

Control of Piecewise-Affine Hybrid Systems - Extended abstract^{*}

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Abstract: The character of this paper is an extended abstract of the approach developed by the authors for control of piecewise-affine hybrid systems. Control of hybrid systems is a wide ranging research topic with many theoretical problems. The authors have therefore decided early on to restrict attention to piecewise-affine hybrid systems on polytopes. The approach developed includes for affine systems on polytopes sufficient conditions for the problems (1) leaving the polytope in finite time through a prespecified set of facets; (2) remaining inside a particular polytope forever; and (3) converging to a fixed state inside a particular polytope. For piecewise-affine hybrid system on polytopes a procedure for reachability and control synthesis is proposed consisting of (1) for each discrete state the computation of all possible exit facets and exit events; (2) abstraction of the continuous dynamics to an automaton; and (3) the search of a path in the automaton from an initial discrete state to a target discrete state. Open research issues for control of hybrid systems are stated.

Keywords: Piecewise-affine hybrid system. Control system synthesis. Control-to-facet.

1. SUMMARY

The purpose of this paper is to present a summary of control synthesis of piecewise-affine hybrid systems on polytopes developed by the authors and to put the approach in the perspective of control theory of hybrid systems.

1.1 Need for control of hybrid systems

Control engineering is since about the 1970's faced with the problem that on-line control algorithms are implemented by computers and that therefore (1) the continuous dynamics of the technological systems and (2) the discrete dynamics of the computer have to be treated jointly. The problem existed earlier but with the advance of computer and communication technology of the late twentieth century it became more pressing and it received attention at the academical level. Systems that combine continuous and discrete dynamics are called *hybrid systems* precisely because of this combination.

1.2 Approaches to control of hybrid systems

Control and analysis of hybrid system has received considerable attention from researchers in control engineering, control theory, computer science, and computer engineering. There are conference series for the topic including the *Analysis and Design of Hybrid Systems* (ADHS) and *Hybrid Systems - Computation and Control* (HSCC). Major

conference series as the *Conference on Decision and Control* and the *European Conference on Control* have in the recent past included in their scientific program sessions on control of hybrid systems. Computer science conferences have similarly been presenting lectures on verification of hybrid systems.

Control design and control synthesis approaches of hybrid systems include: (1) The discrete-time piecewise-affine hybrid systems in which control design is reduced to computing the solution to a mixed-integer nonlinear-programming problem. This approach was developed by A. Bemporad, M. Morari, and their coworkers, see for example Bemporad and Morari (1999). (2) A game theoretic approach to control of continuous-time hybrid systems. This approach was developed by S. Sastry and coworkers see Tomlin et al. (2000); Lygeros et al. (1995).

There is quite a set of papers and books on stability analysis of hybrid systems, for example see Liberzon (2003); Matveev and Savkin (2000). Control design of linear switched systems is treated in Matveev and Savkin (2000); Sun and Ge (2005). Finite-state approximation of hybrid systems may be found in the references Niinomi et al. (1995); Förstnera et al. (2002); Nerode and Kohn (1992); Raisch and O'Young (1995); Moor et al. (2002); Raisch (2000); Alur et al. (2000); Tabuada and Pappas (2005); Belta et al. (2005).

In addition to research on control synthesis of hybrid systems, computer scientists have investigated problems of reachability of hybrid systems and explored the boundary of decidable and undecidable problems, see Henzinger et al. (1998).

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1.3 The current state of control theory of hybrid systems

It seems fair to state that there is still a considerable gap between what is needed in control engineering by companies and by academic research groups in control engineering on one hand, and what control theory can offer by the way of control synthesis methods. Control synthesis of *discrete-time* piecewise affine hybrid systems developed by A. Bemporad and M. Morari is a promising approach in the opinion of the authors. But the outcome is a control law of enormous complexity. Reducing the complexity of the control law is a nontrivial task. The game theoretic approach has been shown to be effective for a small number of academic examples but does not promise to be useful for a variety of control engineering problems. Another issue of control of hybrid systems is that it is inherently discontinuous, that therefore the closed-loop control systems are not as robust as is needed by control engineering, and therefore may require more expensive tuning and maintenance.

The basic underlying problem is that control theory of hybrid systems has not made serious progress on the complexity issues of control synthesis of hybrid systems. A hybrid system is a particular class of nonlinear systems. It is well known from other areas of mathematics that analysis and control of nonlinear systems may be undecidable and, if decidable, can be of extremely high complexity. Control theory of hybrid systems has therefore to take account of the computability and of the complexity issues at the fundamentals of the problem. One way to proceed is to restrict attention to a subclass of hybrid systems and to control problems of which the solution can be shown to be decidable and of manageable complexity. The approach proposed by Eduardo D. Sontag for control of piecewise-linear systems on polytopes was intended to make the computability and the complexity manageable, see Sontag (1981, 1982). But even that class was too large, see Sontag (1995); Vidal et al. (2003); Collins and van Schuppen (2004). Therefore the authors of this paper have decided to restrict the attention further as described below.

1.4 The control-to-facet approach

The approach developed by the authors in regard to the computability and the complexity issues is in restricting attention to continuous-time piecewise-affine hybrid systems on polytopes and to control-to-facet problems. Many control engineering systems can be modeled by a system in this class. For arbitrary initial states the reachability problem for piecewise-affine hybrid systems is undecidable. For particular initial states it may be decidable. The control-to-facet problem applied to particular examples seems to result in control problems of manageable complexity. The class of systems selected is inspired by the paper of E.D. Sontag in Sontag (1981, 1982). In hindsight, the decidability has been achieved. The complexity is manageable but further research is required for this issue.

Attention will therefore be restricted to continuous-time piecewise-affine hybrid systems on polytopes, with as particular cases such systems on simplices. The class of multi-affine systems on rectangles will also be mentioned. The main control problem treated is controllability of the

systems mentioned above. The control synthesis method described in this paper is the control-to-facet approach, the abstraction of continuous dynamics to discrete dynamics, and the control synthesis of hybrid systems at the discrete level.

1.5 Information on the approach

The approach described in this paper was developed at the research institute Centrum Wiskunde en Informatica (CWI) in Amsterdam, The Netherlands by the authors. The approach was inspired by an example of a conveyor belt, see van Schuppen (2000), provided by the research group of J.E. Rooda (Eindhoven University of Technology). The approach to control of hybrid systems was first described in the paper van Schuppen (1998), where the terms arrival and departure sets were used. Later this was developed for piecewise-affine systems on polytopes and, in particular, simplices. See the references, Habets and van Schuppen (2001a,b, 2002, 2004, 2005). The main reference for this extended abstract is the journal paper Habets et al. (2006a). and the conference paper for control-to-facet with partial observations, Collins et al. (2008). Computer programs to compute affine control laws are described in the paper Collins et al. (2006). The approach of control of piecewise-affine hybrid systems has been applied in several case studies, see Balluchi et al. (2004); Hodrus et al. (2005).

Luc Habets of the CWI team has developed with Calin Belta and co-workers the approach to control of multi-affine systems on rectangles. This class of systems is inspired by models of genetic networks as used in systems biology. Not all models of biochemical reaction networks are modelled by this class of systems but it is a relevant subclass. It is remarkable that the results of piecewise-affine systems on polytopes can be generalized to multi-affine systems on rectangles, a particular combination of algebra and geometry seems to be required for this. References on this approach are Belta et al. (2002, 2004); Belta and Habets (2004, 2006); Habets et al. (2006b).

Hidde de Jong and Jean-Luc Gouzé have developed an abstraction method for affine systems on rectangles for analysis of genetic networks, see de Jong et al. (2004, 2003). That approach does not address control and the underlying approximation procedure is not related to the control approach developed by the authors of this paper. A Japanese group has developed controllability of piecewise-affine hybrid systems for genetic networks inspired by the approach of Calin Belta and coworkers, see Azuma et al. (2008).

2. FURTHER RESEARCH ISSUES

This section provides information on what the authors consider to be important research issues for control and system theory of hybrid systems.

Control engineering

- (1) Still more experience must be gained with concrete examples of control engineering problems for existing control synthesis and control design approaches of control of hybrid systems.

- (2) A software package for control of piecewise-affine hybrid systems is needed. The paper Collins et al. (2006) describes the author's first step in this direction.

Control theory

- (1) Research issues of control of affine systems on polytopes requiring attention include optimal control of such systems, the concept of controllability for particular control objectives, and robustness properties of closed-loop systems.
- (2) Research issues of control of piecewise-affine systems on polytopes requiring attention include extensions from systems on simplices to systems on rectangles and on arbitrary polytopes; the control at the discrete-event level; the computations of the control law; and the complexity of the control law.
- (3) The control-to-facet problem of an affine system on a polytope with partial observations, both static output feedback and dynamic output feedback has been developed by the authors. A first paper is Collins et al. (2008) and a second paper is in preparation. Further research is required for the extension to systems on rectangles and on polytopes.
- (4) Control of piecewise-multi-affine hybrid systems on rectangles as developed by Luc Habets and Calin Belta.

System theory

- (1) Approximation procedures for approximating a nonlinear system by a piecewise-affine hybrid system requires attention. In particular, approximation procedures, approximation bounds, and optimality of approximations have to be investigated.
- (2) Realization theory for piecewise-affine systems needs to be developed in order to obtain model reduction and system identification methods, along with concepts of observability and controllability. Realization theory has been developed for linear and bilinear hybrid and switched systems without guards, Petreczky (2006). Preliminary results for discrete-time piecewise-affine and semi-algebraic hybrid systems are also available Petreczky (2006) and Petreczky and Vidal (2008) However, the case of general piecewise-affine system remains open; the presence of autonomous switching seems to be the major bottleneck.
- (3) Based on realization theory, model reduction and system identification methods for hybrid system need to be developed. The former is important because of the complexity of both the system models arising in practice and the computation complexity of control synthesis algorithms. System identification could be useful for applications in systems biology, computer vision and fault detection. There are also examples of engineering problem, where one needs to identify an unknown hybrid plant. A major challenge is to come up with model reduction and system identification methods which are consistent with the needs of control synthesis methods for hybrid systems. There are several results on system identification and model reduction of hybrid systems, see the work of Juloski, Vidal, Ferrari-Trecate, Paoletto, and the tutorial Juloski et al. (2005).

Computability of nonlinear hybrid systems

- (1) Basic research is required into the computability and the complexity issues of control of hybrid systems. It will be useful to know how far the boundary of systems and of control problems can be pushed when the constraints are computability and manageable complexity of these problems. The combination of algebra, geometry, and analysis plays a role in this, as is clear from the contents of this paper.
- (2) Computability of control synthesis of hybrid systems is related to that of nonlinear systems. Computability of subsets of the real numbers has developed by Klaus Weihrauch, see Weihrauch (1999), and coworkers. Pieter Collins and his coworkers are developing this approach to computability for hybrid system and this may well provide further results on what is computable and of low complexity for control of hybrid systems. In regard to complexity, facet blocking can be solved by linear programming hence is of polynomial complexity.

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