Modelling Legal Relations

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Abstract

We use propositional dynamic logic and ideas about propositional control from the agency literature to construct a formal model of how legal relations interact with actions that change the world, and with actions that change the legal relations. Our conceptual model also allows us to study the interplay of obligation, knowledge, and ignorance, and to model knowledge based obligation.

1 Introduction

Morality and (civil and penal) law are concerned with what individuals (and, in the case of law, institutions) are permitted or forbidden to *do*, so they are concerned with action and forbearance (restraining impulses to prevent others from acting in certain ways).

How one views the relation between the moral and legal realms depends on one's philosophy: for Jeremy Bentham legal rights are the essence of rights, while Thomas Aquinas maintained that societal law should be grounded in natural law, that is, the fundamental law of morality. For most people, morality is wider than what the laws of society forbid or allow. After all, laws can be criticized and amended if they fail to reflect the dictates of our moral conscience. Also, acts that are not strictly against the law may still be morally reprehensible. Breaking a promise to help a friend is most often not punishable by civil law, but it still counts as an offence against moral law.

Instead of trying to chart the differences and commonalities between what is legally permissible and what is just, we will aim in this paper at clarification of legal relations in their connection with ability to act, knowledge and ignorance about the effects of actions, and harm or benefit that various actions might have for individuals or for society as a whole. Our point of departure here is the emphasis on action. We hope to show in this paper that the very general methods that were developed in theoretical computer science to structure action allow us to say something relevant about legal acceptability of composite actions.

To give an example, when do we say that a composite action consisting of a first part α and a second part β is legally permitted for an agent? Clearly, there are two conditions:

- The first part α should be legally permitted in the current situation.
- In the state that results from action α, the second part β should be legally permitted.

To see that this make sense, consider the example of first buying a train ticket and next boarding the train. Doing the second part without the first may well be against the law, but in the state that results from having obtained the ticket, it is legally permitted to board the train.

2 Two Motivating Examples

As a first motivating example, we take the following case of *knowledge based obligation* taken from [PPC06]. Suppose Anne is a medical doctor and Bob is having a heart attack. Anne has a moral or legal obligation to help. But now we can distinguish between *obligation per se*, and *knowledge based obligation*. If Anne knows that Bob is in danger it is clear what is her duty. But how about the case when she does not know? Here is a picture of that situation:

$$
\begin{array}{ccc}
\Downarrow & & \\
p & \overset{a}{\text{———————}} & \overline{p} \\
w & & w' \\
C^+_{ba} = \{p\} & & C^+_{ba} = \emptyset \\
A_a = \{p\}, A_b = \emptyset & & A_a = \{p\}, A_b = \emptyset
\end{array}
$$

Agent a is a doctor, and p means that agent b is in danger. In the actual situation w, b is in danger. a is able to save b (putting him out of danger), and b cannot save himself. This is indicated by the ability table $A_a = \{p\}, A_b = \emptyset$. Abilities of agents are their powers to change the values of basic propositions. The duty of a to save b is represented as a claim of b that a changes the value of p, indicated by $C^+_{ba}(w) = \{p\}$. In case b is not in danger there is no such claim: $C^+_{ba}(w') = \emptyset$. Knowledge and ignorance are represented by accessibility relations between worlds (with reflexive arrows not drawn). The a-arrow linking w and

w' indicates that b can distinguish the two worlds but a cannot. The following statements are all true in the actual world w of the model (these claims will be made precise in our formal treatment):

$$
\begin{array}{lcll}
w & \models & p & \text{“}b \text{ is in danger”} \\
w & \models & \neg\Box_a p & \text{“}a \text{ does not know that } b \text{ is in danger”} \\
w & \models & D^+_{ab} p & \text{“}a \text{ has a duty to get } b \text{ out of danger”} \\
w & \models & \neg\Box_a D^+_{ab} p & \text{“}a \text{ does not know that } a \text{ has a duty to help } b\text{”}
\end{array}
$$

Suppose b is in a position to inform a that p. Then we can assume that the result of this action, the a-link between w and w' is cut. In the new situation, a has a duty to get b out of danger, and moreover, a knows that she has a duty to get b out of danger.

$$
\begin{array}{lcll}
w & \models & [(b, I_a p)]\Box_a D^+_{ab} p & \text{“After being informed by } b \text{ about } p \\
 & & & \; a \text{ knows her duty towards } b \text{ to act on } p\text{”}
\end{array}
$$

As a second example, consider a case where b has a duty towards a of refraining from acting on p, in case p is true, and a duty towards a to act on p (restoring its old value) if p is false.

$$
\boxed{
\begin{array}{cc}
\Downarrow & \\
\overline{p} & p \\
w & w' \\
C^-_{ab} = \{p\} & C^+_{ab} = \{p\} \\
A_a = A_b = \{p\} & A_a = A_b = \{p\}
\end{array}
}
$$

In the actual world b has the duty to forbear on p. But if a acts on p, or if b acts contrary to duty by operating on p, then b incurs the new duty of restoring the old situation. After this repair action, the old duty holds again. And so on.

$$
\begin{array}{lcl}
w & \models & D^-_{ba} p \\
w & \models & [(a, p)]D^+_{ba} p \\
w & \models & [(a, p); b, p)]D^-_{ab} p \\
w & \models & [((a, p); (b, p))^*]D^-_{ab} p
\end{array}
$$

3 Agency, Abilities, and Legal Claims

We adopt a relational view on legal relations inspired by the work on the foundations of legal reasoning by Wesley Newcomb Hohfeld [Hoh13]. For earlier formal representations of this in logic we refer to [Kan72, Mak86] and the references given there.

Agency is crucial in Hohfeld-style legal relations, for Hohfeld insists that any legal relation is a relation between precisely two agents. Another aspect that is crucial, according to us, is that legal relations between agents are about *what agents are allowed to do* rather than about *what state the world should be in*. It is here that we diverge from earlier approaches at formalization.

Our intention is to keep matters concerning the actions in the world that the legal relations pertain to as simple as possible (or possibly even simpler). Assume, therefore, that basic actions are of an extremely simple kind: all an agent can do is change basic facts about the world, or reset the value of a basic proposition, or as we will say, *operate on the value of* p. But we need an extra element, to indicate the *author* of the action [Eij00, vdHW05]. We will use (a, Q) for "agent a acts on all $q \in Q$" (making q false in case it is true, true if it is false). We need the notion of simultaneous action on a set of basic propositions because we want to model obligation to act, and for such obligations to have force we need a *deadline* for them. In our simple set-up the deadline is: obligations have to be fulfilled immediately, that is, in the next step. The only way to fulfil multiple obligations in the next step is to perform them simultaneously.

Next, we need a distinction between *abilities of agents* and *moral or legal claims* or *moral or legal duties* of agents. We want to be able to say: "agent a *can* perform action A but *does not have the right* to perform A." After all, legal or moral issues cannot arise if agents do not have the ability to deviate from what the moral law or the societal law prescribes.

To model *ability*, we assume that for every agent a and every valuation Q there is some subset $A_a(Q)$ of P (the set of basic propositions) that the agent is able to change (from false to true, or from true to false, as the case may be). If some p is in Q and in $A_a(Q)$ then this means a can set p to false, if some p is in $A_a(Q)$ and not in Q then this means a can set p to true. If $p \in Q$ and $p \in A_a(Q)$, and for all $R \subseteq P$ with $p \notin R$ it holds that $p \notin A_a(R)$ then this means that the action of making p false cannot be undone by agent a (the action of making p false is like breaking an egg or wrecking a car).

There may be propositions that can be made true or false by more than one agent in some situation, that is, there may be Q, $a \neq b$ with $A_a(Q) \cap A_b(Q) \neq \emptyset$.

To model *legal claims*, we assume for simplicity that all claims are individual (as opposed to group claims). Hohfeld makes a crucial distinction between two kinds of performance of an agent: actions and forbearances. To *forbear* is to restrain an impulse to act, or to restrain an inpulse to prevent another agent agent from acting. If the law says that tenants have to forbear property inspections by their landlords, this means that the tenants act against the law when they do not allow such inspections.

If we assume that basic actions are changes in the truth values of atomic propositions, then we can distinguish between actions and forbearances by making the following distinction:

- $C^+_{ab}(Q) \subseteq P$ are the actions that are claimed in situation Q by a against b, in the sense that a claims that b has the duty to perform them. If $p \in C^+_{ab}(Q)$ then the action that is meant is the action of b of flipping the truth value of p (from the value p has in Q to the opposite value).

- $C^-_{ab}(Q) \subseteq P$ are the propositions that are claimed in Q as forbearances by a against b, in the sense that a claims that b has the duty to forbear operating on p. If $p \in C^-_{ab}(Q)$, then by forbearance is meant that b should *not* flip the truth value of p.

In the Hohfeld perspective, claims and duties are legal correlates, so $C^+_{ba}(Q) \subseteq P$ can be viewed as the propositions that a has, in w, a duty towards b to operate on, and $C^-_{ba}(Q) \subseteq P$ are the actions for which a in Q has a duty towards b to forbear.

For a system of claims tables to be consistent, there should be **no conflict of duties**:

$$C^+_{ba}(Q) \cap C^-_{ca}(Q) = \emptyset \text{ for all } Q \subseteq P, \text{ and for all agents } a, b, c.$$

This says that if a has a duty towards b to act on p in some situation Q, then it cannot be the case in the same situation that a has a duty towards an agent c to forbear on p, and vice versa.

Conflicts of duties can arise when an agent is playing different roles, say as a family member and as an employee in a demanding profession. Such situations are outside the scope of this paper (but see [HLMT11, HLT11]).

We will assume throughout the paper that abilities of agents may be different in different worlds, but that they depend on the state of affairs of the world (as given by the valuation) only. Similarly, claims (duties) may be different in different situations. It is because of this that issues of *knowledge and ignorance of the facts* are crucial in moral and legal reasoning.

Situations can occur where an agent is legally obliged to act, but unable to perform the required action: think of a legal obligation to pay off a debt for an agent who is broke.

Legal Powers Legal powers, in our simple framework, are the powers change the C^+_{ab} and C^-_{ab} relations. Change of legal relations occur with every act of buying and selling. For simplicity, we will assume that all agents can give up claims and impose new duties, but an agent a is unable to change the claims that another agent b has on a.

4 DynaLex for Legal Specification

Legal argument between parties involves disagreement about the claims of the parties regarding what the parties are allowed or forbidden to do. There is a *level of description* and a level of *action specification*, and these two levels interact. So we define formulas ϕ and actions α. If an atomic formula mentions two agents a and b, then it is always assumed that a and b are different agents.

As we have seen, the primitive actions for an agent a are:

- Making a basic proposition p true, or making a basic proposition p false. We use (a, Q) for a swapping the values of all $p \in Q$.
- Informing an agent b of the value of p (true or false, as the case may be). We use $(a, I_b p)$ for this.
- Changing a claim against another agent to act on p. We will use $(a, C_b^+ p)$ for an action of a of changing the claim to act against b with respect to p. We can also view this as changing the duties to act of b towards a regarding p. A constraint on this change is that it does not conflict with a duty of b not to act on p.
- Changing a claim against another agent to forbear on p. We will use $(a, C_b^- p)$ for an action of a of changing the claim to forbear against b with respect to p. We can also view this as changing the duties to forbear of b regarding p. A constraint on this change is that it does not conflict with a duty of b to act on p.

It makes sense for several agents to perform their fact-changing actions in parallel. We will allow for simultaneous actions σ, in the style of lists of bindings to model parallel substitution. We will assume that each basic proposition p only has one assignment or power transfer in a substitution σ.

Definition 1 (Dynalex Language $\mathcal{L}$)**.** *Let a, b range over a set of agents I, p over a set of propositions P, and Q over subsets of P.*

$$
\begin{aligned}
\pi &::= p \mid A_a p \mid C^+_{ab} p \mid C^-_{ab} p \\
\phi &::= \pi \mid \neg\phi \mid \phi \wedge \phi \mid \Box_a \phi \mid [\alpha]\phi \\
A &::= (a, Q) \mid (a, C^+_b p) \mid (a, C^-_b p) \\
\sigma &::= [A, \ldots, A] \\
\alpha &::= \sigma \mid (a, I_b \pi) \mid ?\phi \mid \alpha;\alpha \mid \alpha \cup \alpha \mid \alpha^*
\end{aligned}
$$

We call σ a *parallel action* or a *substitution*, and we use ϵ for the empty substitution $[]$. We will assume that in a parallel action σ, a occurs in a pair $(a, \_)$ at most once, and moreover, if $(a, Q), (b, R) \in \sigma$ then $Q \cap R = \emptyset$. This guarantees that for every fact-changing action there is a unique responsible agent.

We assume the usual definitions of $\top$, $\bot$, $\vee, \to, \leftrightarrow$ and of the duals $\Diamond_a$ of $\Box_a$ and $\langle\alpha\rangle$ of $[\alpha]$. In addition we define $D^+_{ab}p$ as $C^+_{ba}p$, $D^-_{ab}p$ as $C^-_{ba}p$.

Models M for $\mathcal{L}$ will involve not only worlds, epistemic accessibilities and valuations, but also ability and claims tables. Interpretation of $A_a p$ uses the ability table. Interpretation of $C^+_{ab}p$ and $C^-_{ab}p$ uses the claim table. Interpretation of basic actions (a, Q) uses the ability table and may change the valuation, and interpretation of basic actions $(a, C^+_b p)$ and $(a, C^-_b p)$ changes the claims table.

Here is the definition of DynaLex models that allows us to interpret the Dynalex language $\mathcal{L}$.

Definition 2 (DynaLex Model)**.** *A DynaLex model M for a set of propositions P and a set of agents I is a tuple $M = (W, R, V, A, C^+, C^-)$ where*

- *W is a non-empty set of worlds,*
- *$R : I \to \mathcal{P}(W^2)$ is a function that maps each agent a to an equivalence relation $R(a) = \sim_a$ on W,*
- *$V : W \to \mathcal{P}(P)$ is a valuation function,*
- *$A : I \to \mathcal{P}(P) \to \mathcal{P}(P)$ is an ability table, listing for each agent and each valuation Q the propositions that the agent is able to change (from true to false if the proposition is in Q, from false to true otherwise).*
- *$C^+ : I^2 \to \mathcal{P}(P) \to \mathcal{P}(P)$ and $C^- : I^2 \to \mathcal{P}(P) \to \mathcal{P}(P)$ are the claims tables. $C^+_{ab}Q$ lists the propositions for which a claims that b should change their values, and $C^-_{ab}Q$ lists the propositions for which a claims that b should leave the values unaffected.*

Formally, the claims table depends on the valuation of a world w rather than on w itself. Whenever we use $C^s_{ab}(w)$, this is an abbreviation of $C^s_{ab}(V(w))$, where V is the relevant valuation.

We are now in a position to define truth of Dynalex formulas in DynaLex models.

First, we interpret each parallel action σ as a substitution on atoms of $\mathcal{L}$.

Definition 3 (Substitution for $\mathcal{L}$ atoms)**.**

$$
\begin{array}{rcl}
\sigma(p) & = & \begin{cases} \neg p \leftrightarrow A_a p & \textit{if } (a, Q) \in \sigma \textit{ for some } Q \textit{ with } p \in Q \\ p & \textit{otherwise} \end{cases} \\
\sigma(A_a p) & = & A_a p \\
\sigma(C^+_{ab}p) & = & \begin{cases} C^+_{ab}p \leftrightarrow \bigvee_{c \in I} C^-_{cb}p & \textit{if } (a, C^+_b p) \in \sigma \\ C^+_{ab}p & \textit{otherwise} \end{cases} \\
\sigma(C^-_{ab}p) & = & \begin{cases} C^-_{ab}p \leftrightarrow \bigvee_{c \in I} C^+_{cb}p & \textit{if } (a, C^-_b p) \in \sigma \\ C^-_{ab}p & \textit{otherwise} \end{cases}
\end{array}
$$

Explanation of the clauses for $\sigma(C^+_{ab}p)$ and $\sigma(C^-_{ab}p)$: these clauses take the constraint into account that newly imposed duties should not conflict with existing duties.

Using this, we can define the result of executing σ on a DynaLex model. The following definition is by mutual recursion with Definitions 5 and 6.

Definition 4 (Model Substitution). *Let $M = (W, R, V, A, C^+, C^-)$ be a DynaLex model and let σ be a $\mathcal{L}$ substitution. Then V^σ is the valuation given by:*

$$V^\sigma(w) = \{p \in P \mid M, w \models \sigma(p)\},$$

where $M, w \models \phi$ is given by Definition 5. $C^{+\sigma}$ is the claims table given by:

$$C^{+\sigma}_{ab}(V(w)) = \{p \in P \mid M, w \models \sigma(C^+_{ab}p)\}.$$

$C^{-\sigma}$ is the claims table given by:

$$C^{-\sigma}_{ab}(V(w)) = \{p \in P \mid M, w \models \sigma(C^-_{ab}p)\}.$$

M^σ is given by $(W, R, V^\sigma, A, C^{+\sigma}, C^{-\sigma})$.

We are ready for the definitions of truth and action interpretation, for formulas and action of expressions of the DynaLex language.

Definition 5 (Truth). *Let $M = (W, V, A, C^+, C^-)$ be a DynaLex model, let $w \in W$. Then:*

$$\begin{array}{rll}
M, w \models p & \textit{iff} & p \in V(w) \\
M, w \models A_a p & \textit{iff} & p \in A_a(V(w)) \\
M, w \models C^+_{ab} p & \textit{iff} & p \in C^+_{ab}(V(w)) \\
M, w \models C^-_{ab} p & \textit{iff} & p \in C^-_{ab}(V(w)) \\
M, w \models \neg\phi & \textit{iff} & \textit{not } M, w \models \phi \\
M, w \models \phi_1 \wedge \phi_2 & \textit{iff} & M, w \models \phi_1 \textit{ and } M, w \models \phi_2 \\
M, w \models \Box_a \phi & \textit{iff} & \textit{for all } v \textit{ with } (w, v) \in \sim_a\text{: } M, v \models \phi \\
M, w \models [\alpha]\phi & \textit{iff} & \textit{for all } (M', w') \textit{ with } (M, w)[\![\alpha]\!](M', w') : M', w' \models \phi \\
 & & \textit{where } [\![\alpha]\!] \textit{ is as given in Def 6.}
\end{array}$$

Definition 6 (Action Interpretation)**.** *This uses Def 4 for M^σ.*

$$
\begin{array}{rcl}
(M,w)[\![\sigma]\!](M',w') & \textit{iff} & M' = M^\sigma \textit{ and } w = w' \\
(M,w)[\![(a, I_b\pi)]\!](M',w') & \textit{iff} & M, w \models \Box_a\pi \vee \Box_a\neg\pi,\ w = w' \\
 & & \textit{and } M' = (W', R', V', A', C^{+'}, C^{-'}) \\
 & & \textit{where } W' = \{v \in W \mid M, v \models \Box_a\pi \vee \Box_a\neg\pi\}, \\
 & & R' \textit{ is given by } \sim'_c = \sim_c \cap W'^2 \textit{ for } c \neq b, \\
 & & \sim'_b = \sim_b \cap W'^2 \cap\ \{(u,v) \mid M, u \models \pi \textit{ iff } M, v \models \pi\}, \\
 & & A' = A, C^{+'} = C^+, C^{-'} = C^- \\
(M,w)[\![\ ?\phi]\!](M',w') & \textit{iff} & (M,w) = (M',w') \textit{ and } M, w \models \phi \\
(M,w)[\![\ \alpha_1;\alpha_2]\!](M',w') & \textit{iff} & \textit{for some } N, v: \\
 & & (M,w)[\![\ \alpha_1]\!](N,v) \textit{ and } (N,v)[\![\ \alpha_2]\!](M',w') \\
(M,w)[\![\ \alpha_1 \cup \alpha_2]\!](M',w') & \textit{iff} & (M,w)[\![\ \alpha_1]\!](M',w') \textit{ or } (M,w)[\![\ \alpha_2]\!](M',w') \\
(M,w)[\![\ \alpha^*]\!](M',w') & \textit{iff} & \textit{there are } (N_0,w_0),\ldots,(N_n,w_n) \\
 & & \textit{with } (M,w) = (N_0,w_0), (M',w') = (N_n,w_n), \\
 & & \textit{and } (N_i,w_i)[\![\ \alpha]\!](N_{i+1},w_{i+1}) \\
 & & \textit{for all } i \textit{ with } 0 \leq i < n.
\end{array}
$$

5 Axioms

Crucial are the two "no conflict of duties" axioms:

No CoD for action	$C^+_{ba}p \to \neg \bigvee_{c\in I} C^-_{ca}p$
No CoD for restraint	$C^-_{ba}p \to \neg \bigvee_{c\in I} C^+_{ca}p$

S5 axioms for knowledge:

Normality	$\Box_a(\phi \to \psi) \to \Box_a\phi \to \Box_a\psi$
Truth	$\Box_a\phi \to \phi$
Pos Introspection	$\Box_a\phi \to \Box_a\Box_a\phi$
Neg Introspection	$\neg\Box_a\phi \to \Box_a\neg\Box_a\phi$

Axioms for substitution actions: derive from the definition of M^σ, in particular the definition of $\sigma(\mathrm{At})$.

$$
\begin{array}{rcl}
[\sigma]\pi & \leftrightarrow & \sigma(\pi) \text{ (Def 3)} \\
[\sigma](\neg\phi) & \leftrightarrow & \neg([\sigma]\phi) \\
[\sigma](\phi \wedge \psi) & \leftrightarrow & [\sigma]\phi \wedge [\sigma]\psi \\
[\sigma](\Box_a\phi) & \leftrightarrow & \Box_a[\sigma]\phi \\
[\sigma]([\alpha]\phi) & \leftrightarrow & [\sigma][\alpha]\phi.
\end{array}
$$

Reduction axioms for informing actions: use the method of LCC [BvEK06]:

$$
\begin{aligned}
[(a, I_b\pi)]\pi' &\leftrightarrow (\Box_a\pi \vee \Box_a\neg\pi) \rightarrow \pi' \\
[(a, I_b\pi)]\neg\phi &\leftrightarrow (\Box_a\pi \vee \Box_a\neg\pi) \rightarrow \neg[(a, I_b\pi)]\phi \\
[(a, I_b\pi)](\phi \wedge \psi) &\leftrightarrow [(a, I_b\pi)]\phi \wedge [(a, I_b\pi)]\psi \\
[(a, I_b\pi)]\Box_a\phi &\leftrightarrow (\Box_a\pi \vee \Box_a\neg\pi) \rightarrow \Box_a[(a, I_b\pi)]\phi \\
[(a, I_b\pi)]\Box_b\phi &\leftrightarrow (\Box_a\pi \rightarrow \Box_b(\pi \wedge [(a, I_b\pi)]\phi)) \wedge (\Box_a\neg\pi \rightarrow \Box_b(\neg\pi \wedge [(a, I_b\pi)]\phi)) \\
[(a, I_b\pi)]\Box_c\phi &\leftrightarrow (\Box_a\pi \vee \Box_a\neg\pi) \rightarrow \Box_c[(a, I_b\pi)]\phi
\end{aligned}
$$

Axioms for the regular operations: use the PDL axioms ([Seg82, KP81]):

Normality	$\vdash [\alpha](\phi \rightarrow \psi) \rightarrow ([\alpha]\phi \rightarrow [\alpha]\psi)$
Test	$\vdash [?\phi]\psi \leftrightarrow (\phi \rightarrow \psi)$
Sequence	$\vdash [\alpha_1; \alpha_2]\phi \leftrightarrow [\alpha_1][\alpha_2]\phi$
Choice	$\vdash [\alpha_1 \cup \alpha_2]\phi \leftrightarrow ([\alpha_1]\phi \wedge [\alpha_2]\phi)$
Mix	$\vdash [\alpha^*]\phi \leftrightarrow (\phi \wedge [\alpha][\alpha^*]\phi)$
Induction	$\vdash (\phi \wedge [\alpha^*](\phi \rightarrow [\alpha]\phi)) \rightarrow [\alpha^*]\phi$

Inference rules:

- Modus Ponens: from $\vdash \phi \rightarrow \psi$ and $\vdash \phi$ infer $\vdash \psi$.
- Knowledge Generalization: from $\vdash \phi$ infer $\vdash \Box_a\phi$.
- Action Generalization: from $\vdash \phi$ infer $\vdash [\alpha]\phi$.

These axioms and inference rules are sound, and we conjecture that they are also enough for completeness.

6 Obligation, Permission, Acceptability

Obligation, permission and legal acceptability are properties of actions, not of propositions. These properties can be defined by recursion on action structure.

Knowledge Based Obligation to Act We define $Q_a(\alpha)$, for "Agent a has a knowledge-based legal obligation to perform action α". We restrict attention to fact-changing actions with a as agent.

$$
\begin{aligned}
Q_a(a, Q) &\leftrightarrow \bigwedge_{q \in Q}((\Box_a q \vee \Box_a\neg q) \wedge \bigvee_{b \in I} C^+_{ba} q) \\
Q_a(\alpha; \beta) &\leftrightarrow Q_a(\alpha) \wedge [\alpha]Q_a(\beta) \\
Q_a(\alpha \cup \beta) &\leftrightarrow Q_a(\alpha) \wedge Q_a(\beta) \\
Q_a(\alpha^*) &\leftrightarrow Q_a(\alpha) \wedge [\alpha]Q_a(\alpha^*)
\end{aligned}
$$

Obligation to Act Per Se We define $O_a(\alpha)$, for "Agent a has a legal obligation to perform action α". We restrict attention to fact-changing actions with a as agent.

$$\begin{array}{rcl} O_a(a, Q) & \leftrightarrow & \bigwedge_{q \in Q} \bigvee_{b \in I} C^{+}_{ba} q \\ O_a(\alpha; \beta) & \leftrightarrow & O_a(\alpha) \wedge [\alpha] O_a(\beta) \\ O_a(\alpha \cup \beta) & \leftrightarrow & O_a(\alpha) \wedge O_a(\beta) \\ O_a(\alpha^*) & \leftrightarrow & O_a(\alpha) \wedge [\alpha] O_a(\alpha^*) \end{array}$$

Permission $P_a(\alpha)$ for "Agent a has legal permission to perform action α". We restrict attention to fact-changing actions with a as agent.

$$\begin{array}{rcl} P_a(a, Q) & \leftrightarrow & \bigwedge_{q \in Q} \bigwedge_{b \in I} \neg C^{-}_{ba} q \\ P_a(\alpha; \beta) & \leftrightarrow & P_a(\alpha) \wedge [\alpha] P_a(\beta) \\ P_a(\alpha \cup \beta) & \leftrightarrow & P_a(\alpha) \wedge P_a(\beta) \\ P_a(\alpha^*) & \leftrightarrow & P_a(\alpha) \wedge [\alpha] P_a(\alpha^*) \end{array}$$

Legal Acceptability $L_a(\alpha)$ for "The legal rights of agent a are not infringed by action α". This time, we focus on fact-changing actions by agents different from a.

$$\begin{array}{rcl} L_a(b, Q) & \leftrightarrow & \bigwedge_{q \in Q} \neg C^{-}_{ba} q \\ L_a(\alpha; \beta) & \leftrightarrow & L_a(\alpha) \wedge [\alpha] L_a(\beta) \\ L_a(\alpha \cup \beta) & \leftrightarrow & L_a(\alpha) \wedge L_a(\beta) \\ L_a(\alpha^*) & \leftrightarrow & L_a(\alpha) \wedge [\alpha] L_a(\alpha^*) \end{array}$$

Next, one can prove by induction:

- knowledge based obligation implies obligation;
- obligation implies permittedness;
- permittedness implies legal acceptability for any agent.

We can now also make precise when an agent a has a (legal or moral) duty to inform another agent b of a fact. This is the case precisely when this information has the effect of turning an obligation of b into a knowledge based obligation.

7 Further Work

What is still lacking in our set-up is *incentives* for agents to act. One way of incorporating these is by equipping each agent with a *goal formula*, as in the Boolean games defined in [HvdHMW01, GHW15]. Goal formulas divide the space of possibilities between acceptable and unacceptable for an agent. An action σ is better that σ' for agent a with goal formula ϕ iff it holds that $[\sigma']\phi$ implies $[\sigma]\phi$. Thus, goal formulas can be viewed as defining a crude utility function U, with $U_a(w) = 1$ for worlds w that do satisfy the goal formula of a, and $U_a(w) = 0$ for worlds w that do not.

Alternatively, one could adopt more elaborate payoff functions U_a. This would lead to an orthogonal system of utilitarian arguments, where operations on propositions p have payoffs for the various agents. Once there is a payoff table, one can define notions of *harm of an action to society* (harmful actions are actions that decrease the societal payoff, that is: the sum of the individual payoffs).

A payoff function or utility function U could take the form of a map from agents to functions from valuations to values in the real numbers. Using U_a for the payoff function for agent a, the payoff for agent a of an action of changing the value of p could be computed as the difference

$$U_a(V') - U_a(V)$$

where V is the old valuation and V' the new valuation (after the update of the value of p).

The societal payoff of an action that changes V to V' can be computed as

$$\sum_{a \in I} U_a(V') - \sum_{a \in I} U_a(V).$$

How can we incorporate this? Is there an obvious way to connect up between utilitarian concepts and deontological concepts?

Actions may have *benefits* for one agent and *costs* for another agent, *benefits* for society or *costs* for society. How do these affect the legal relations? If we have a payoff function, then we can formalize the notion of agent a having an *interest* to take legal action against agent b. The fact that someone *infringes on my rights* is one element in this. The other element is the fact that someone *causes harm to me* by his action, in the very precise sense of decreasing my individual payoff.

There is a second level where utilities come in. Society imposes a punishment on those who transgress the law, and this punishment can be viewed as a negative utility. A classic on the theme of just punishment is [Bec64]. A society where punishment is in exact proportion to the gravity of a legal offense would be one where actions that are legally obliged have higher utility for the agent than actions that are not. But this is not how society is organized. Many crimes go unpunished, and for a sophisticated treatment one would not only have to take severity of punishment into account but also probability of getting caught in a crime. See [Eij15] for some hints of how this might be done.

In any case, the utilitarian element of individual action deserves a role in our set-up. We would like to model the fact that agent a has an incentive to take legal action against agent b when the following conditions are fulfilled (concerning operation on some p):

- Agent a has a claim against b that b should forbear operating on p.
- Agent b operates on p.
- The result of b's action is that a suffers a loss in utility (or: the action of b has caused individual harm to a).

Utility transfers from valuations to basic actions as follows. Suppose $p \notin Q \subseteq P$, that is, Q is a valuation that makes p false. Suppose $U_a(Q) = x$ and $U_a(Q \cup \{p\} = y$. Then the a-value of the action of operating on p in a world with valuation Q is $y - x$, and the a-value of the action p in a world with valuation $Q \cup \{p\}$ is $x - y$. Note that the skip action always has utility 0, and that a basic change action may have negative utility. In cases where the a-value of acting on p is negative, it is better for a to skip than to change the value of p.

Actions that *transfer utility* from agent a to agent b (actions where the decrement in utitility for a matches the increment in utility for b, and the utilities for other agents do not change) can be viewed as abstract versions of *acts of payment* with donor a and beneficiary b.

In many cases, rights can be bought and sold. When I buy a house, I obtain property rights and when I sell it again I lose these rights. The example illustrates that legal rights also have utilities, and that transfer of legal rights involves changes in the utilities for the agents involved in the transfer. In general, a change in legal rights of agent a against b will be reflected in a change in the utilities of these two agents.

Duty to forbear in the above is modelled as duty not to act. This is perhaps too narrow. Duty to forbear could also mean: duty not to prevent another agent from acting in certain ways. In a future extension, we plan to model actions that change the ability table, so that $(a, A_b q)$ is an action that flips the abilities of b with respect to q: in case b can act on q, b a makes b lose that ability after $(a, A_b q)$; in case b cannot act on q, a makes b acquire that ability after $(a, A_b q)$. Now a duty of a towards b to forbear on q in a situation where b has the ability to act on q could mean that $(a, A_b q)$ is forbidden: the action $(a, A_b q)$ can be viewed as a preventing b from acting on q. See [HLMT11] for an account of such ability-changing actions.

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