Gossip in Dynamic Networks

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Abstract

A gossip protocol is a procedure for spreading secrets among a group of agents, using a connection graph. In this talk the problem of designing and analyzing gossip protocols is given a dynamic twist by assuming that when a call is established not only secrets are exchanged but also contact lists, i.e., links in the gossip graph. Thus, each call in the gossip graph changes both the graph and the distribution of secrets. In the talk, we give a full characterization for the class of dynamic gossip graphs where the Learn New Secrets protocol (make a call to an agent if you know the number but not the secret of that agent) is successful.

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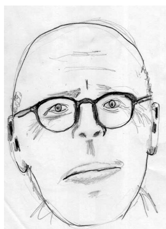
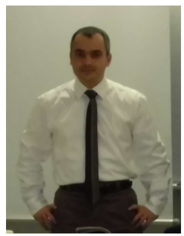
This is joint work with Hans van Ditmarsch, Pere Pardo, Rahim Ramezanian and François Schwarzentruber

Overview

- What are Gossip Protocols? Brief History
- Gossip in Totally Connected Networks
- Distributed Protocols
- The Dynamic Turn
- Examples of Gossip Graph Completion
- The Learn New Secrets Protocol
- Results
- Further Questions

An Abstract Perspective on Gossip Spreading

Key notions: gossip graph, gossip secret, gossip call, gossip protocol.

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- What do we assume about the graph?
- What do we assume about the secrets?
- What do we assume about the protocol? In particular: is there a central authority, or is the protocol distributed?

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- Distributed gossip protocols: dynamic turn (this talk) [DvEP$^+$15a, DvEP$^+$15b]

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Assumptions: graph totally connected, during a call all secrets are exchanged.

Key question: find a minimal sequence of calls to achieve a state where all agents know all secrets. What are the lengths of these minimal sequences?

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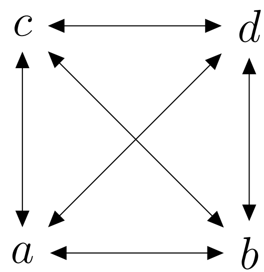
Fact: In a totally connected graph with $n > 3$ agents, $2n - 4$ calls are sufficient.

Gossip in Totally Connected Graphs: Example

Consider the totally connected graph with four agents, each with a unique secret.



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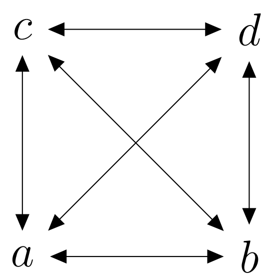

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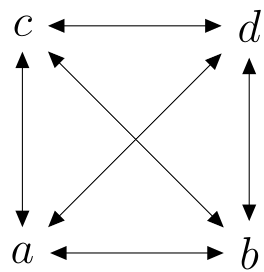

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Now suppose e is also present. Consider ae, next $ab; cd; ac; bd$, finally ae. Two extra calls are enough to accommodate one extra agent.

For n agents, $2n-4$ calls are enough

Let $n > 3$.

Basis: $n = 4$. We have seen that $4 = 8-4$ calls are enough.

Induction step: Assume for n agents $2n-4$ calls are enough. Next, add one extra agent x. Start with call from x to a, end with call from a to x, and all secrets are shared. This shows that $(2n-4)+2 = 2(n+1)-4$ calls are enough. $\square$

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Note: in graphs that are weakly but not totally connected, the minimum number of calls to distribute all gossip may be larger than $2n - 4$ [HHL88].

Distributed Protocols

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Possible distributed protocol for gossip spreading:

Search For Secrets

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- When a call is established not only secrets are exchanged but also contact lists (or: information about the graph).
- Calls in the gossip graph are constrained by the current distribution of numbers, and each call changes both the graph and the distribution of secrets.
- The network is given in distributed fashion: $(x, y) \in N$ iff y is in the contact list of x (think of contact lists in smartphones). These contact lists are exchanged (merged) when a call is made.

Representing Gossip Graphs

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- Nxy expresses that x has a link to y (or: x does know the phone number of y).
- Sxy expresses that x does know the secret of y.
- Alternatively, we can think of N and S as functions in $A \to \mathcal{P}A$, so that N_x is the set of agents whose numbers are known by x, and S_x is the set of agents whose secrets are known by x.

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If $G = (A, N, S)$ and $x, y \in A$, then $G^{xy} = (A, N', S')$ where

- N' is $N \cup \{(x, y), (y, x)\} \circ N$,
- S' is $S \cup \{(x, y), (y, x)\} \circ S$.

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Accessible secrets $G = (A, N, S)$ has accessible secrets if $I_A \subseteq S \subseteq N$, where $I_A = \{(a, a) \mid a \in A\}$.

Thus, G has accessible secrets iff every agent knows her own secret and moreover, if agent x knows the secret of y, x also knows the number of y.

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This shows that old questions about minimum lengths of calling sequences can receive new answers in this dynamic setting.

Gossip and Weakly Connected Components

Proposition 1 *Let* $G = (A, N, S)$*, and let* σ *be a possible calling sequence for* G*. Then* $N^{\sigma} \subseteq (N \cup N^{-1})^{*}$.

Intuitively, this says that gossip can only spread within the weakly connected components of G.

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Theorem 2 *If σ is a possible calling sequence for $G = (A, N, S)$, then G is weakly connected iff G^σ is weakly connected.*

Intuition: by proposition 1, gossip cannot create weak connectedness.

Gossip Graph Completion

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Key question 2: given some distributed protocol P, what is the class of graphs that can be completed by P?

Search for Secrets as a Distributed Protocol

Search For Secrets

While not every agent knows all secrets, randomly select a pair xy such that Nxy and let x call y.

This will complete any weakly connected graph, but it is not efficient.

Learn New Secrets

The following protocol is studied in [AvDGvdH14, AGvdH15] in the context of totally connected graphs.

> **Learn New Secret Protocol (LNS)**
>
> While not every agent is an expert, let an agent x that is not an expert randomly choose an agent y from the list of agents for which Nxy but not Sxy, and perform the call xy.

LNS-permitted and LNS-stuck Sequences

LNS-permitted calling sequences:

- ϵ is LNS-permitted on any G,
- $\sigma; xy$ is LNS-permitted on G iff σ is LNS-permitted on G and xy is LNS-permitted on G^{σ}.

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A calling sequence σ is **LNS-stuck** on G if σ is LNS-permitted on G, G^σ is not complete, and no call is LNS-permitted on G^σ.

Example of LNS-stuck Sequence

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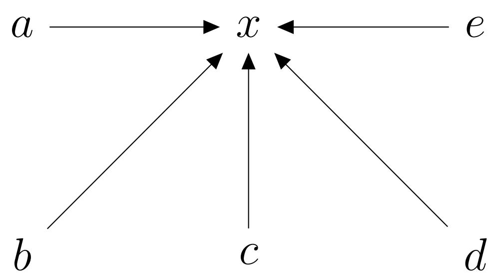
Still, the sequence $ab; bc; ca$ is LNS permitted.

Example Graph that Cannot be Completed by LNS



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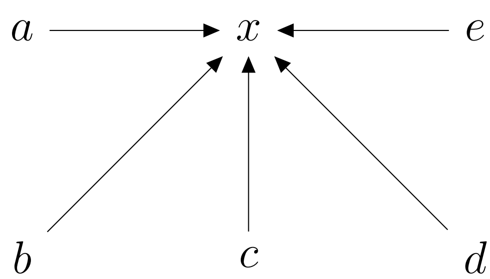

Whoever starts the calling sequence, it can not be LNS completed.

The reason is that x, the spider in the web, never learns enough about the network structure to be able to make a useful call (x only learns contact info of agents whose secret x also learns.

The LNS-permitted sequences are all the permutations of $ax; bx; cx; dx; ex$, and they all get stuck.

Graphs Where LNS is Successful

The LNS protocol is successful on G if either G is complete, or there is an LNS-permitted call xy, and after any LNS-permitted call xy the LNS protocol is successful on G^{xy}.

It follows that LNS is successful on G iff every sequence of LNS-permitted calls σ results in a graph G^{σ} that is complete, or is such that there is an LNS-permitted call, and after any LNS-permitted call xy, LNS is successful on $G^{\sigma;xy}$.

> **LNS gossip graph algorithm**
>
> Search for an LNS-stuck calling sequence in depth-first fashion, and declare success if no such calling sequence can be found [EG15a].

LNS-maximal Sequences

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Proposition 3 *If σ is an LNS-maximal calling sequence for G, and G has accessible secrets, then $S^{\sigma} = N^{\sigma}$.*

Proposition 4 *If σ is an LNS-maximal calling sequence for G, and G satisfies $I_A = S \subseteq N$, then $S^{\sigma} \circ N^{*} = S^{\sigma}$.*

Terminal Points, Skin, Sun Graphs

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Let $s(G)$ be the result of skinning graph G, i.e. removing all terminal points from G. That is, $s(G) = (B, N', S')$ where

$$B = \{x \in A \mid N_x - \{x\} \neq \emptyset\}, N' = N \cap B^2, S' = S \cap B^2.$$

Note that skinning a graph is not a closure operation: there are graphs with $s(s(G)) \neq s(G)$.

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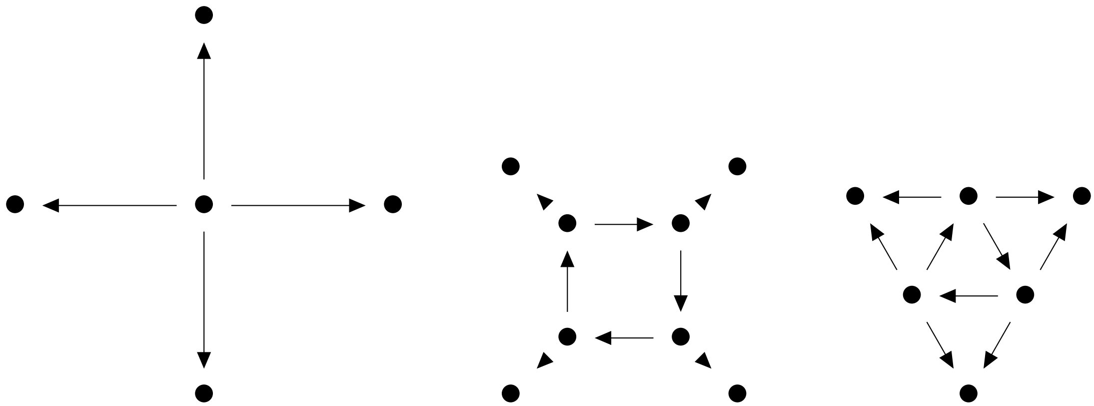
Call a graph $G = (A, N, S)$ a **sun** if $S = I_A \subseteq N$, N is weakly connected on G, and N is strongly connected on $s(G)$.

Examples of Sun Graphs



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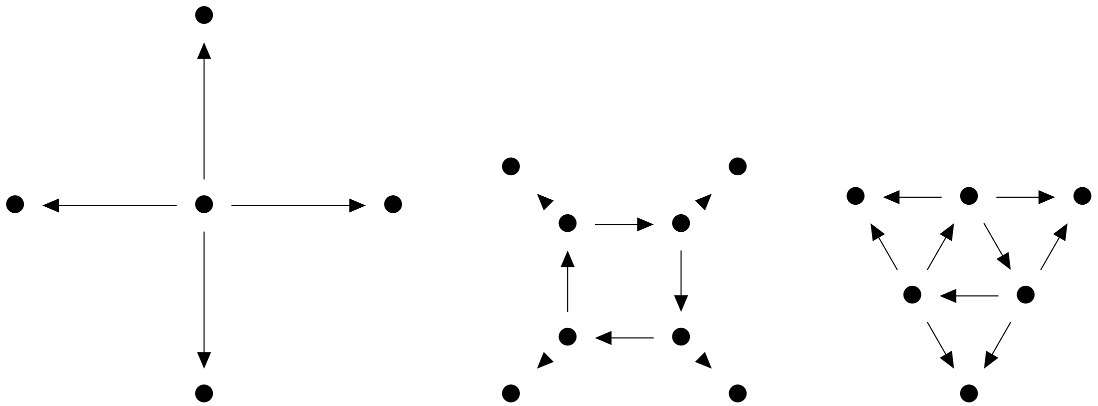

Theorem 5 *The LNS protocol is successful for any sun G.*

Strongly Connected Components of a Graph

Let $\sim$ be the relation on $G = (A, N, S)$ given by $x \sim y$ iff there is an N-path from x to y and there is an N-path from y to x. Then $\sim$ is an equivalence relation, and a cell in the partition induced by $\sim$ is called a strongly connected component of G. Use $[x]_\sim$ for the strongly connected component of G that contains x.

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Theorem 6 *If* $G = (A, N, S)$ *is a gossip graph with the property that* $s(G)$ *is connected but not strongly connected, then there are* $x, y \in A$ *such that*

1. $\hat{N}xy$*,*
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Theorem 8 *For any connected graph $G = (A, N, S)$ with $I_A = S \subseteq N$ the following holds: $s(G)$ is strongly connected iff the LNS protocol is successful for G.*

LNS-impossible Graphs

Question Characterize the graphs where **no** CNS sequence is successful.

Answer See [DvEP+15a].

Note: the conditions for this are quite strong. Spider in the web above is an example.

Other Protocols: HYN

Help Your Neighbour (HYN) [Her15]

Everyone who ever contacted you becomes your neighbour. Every time you learn new secrets, you incur an obligation to inform them …

Theorem 9 *HYN completes any weakly connected graph. [Her15]*

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- Further features from actual life could be imported. An important one that comes to mind is caller blocking. Which blocking patterns on which graphs can be overcome by which protocols?
- In [EG15b] we try to harness PDL as a logic for dynamic gossip. Details will be given by Malvin in the next talk.


2015
Mark Zuckerberg


facebook, a great place to
spread gossip

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