Belief, Probabilities, Updates, and Model Checking

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Abstract

The talk will present epistemic probability models with probabilistic updates, and will discuss an implementation that allows model checking the results of updates in a multi-agent setting. I will do my best to connect this to the themes of the workshop.

Belief and Probability

In the perspective of epistemic logic, our body of knowledge consists of true facts that we are certain about. But in the practice of everyday life and in the pursuit of science such absolute certainty is very rare.

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- Can I safely cross this road?
- Should I bring my umbrella?
- Can I trust this bank?
- Is it safe to order from this cheap website?
- Can I trust this estimate of the mass of the planet Saturn?[1]

[1] Pierre Simon Laplace made a famous calculation of this, including an estimate for the uncertainty, using the astronomical data that were available to him in the early Nineteenth Century.

The Lottery Puzzle

If Alice believes of each of the tickets 000001 through 111111 that they are not winning, then this situation is described by the following formula:

$$\bigwedge_{t=000001}^{111111} B_a \neg t.$$

If her beliefs are closed under conjunction, then this follows:

$$B_a \bigwedge_{t=000001}^{111111} \neg t.$$

But actually, she believes, of course, that one of the tickets is winning:

$$B_a \bigvee_{t=000001}^{111111} t.$$

This is a contradiction. The difficulty arises if we assume belief is closed under conjunction.

So it seems we need an operator B_i that does not satisfy (Dist).

$$B_i(\varphi \to \psi) \to B_i\varphi \to B_i\psi \tag{Dist-B}$$

This means: B_i is not a normal modal operator.

Epistemic Neighbourhood Models

An **Epistemic Neighbourhood Model** $\mathcal{M}$ is a tuple

$$(W, R, N, V)$$

where

- W is a non-empty set of worlds.
- R is a function that assigns to every agent $i \in Ag$ an equivalence relation $\sim_i$ on W. We use $[w]_i$ for the $\sim_i$ class of w, i.e., for the set $\{v \in W \mid w \sim_i v\}$.
- N is a function that assigns to every agent $i \in Ag$ and world $w \in W$ a collection $N_i(w)$ of sets of worlds—each such set called a neighbourhood of w—subject to a set of conditions.
- V is a valuation function that assigns to every $w \in W$ a subset of *Prop*.

Conditions

(c) $\forall X \in N_i(w) : X \subseteq [w]_i$. This ensures that agent i does not believe any propositions $X \subseteq W$ that she knows to be false. If X contains a world in $w' \in W - [w]_i$ that the agent knows is not possible with respect to the actual world w, then she knows that X cannot be the case and hence she does not believe X.

(f) $\emptyset \notin N_i(w)$. This ensures that no logical falsehood is believed.

(n) $[w]_i \in N_i(w)$. This ensures that what is known is also believed.

(a) $\forall v \in [w]_i : N_i(v) = N_i(w)$. This ensures that if X is believed, then it is known that X is believed.

(m) $\forall X \subseteq Y \subseteq [w]_i :$ if $X \in N_i(w)$, then $Y \in N_i(w)$. This says that belief is monotonic: if an agent believes X, then she believes all propositions $Y \supseteq X$ that follow from X.

(d) If $X \in N_i(w)$ then $[w]_i - X \notin N_i(w)$. This says that if i believes a proposition X then i does not believe the negation of that proposition.

Language

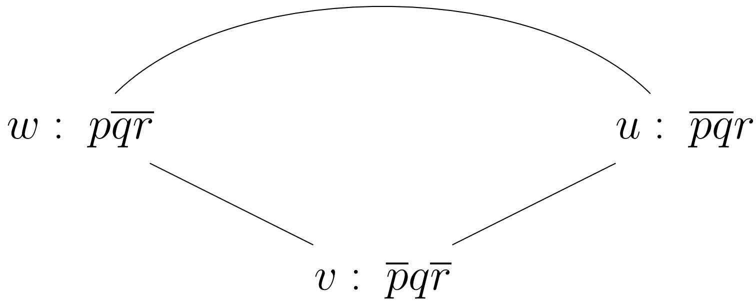
$$\varphi ::= \top \mid p \mid \neg\varphi \mid (\varphi \wedge \varphi) \mid K_i\varphi \mid B_i\varphi.$$

Semantics:

$$\mathcal{M}, w \models K_i\varphi \text{ iff } \quad \text{for all } v \in [w]_i : \mathcal{M}, v \models \varphi.$$

$$\mathcal{M}, w \models B_i\varphi \text{ iff } \quad \text{for some } X \in N_i(w), \text{for all } v \in X : \mathcal{M}, v \models \varphi.$$

Example



$$N(w) = N(v) = N(u) = \{\{w, v\}, \{v, u\}, \{w, u\}, \{w, v, u\}\}$$

Example

$$w : \; p\overline{qr} \qquad\qquad u : \; \overline{pq}r$$

$$v : \; \overline{p}q\overline{r}$$

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In all worlds, $K(p \vee q \vee r)$ is true.

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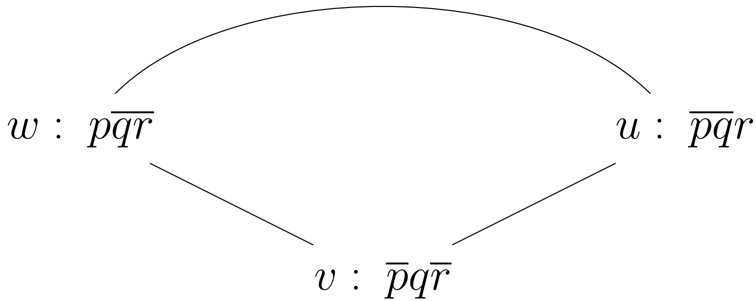

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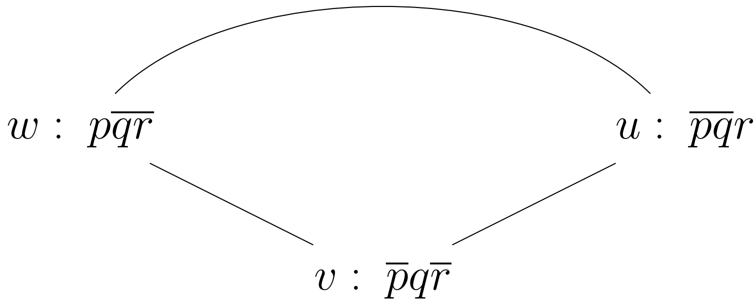

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In all worlds $B\neg p$, $B\neg q$, $B\neg r$ are true.

In all worlds $B(\neg p \wedge \neg q)$, $B(\neg p \wedge \neg r)$, $B(\neg q \wedge \neg r)$ are false.

Axioms

(Taut)	All instances of propositional tautologies
(Dist-K)	$K_i(\varphi \to \psi) \to K_i\varphi \to K_i\psi$
(T)	$K_i\varphi \to \varphi$
(PI-K)	$K_i\varphi \to K_iK_i\varphi$
(NI-K)	$\neg K_i\varphi \to K_i\neg K_i\varphi$
(F)	$\neg B_i\bot.$
(PI-KB)	$B_i\varphi \to K_iB_i\varphi$
(NI-KB)	$\neg B_i\varphi \to K_i\neg B_i\varphi$
(KB)	$K_i\varphi \to B_i\varphi$
(M)	$K_i(\varphi \to \psi) \to B_i\varphi \to B_i\psi$
(D)	$B_i\varphi \to \neg B_i\neg\varphi.$

RULES

$$\frac{\varphi \to \psi \qquad \varphi}{\psi}\ \text{(MP)} \qquad\qquad \frac{\varphi}{K_i\varphi}\ \text{(Nec-K)}$$

Further details: see [ER14] and [BvBvES14].

Bisimulation for epistemic-doxastic neighbourhood models

Let $\mathcal{M} = (W^{\mathcal{N}}, R^{\mathcal{M}}, V^{\mathcal{M}}, N^{\mathcal{M}})$ and $\mathcal{N} = (W^{\mathcal{N}}, R^{\mathcal{N}}, V^{\mathcal{N}}, N^{\mathcal{N}})$ be two epistemic-doxastic neighbourhood models, and let C be a relation on $W^{\mathcal{M}} \times W^{\mathcal{N}}$. Then C is a bisimulation if wCv implies that the following hold:

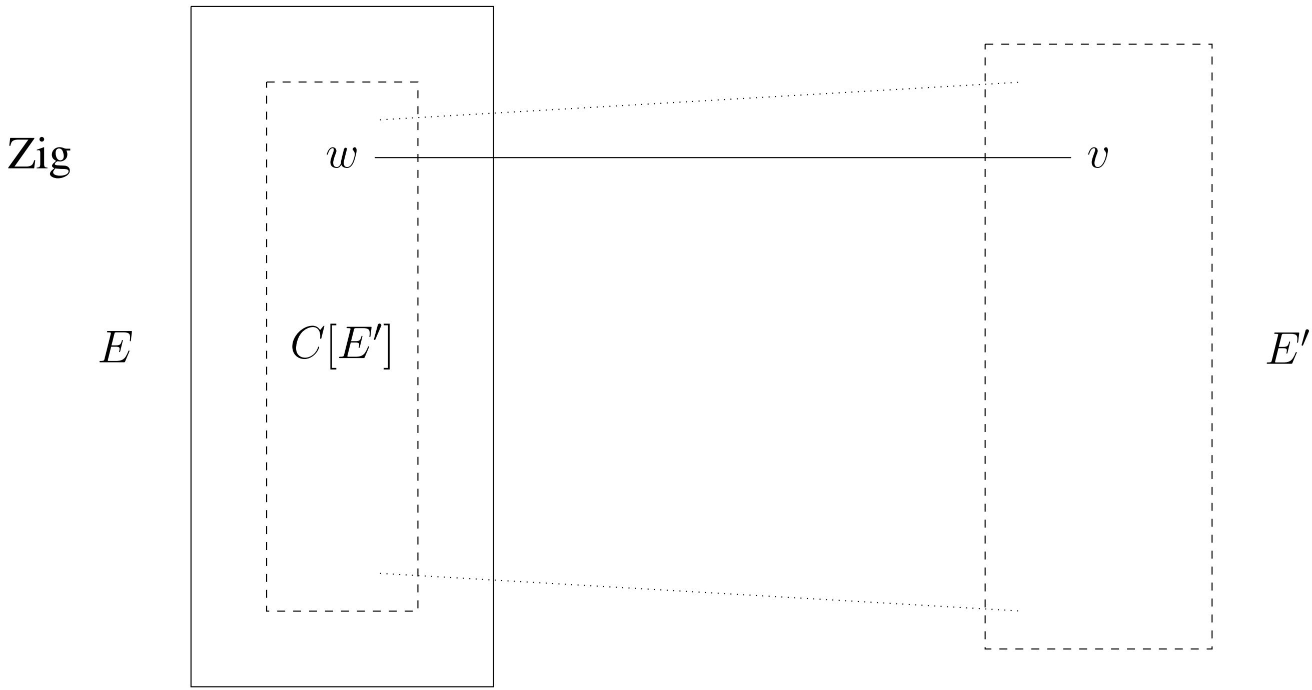
Invariance $V^{\mathcal{M}}(w) = V^{\mathcal{N}}(v)$, i.e., the two worlds have the same valuation.

Zig

1. If $w' \in R^{\mathcal{M}}(w)$ then there is a $v' \in R^{\mathcal{N}}(v)$ with $w'Cv'$,
2. for all subsets $E \subseteq R_i^{\mathcal{M}}(w)$ there exists a subset $E' \subseteq R_i^{\mathcal{N}}(v)$ such that for all $u' \in E'$ there exists $u \in E$ with uCu'.

Zag 1. and 2. vice versa.

$$\mathcal{M}, w \leftrightarroweq \mathcal{N}, v.$$



Knowledge, Certainty, Belief

One way to make the connection between epistemic logic and probability theory is by interpreting $K_i\varphi$ as "agent i assigns φ probability 1", or, "agent i is certain that φ is true."

Interpret $B_i\varphi$ as "agent i assigns φ higher probability than $\neg\varphi$", or, "agent i assigns φ probability greater than $\frac{1}{2}$."

As it turns out, the only thing we have to do is remove the neighbourhood function and add a lottery function to an epistemic model.

If W is the set of worlds of an epistemic model, a lottery function L assigns to every agent i a function $L_i : W \to \mathbb{Q}^+$, subject to the constraint that the sum of the L_i values over each epistemic partition cell of i is bounded.

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If $X \subseteq W$ then $L_i(X)$ is shorthand for $\sum_{x \in X} L_i(x)$.

Boundedness

The boundedness condition excludes cases where $[w]_i$ is infinite and each v in $[w]_i$ gets the same positive value c. It does not exclude infinite epistemic partition cells, however.

Example 1 *Let* $[w]_i = \mathbb{N}$*, and let* $L_i(n) = \frac{1}{2^n}$*. Then:*

$$L_i([w]_i) = \sum_{n \in \mathbb{N}} \frac{1}{2^n} = 2 < \infty.$$

Epistemic Lottery Models

An **Epistemic Lottery Model** $\mathcal{M}$ is a tuple (W, R, V, L), where

- W is a non-empty set of worlds.
- R is a function that assigns to every agent $i \in Ag$ an equivalence relation $\sim_i$ on W.
- V is a valuation function that assigns to every $w \in W$ a subset of *Prop*.
- L is a function that assigns to every agent $i \in Ag$ a lottery L_i, where L_i is a function from W to $\mathbb{Q}^+$, the set of positive rationals, with the constraint that for each $w \in W$,

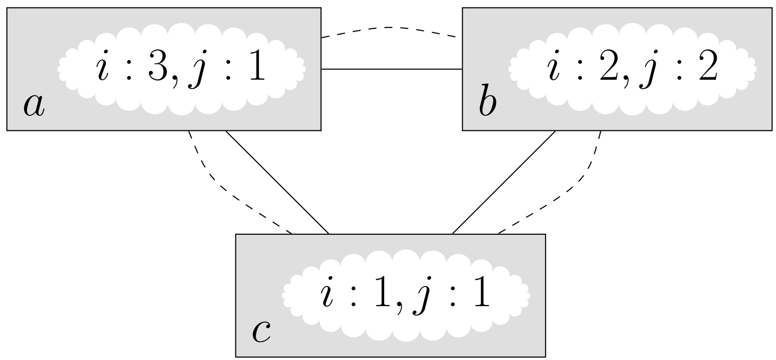
$$L_i([w]_i) < \infty.$$

Single Lottery Models

An epistemic lottery model $\mathcal{M} = (W, R, V, L)$ is single (or: a single lottery model) if for all $i, j \in Ag$ it holds that $L_i = L_j$.

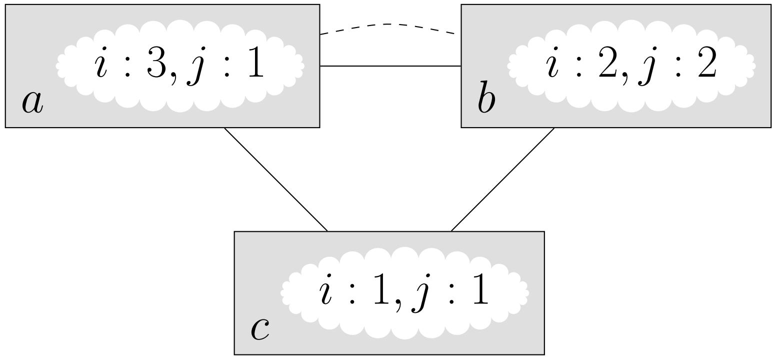
Example 2 *Take any epistemic model* $\mathcal{M} = (W, R, V)$ *with* W *finite. Let* L *be the function that maps* i *to the lottery* $L_i = \lambda w.1$. *Then* (W, R, V, L) *is an epistemic single lottery model.*

Example 3 *Two agents i, j consider betting on a horse race. Three horses take part in the race, and there are three possible outcomes: a for "a wins the race", b for " b wins the race", and c for "c wins the race." Neither agent knows which horse will win; i takes the winning chances to be $3:2:1$, j takes them to be $1:2:1$. In a picture:*



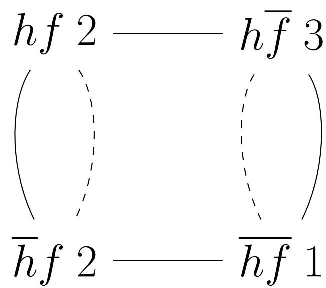
In all worlds, i assigns probability $\frac{1}{2}$ to a, $\frac{1}{3}$ to b and $\frac{1}{6}$ to c, while j assigns probability $\frac{1}{4}$ to a and to c, and probability $\frac{1}{2}$ to b.

Example 4 *Same situation as in example 3, but now agent j (dashed lines) considers c impossible.*



The probabilities assigned by i remain as before. The probabilities assigned by j have changed, as follows. In worlds a and b, j assigns probability $\frac{1}{3}$ to a and $\frac{2}{3}$ to b. In world c, j is sure of c.

Example 5 *Two agents i (solid lines) and j (dashed lines) are uncertain about the toss of a coin. i holds it for possible that the coin is fair f and that it is biased $\overline{f}$, with a bias $\frac{2}{3}$ for heads h. j can distinguish f from $\overline{f}$. The two agents share the same lottery (so this is a single lottery model), and the lottery values are indicated as numbers in the picture.*



In world hf, i assigns probability $\frac{5}{8}$ to h and probability $\frac{1}{2}$ to f. In world hf, j assigns probability $\frac{1}{2}$ to h and probability 1 to f. In other words, j is certain that the coin is fair.

Epistemic Probability Language

Let i range over *Ag*, p over *Prop*, and q over $\mathbb{Q}$. Then the language of epistemic probability logic is given by:

$$\begin{array}{rcl} \varphi & ::= & \top \mid p \mid \neg\varphi \mid (\varphi \wedge \varphi) \mid t_i \geq 0 \mid t_i = 0 \\ t_i & ::= & q \mid q \cdot P_i\varphi \mid t_i + t_i \text{ where all indices } i \text{ are the same.} \end{array}$$

Truth for Epistemic Probability Logic

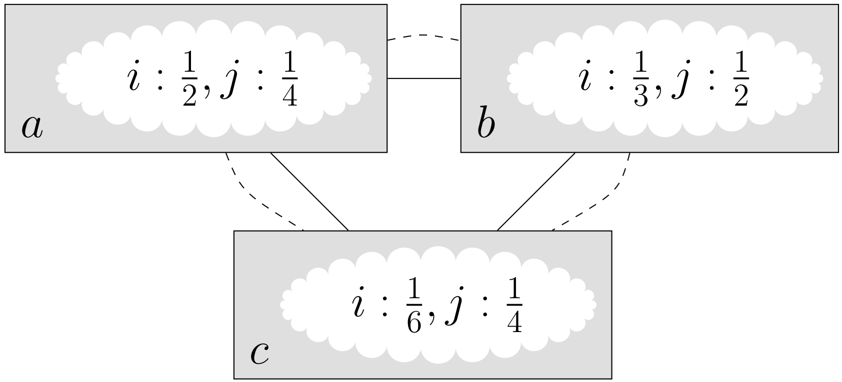
Let $\mathcal{M} = (W, V, R, L)$ be an epistemic lottery model and let $w \in W$.

$$\begin{array}{rcl}
\mathcal{M}, w \models \top & & \text{always} \\
\mathcal{M}, w \models p & \text{iff} & p \in V(w) \\
\mathcal{M}, w \models \neg\varphi & \text{iff} & \text{it is not the case that } \mathcal{M}, w \models \varphi \\
\mathcal{M}, w \models \varphi_1 \wedge \varphi_2 & \text{iff} & \mathcal{M}, w \models \varphi_1 \text{ and } \mathcal{M}, w \models \varphi_2 \\
\mathcal{M}, w \models t_i \geq 0 & \text{iff} & [\![t_i]\!]_w^{\mathcal{M}} \geq 0 \\
\mathcal{M}, w \models t_i = 0 & \text{iff} & [\![t_i]\!]_w^{\mathcal{M}} = 0.
\end{array}$$

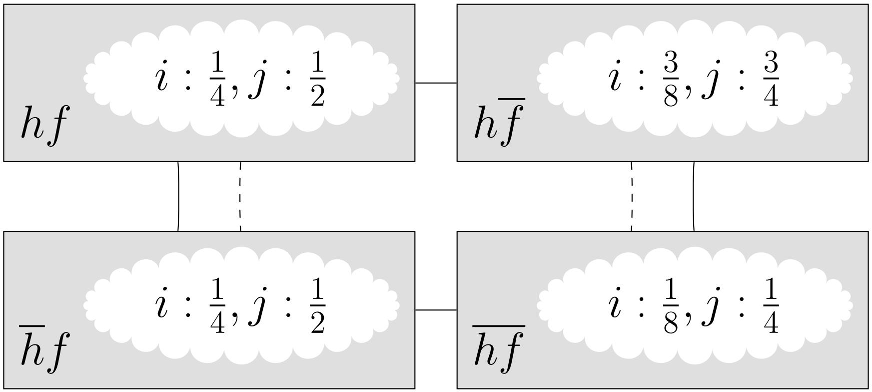
$$\begin{array}{rcl}
[\![q]\!]_w^{\mathcal{M}} & := & q \\
[\![q \cdot P_i\varphi]\!]_w^{\mathcal{M}} & := & q \times P_{i,w}^{\mathcal{M}}(\varphi) \\
[\![t_i + t_i']\!]_w^{\mathcal{M}} & := & [\![t_i]\!]_w^{\mathcal{M}} + [\![t_i']\!]_w^{\mathcal{M}}
\end{array}$$

$$P_{i,w}^{\mathcal{M}}(\varphi) = \frac{L_i(\{u \in [w]_i \mid \mathcal{M}, u \models \varphi\})}{L_i([w]_i)}.$$

Example 6 *A normalized model for the horse racing situation from Example 3 is given in the picture:*



Example 7 *[Continued from Example 5] The model from Example 5 is an epistemic lottery model where the two agents share the same lottery. It is also possible to give each agent its own lottery, and to normalize the lotteries using the epistemic accessibilities.*



Bisimulation for Epistemic Lottery Models

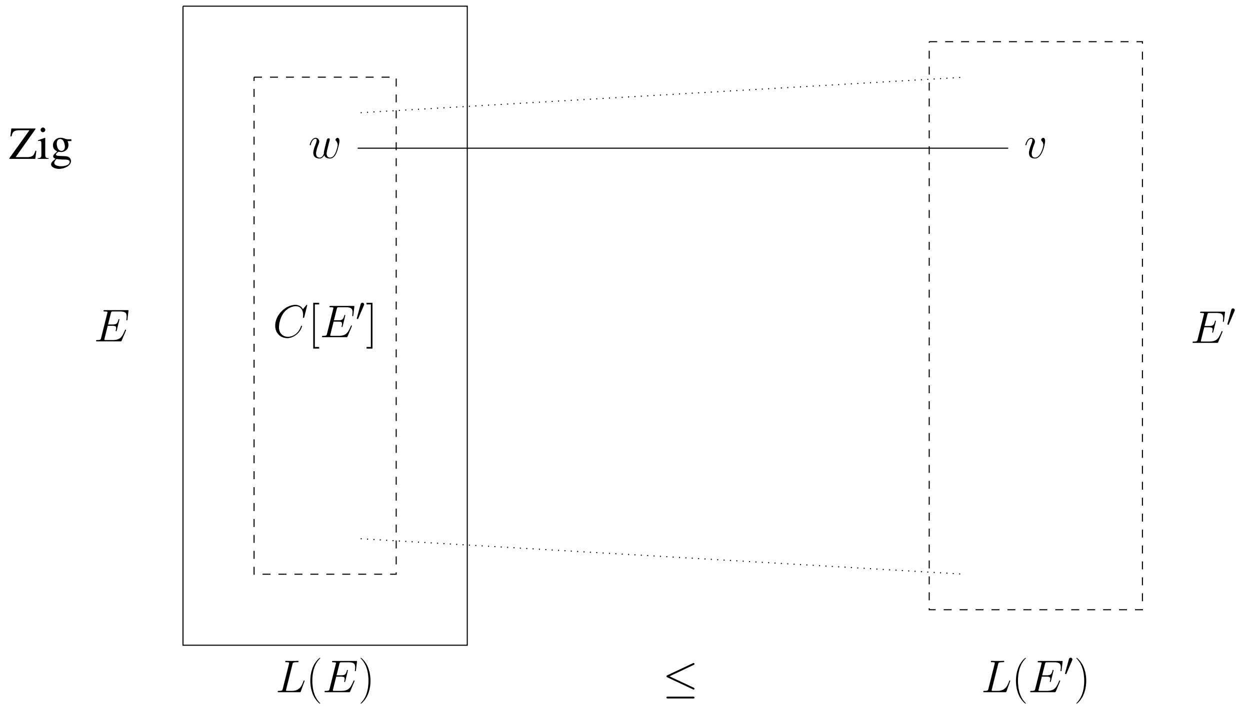
Let $\mathcal{M} = (W^{\mathcal{N}}, R^{\mathcal{M}}, V^{\mathcal{M}}, L^{\mathcal{M}})$ and $\mathcal{N} = (W^{\mathcal{N}}, R^{\mathcal{N}}, V^{\mathcal{N}}, L^{\mathcal{N}})$ be two epistemic lottery models, and let C be a relation on $W^{\mathcal{M}} \times W^{\mathcal{N}}$. Then C is a bisimulation if wCv implies that the following hold:

Invariance $V^{\mathcal{M}}(w) = V^{\mathcal{N}}(v)$, i.e., the two worlds have the same valuation.

Zig For all subsets $E \subseteq R_i^{\mathcal{M}}(w)$ there exists a subset $E' \subseteq R_i^{\mathcal{N}}(v)$ such that:

1. for all $u' \in E'$ there exists $u \in E$ with uCu';
2. $L_{i,w}^{\mathcal{M}}(E) \leq L_{i,v}^{\mathcal{N}}(E')$.

Zag Same conditions vice versa.



See [ES14]. This is a variation on (and simplification of) the bisimulation notion in [Koo03].

Fact 1 *Formulas of epistemic probability logic are invariant for bisimulation [ES14].*

Fact 2 *On epistemic lottery models with finite epistemic partition cells for every agent, invariance for formulas of epistemic probability logic implies bisimilarity [ES14].*

Fact 3 *A sound and complete for the language of epistemic probability logic, interpreted in epistemic probability models, is given in [ES14].*

Axioms

(Taut)	All instances of propositional tautologies
(Linear)	All instances of valid formulas about linear inequalities
(ProbNonNeg)	$P_i\varphi \geq 0$
(ProbTrue)	$P_i\top = 1$
(ProbAdd)	$P_i(\varphi_1 \wedge \varphi_2) + P_i(\varphi_1 \wedge \neg\varphi_2) = P_i\varphi_1$
(ProbProbGeq)	$t_i \geq 0 \rightarrow P_i(t_i \geq 0) = 1$
(ProbProbEq)	$t_i = 0 \rightarrow P_i(t_i = 0) = 1$
(ProbT)	$P_i\varphi = 1 \rightarrow \varphi$

Rules

$$\frac{\varphi \rightarrow \psi \quad \varphi}{\psi}\ \text{(MP)} \qquad \frac{\varphi_1 \leftrightarrow \varphi_2}{P_i\varphi_1 = P_i\varphi_2}\ \text{(ProbRule)}$$

From Lottery Models to Neighbourhood Models

If $\mathcal{M} = (W, R, V, L)$ is an epistemic lottery model, then $\mathcal{M}^\bullet$ is the tuple (W, R, V, N) given by replacing the lottery function by a function N, where N is defined as follows, for $i \in$ *Ag*, $w \in W$.

$$N_i(w) = \{X \subseteq [w]_i \mid L_i(X) > L_i([w]_i - X)\}.$$

Fact 4 *For any epistemic lottery model $\mathcal{M}$ it holds that $\mathcal{M}^\bullet$ is a neighbourhood model.*

Fact 5 *The calculus of epistemic-doxastic neighbourhood logic is sound for interpretation in epistemic probability models. Probabilistic beliefs are neighbourhoods.*

Translating Knowledge and Belief

If φ is a formula of the language of epistemic/doxastic logic, then $\varphi^\bullet$ is the formula of the language of epistemic probability logic given by the following instructions:

$$\begin{aligned}
\top^\bullet &= \top \\
p^\bullet &= p \\
(\neg\varphi)^\bullet &= \neg\varphi^\bullet \\
(\varphi_1 \wedge \varphi_2)^\bullet &= \varphi_1^\bullet \wedge \varphi_2^\bullet \\
(K_i\varphi)^\bullet &= P_i(\varphi^\bullet) = 1 \\
(B_i\varphi)^\bullet &= P_i(\varphi^\bullet) > P_i(\neg\varphi^\bullet).
\end{aligned}$$

Theorem 6 *For all formulas of epistemic/doxastic logic φ, for all epistemic lottery models $\mathcal{M}$, for all worlds w of $\mathcal{M}$:*

$$\mathcal{M}^\bullet, w \models \varphi \textit{ iff } \mathcal{M}, w \models \varphi^\bullet.$$

Theorem 7 *Let* $\vdash$ *denote derivability in the calculus of EDNL. Let* $\vdash'$ *denote derivability in the calculus of EPL. Then* $\vdash \varphi$ *implies* $\vdash' \varphi^\bullet$.

Implementation

Building epistemic models from partitions . . .

```
type Erel a = [[a]]

data Agent = Ag Int deriving (Eq,Ord)

a,b,c,d,e :: Agent
a = Ag 0; b = Ag 1; c = Ag 2; d = Ag 3; e = Ag 4

data Prp = P Int | Q Int | R Int | S Int
  deriving (Eq,Ord)
```

Epistemic models

```
data EpistM state = Mo
                    [state]
                    [Agent]
                    [(state,[Prp])]
                    [(Agent,Erel state)]
                    [state]  deriving (Eq,Show)
```

```
example1 :: EpistM Int
example1 = Mo
  [0..3]
  [a,b,c]
  []
  [(a,[[0],[1],[2],[3]]),
   (b,[[0],[1],[2],[3]]),(c,[[0..3]])]
  [1]
```

Epistemic Formulas

```
data Frm a = Tp
           | Info a
           | Prp Prp
           | N (Frm a)
           | C [Frm a]
           | D [Frm a]
           | Kn Agent (Frm a)
          deriving (Eq,Ord,Show)
```

Truth Definition

...

```
isTrueAt :: Ord state =>
         EpistM state -> state -> Frm state -> Bool
isTrueAt m w Tp = True
isTrueAt m w (Info x) = w == x
isTrueAt
  m@(Mo worlds agents val acc points) w (Prp p) =
  let props = apply val w
  in elem p props
isTrueAt m w (N f) = not (isTrueAt m w f)
isTrueAt m w (C fs) = and (map (isTrueAt m w) fs)
isTrueAt m w (D fs) = or  (map (isTrueAt m w) fs)
isTrueAt
 m@(Mo worlds agents val acc points) w (Kn ag f) =
  let
    r = rel ag m
    b = bl r w
  in
    and (map (flip (isTrueAt m) f) b)
```

Public Announcement

```
upd_pa :: Ord state =>
       EpistM state -> Frm state -> EpistM state
upd_pa m@(Mo states agents val rels actual) f =
 (Mo sts' agents val' rels' actual') where
   sts'    = [ s | s <- states, isTrueAt m s f ]
   val'    = [ (s, ps) | (s,ps) <- val,
                             s `elem` sts']
   rels'   = [(ag,restrict sts' r) |
                                  (ag,r) <- rels ]
   actual'= [ s | s <- actual, s `elem` sts' ]

upds_pa ::  Ord state =>
         EpistM state -> [Frm state] -> EpistM state
upds_pa = foldl upd_pa
```

Example: Sum and Product (Hans Freudenthal)

A says to S and P: I have chosen two integers x, y such that $1 < x < y$ and $x + y \leq 100$. In a moment, I will inform S only of $s = x + y$, and P only of $p = xy$. These announcements remain private. You are required to determine the pair (x, y). He acts as said. The following conversation now takes place:

1. P says: “I do not know the pair.”
2. S says: “I knew you didn’t.”
3. P says: “I now know it.”
4. S says: “I now also know it.”

Determine the pair (x, y).

A model checking solution with DEMO [vE05, vE07] (based on a DEMO program written by Ji Ruan) was presented in [DRV05]. An optimized version of that solution is in [vE13].

The list of candidate pairs:

```
pairs :: [(Int,Int)]
pairs  = [ (x,y) |  x <- [2..100], y <- [2..100],
                    x < y, x+y <= 100 ]
```

The solution:

```
solution = upds_pa msnp
           [k_a_statement_1e,statement_2e,statement_
```

This is checked in a matter of seconds:

```
*DEMO_S5> solution
Mo [(4,13)] [a,b] [(a,[[(4,13)]]),(b,[[(4,13)]])]
   [(4,13)]
```

Extending This With Lotteries

- Representation of probabibility information by means of lotteries was designed with implementation of model checking in mind.
- Just extend the epistemic models with a lottery table for each agent.
- Implementations of model checkers for these logics can be found in [Eij13] and in [San14] . . .
- The implementations can deal with Monty Hall style puzzles, urn puzzles, Bayesian updating by drawing from urns or tossing (possibly biased) coins, and 'paradoxes' such as the puzzle of the three prisoners (below).
- Efficiency was not a goal, but these implementation can be made very efficient with a little effort.

Aside: The Puzzle of the Three Prisoners

Alice, Bob and Carol are in prison. It is known that two of them will be shot, the other freed. The warden knows what is going to happen, so Alice asks him to reveal the name of one other than herself who will be shot, explaining to him that since there must be at least one, this will not reveal any new information. The warden agrees and says that Bob will be shot. Alice is cheered up a little by this, for she concludes that her chance of surviving has now improved from $\frac{1}{3}$ to $\frac{1}{2}$. Is this correct? How does this agree with the intuition that the warden has not revealed new information?

Many sources, e.g. [Jef04].

How to Move on From Here

- Towards the goal of the workshop: combine EPL with network information for the agents, where the network is given by a relation, and where links starting from an agent can be added ("start following") and deleted ("stop following, unfollow"). Interpret announcements as group messages to all followers. See [RT11] and current work by Jerry Seligman and Thomas Agotnes. But: this can all be done with epistemic PDL with a binary follow relation F added.

- Further analysis of the connection between neighbourhood logics and probabilistic logics [ER14]. This is also connected to work of Wes Holliday and Thomas Icard.

- Add bias variables X for the representation of unknown biases. Compare Joshua Sack's talk.

- Work with the epistemic PDL version of the probabilistic logic, as an extension of LCC from [BvEK06]. This gives us common knowledge, and a nice axiomatisation by means of epistemic program transformation [Ach14].
- Achieve better efficiency, by teaming up with Kaile Su (next talk).
- Towards analysis of real-life protocols. Compare the use of epistemic model checking by Malvin Gattinger [Gat13, Gat14b, Gat14a].
- Consider weak lottery models, where the lotteries assign pairs of values (x, y), with x giving the lower probability L and $x + y$ the upper probability U. Belief of i in φ is now modelled as $L_i(\varphi) > H_i(\neg\varphi)$. This connects up to weak Bayesianism and imprecise probability theory [Wal91].
- Consolidate what we know about the topic in a state-of-the-art

textbook [BvBvES14].

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