otimes A \models_{(w,e')} \neg C\phi$. By the construction of $\mathbf{M} \otimes A$, we get from this that $\mathbf{M} \otimes A \models_{(w,e)} \langle i \rangle \neg C\phi$, and therefore $\mathbf{M} \otimes A \models_{(w,e)} \neg C\phi$, by the definition of common knowledge.

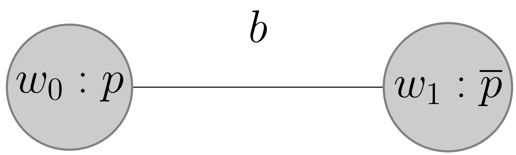
It follows immediately that no protocol built from restricted announcements can create common knowledge.

An Abstract Look at the Coordinated Attack Problem

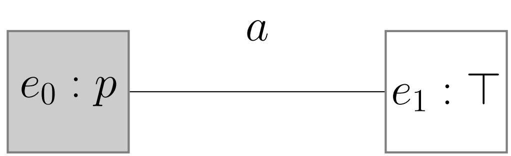
The case of the two generals who fail to achieve common knowledge about a joint attack on the enemy [18, 20] can be viewed as a special case of this theorem.

If there are just two agents, the only way for agent 1 to send a restricted message is by allowing uncertainty about the delivery. If i, j are the only agents, and i knows ϕ then the restricted message $!\phi^{-j}$ conveys no information, so the only reasonable restricted announcement of ϕ is $!\phi^{-i}$. The upshot of this announcement is that the message gets delivered to j, but i remains uncertain about this. According to the theorem, such messages cannot create common knowledge.

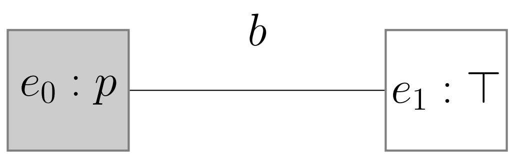
Initial situation:



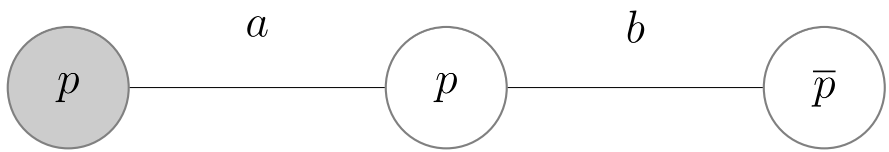
Update action for general a:



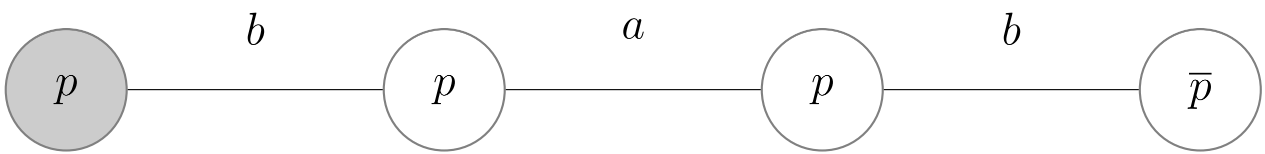
Update action for general b:



Situation after first message from general a:

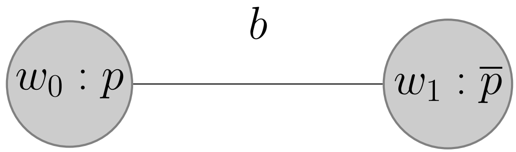


Situation after update by a followed by update by b:

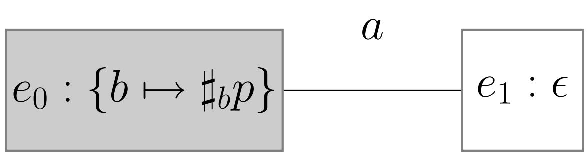


And so on . . .

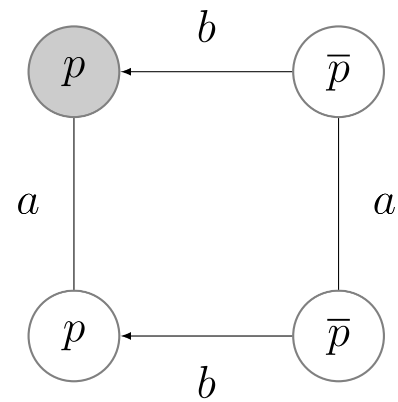
The Coordinated Attack Problem Again: The Power of Restricted Belief Change



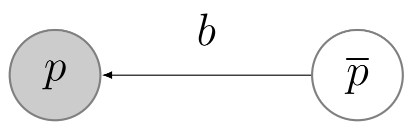
Action model for non-public belief change:



Result of updating with this:



This is bisimilar to:



We have achieved common strong belief in p in a single step, by means of a non-public belief change!

Imposing Conditions on the Basic Accessibilities

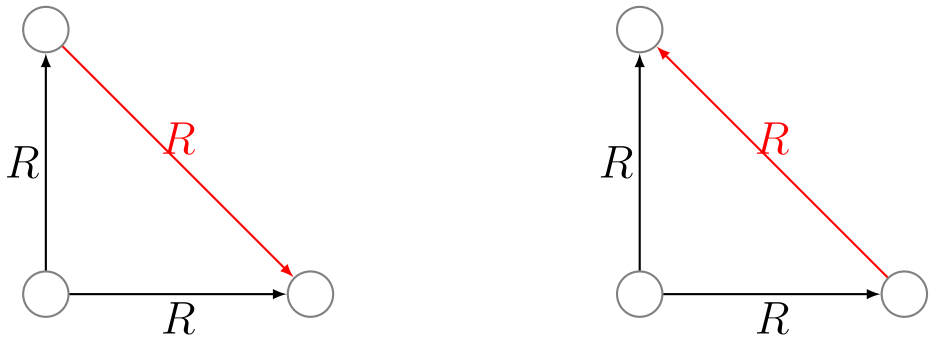
The proto-preference relations that serve as the basis for construction of preference pre-orders leave something to be desired.

Compare an optometrist who collects answers for a number of lenses she tries out on you: “Better or worse?”, (change of lens), “Better or worse?” (change of lens), “Better or worse?”.... If you reply “worse” after a change of x to y, and “worse” after a change from y to z, she will most probably not bother to collect your reaction to a change from x to z. But what if you answer “better” after the second swap? Then, if she is reasonable, she will try to find out how x compares to z. It does make sense to impose this as a requirement on preference relations.

Local Connectedness

A binary relation R is **weakly connected** (terminology of [17]) if the following holds:

$$\forall x, y, z((xRy \wedge xRz) \rightarrow (yRz \vee y = z \vee zRy)).$$



R is **locally connected** if both R and $R^{\smile}$ are weakly connected.

Updates on Locally Connected Models Must Preserve Local Connectedness

Starting from relations that are locally connected, we can upgrade the method from the previous sections to construct 'belief revision models' in the style of Grove [19], Board [11], and Baltag and Smets [4, 5] (who call them 'multi-agent plausibility frames').

Caution: in defining updates for locally connected models one has restrict the update mechanism to ensure that every update result is again locally connected.

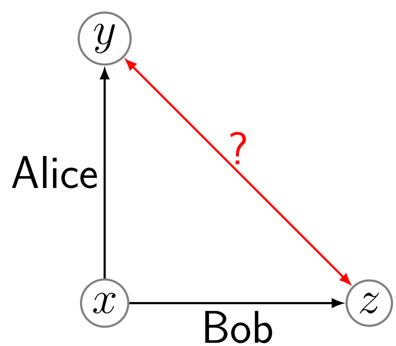
Fact: The following variation on suggestive upgrade (i.e., upgrade with substitution $a \mapsto \sharp_a \phi$) preserves local connectedness.

$$\begin{aligned} \natural_a \phi \; =_{\text{def}} \;\; & ?\phi; a^*; ?\phi \; \cup ?\neg\phi; a^*; ?\neg\phi \\ & \cup \;\; b^*; ?\neg\phi; a^* \cup a^{\smallsmile *}; ?\phi \; \cup \;\; b^{\smallsmile *}; ?\neg\phi; a^* \cup a^{\smallsmile *}; ?\phi \end{aligned}$$

Accessibility Linking

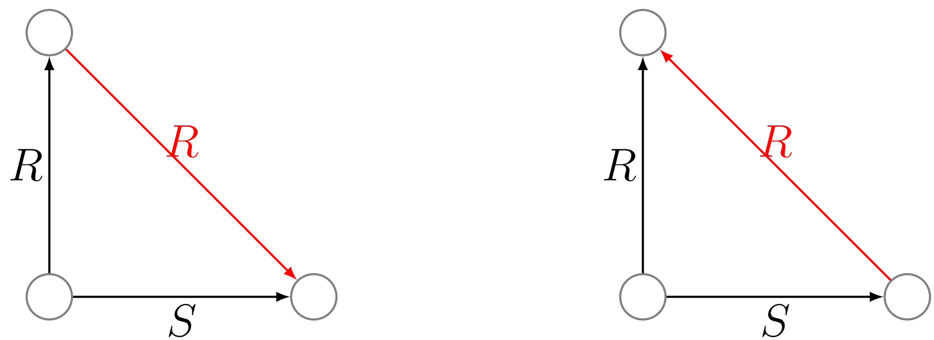
In the multi-agent case there is a further natural constraint.

Consider a situation where Alice and Bob have to decide on the chairperson of a program committee. Carol is mediator. Alice says she prefers y to x. Bob counters by saying that she prefers z to x. What should Carol do? Clearly, she should urge both of them to compare y and z.



Linked Sets of Relations

A set of binary relations **R** on a domain W is **forward linked** if for all R, S in **R**, if xRy and xSz, then either yRz or $z = y$ or zRy.



R is **backward linked** if the set $\{R^{\smile} \mid R \in \mathbf{R}\}$ is forward linked.

R is **linked** if **R** is both forward and backward linked.

It follows from the definition that the set $\{R\}$ is linked iff R is locally connected.

Effect on Common Knowledge, Preservation

If R and S are linked relations then common knowledge equals the union of common strong belief and common strong disbelief:

$$(R \cup R^{\smallsmile} \cup S \cup S^{\smallsmile})^* = (R \cup S)^* \cup (R^{\smallsmile} \cup S^{\smallsmile})^*.$$

Fact: Simultanous suggestive belief upgrade (i.e., upgrade with a list of links $a \mapsto \sharp_a \phi$) preserves linking of relations.

Question: What are natural classes of update actions that preserve linking of relations?

Dutch Meetings ('Vergaderingen')

A Dutch meeting is a simultaneous preference/belief change event where the following happens. Assume an epistemic situation $\mathbf{M}$ with actual world w, and assume proposition ϕ is on the agenda.

- If (a majority prefers ϕ to $\neg\phi$)

  $$\{i \in \mathit{Ag} \mid \mathbf{M} \models_w [\rightarrow_i]\phi\} > \{i \in \mathit{Ag} \mid \mathbf{M} \models_w [\rightarrow_i]\neg\phi\}$$

  then simultaneous belief change $\{i \mapsto \sharp_i\phi \mid i \in \mathit{Ag}\}$ takes place.

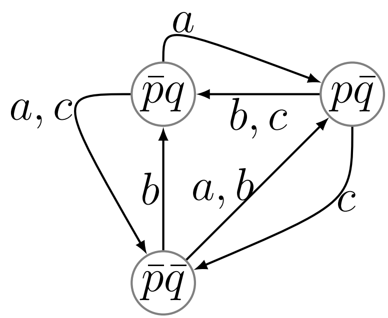
- If (a majority prefers $\neg\phi$ to ϕ)

  $$\{i \in \mathit{Ag} \mid \mathbf{M} \models_w [\rightarrow_i]\phi\} < \{i \in \mathit{Ag} \mid \mathbf{M} \models_w [\rightarrow_i]\neg\phi\}$$

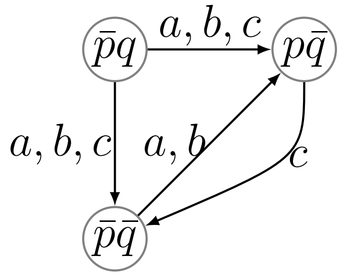
  then simultaneous belief change $\{i \mapsto \sharp_i\neg\phi \mid i \in \mathit{Ag}\}$ takes place.

- If there is no majority either way, nothing happens.

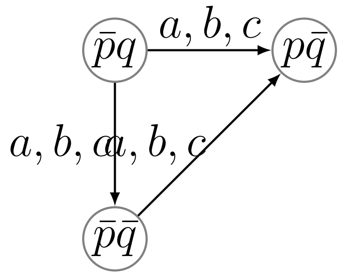
We give an example to illustrate that the outcome of a sequence of Dutch meetings may depend on the way the agenda was set. Consider the following situation.



Suppose a plenary q meeting is scheduled. Then since a majority believes in $\neg q$, the result will be that $\neg q$ worlds get promoted past q worlds:



A subsequent plenary p meeting will create a general belief in p:



Contrast this with the initial situation, where $\neg p$ was a majority belief.

Work, Questions, Challenges

- Dynamic doxastic/epistemic analysis of what can happen during plenary Dutch meetings is work in progress (with Floor Sietsma).
- How does our update/upgrade in the new style compare with Baltag and Smets' action-priority upgrade [4, 5]? Which update mechanism is more expressive?
- We are interested in **model checking** with doxastic/epistemic PDL and updates/upgrades in the new style, and we are currently investigating its complexity.
- We intend to use the logic, and the new update/upgrade mechanism, in the next incarnation of the epistemic model checker DEMO [15].

Reflection

- Why is it that DEL-style epistemic/doxastic analysis has so far only been used for toy applications, analysis of puzzles and brain teasers, and so on? What is it that makes analysis or checking of real life communication protocols so hard?
- DEL with a combined update/upgrade mechanism is still different from game logics, but the differences are getting less and less. The combination of knowledge update, factual change and belief change probably has more applications than we imagine now. What are nice challenges?

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