
Chapter 1. The Language Specification Formalism ASF+SDF

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Warning

This document is work in progress. It is in transition between The Meta-Environment V1.5 and V2.0. It especially needs adaptations in the sections on traversal functions, lexical constructor functions and rewriting layout and source code comments.

Introduction to ASF+SDF

If you want to

- create an Interactive Development Environment (IDE) for some existing or new programming language,
- design and implement your own domain-specific language,

- analyze existing source code, or
- transform existing source code,

then ASF+SDF may be the right technology to use. This document is the reference manual for ASF+SDF. It explains everything about ASF+SDF as a reference for experienced users. Other more introductory material is also available. ASF+SDF is a language that depends heavily on the SDF formalism. We refer to the SDF manual for details. ASF is the actual language that is described here. The complete language is called ASF+SDF because ASF does not exist without SDF.

ASF+SDF programs can be used in connection with The Meta-Environment tools to obtain an IDE for a programming language. Other documentation about The Meta-Environment will tell you more about this.

Why use ASF+SDF?

ASF+SDF is intended for the high-level, modular, description of the analysis and transformation of computer-based formal languages. It is the result of the marriage of two formalisms ASF (Algebraic Specification Formalism) and SDF (Syntax Definition Formalism). ASF+SDF can be used to implement compilers, refactoring tools, reverse engineering tools, re-engineering tools, etc. Basically all meta programs are within the scope of ASF+SDF. It is specifically designed to make the construction of meta programs more easy.

Key features of ASF:

- Concrete syntax for source code patterns, makes writing programs for complex programming languages easy.
- Automated parse tree traversal, keeps meta programs concise.
- Static type system, makes sure you can not generate code that can not be parsed (a.k.a. syntax safety).
- List matching, so any element can be accessed in a list without traversal.
- High fidelity. Access to layout and source code comments in parse trees, to analyze them, generate them, or to simply keep them.

How to use ASF+SDF?

There are two ways to write ASF+SDF specifications and to execute them:

- By far the simplest way is to use the ASF+SDF Meta-Environment that enables the interactive editing and execution of ASF programs. It also supports the compilation of specifications and the generation of all necessary information (including parse tables and compiled specifications) to execute specifications from the command line.
- Command line tools including **asfe** (for interpreting ASF+SDF specifications) and **asfc** (for compiling ASF+SDF specifications to C) enable the execution of ASF+SDF specifications from the command line. The command line tools for SDF are also needed: i.e. **sglr** (to parse a file), **addPosinfo** (to annotate a parse tree with location info) and **unparsePT** (to yield a program text).

Learning more

Warning

Add links to the mentioned documents.

- For a detailed description of SDF see The Syntax Definition Formalism SDF.
- In Term Rewriting the notions *rewrite rules* and *term rewriting* are explained that play an important role when executing ASF+SDF specifications.
- An explanation of error messages of ASF
- An explanation of error messages of SDF

The connection between ASF and SDF

SDF is used to describe syntax of (programming) languages using context-free production rules. The parsers generated by SDF produce parse trees. The nodes of the parse trees are the productions of the SDF specification. The leaves of the parse trees are the characters of the input text. ASF programs take parse trees as input and produce parse trees as output. A node in the parse tree is a "function" for ASF. All ASF does is replace functions by other functions, generating a new parse tree that can be unparsed to obtain an output text.

ASF stands for Algebraic Specification Formalism. This paragraph describes ASF in terms of algebras. ASF can be used to define many-sorted algebra's. Many sorted algebra's have so-called signatures to describe the shape of algebraic terms. In ASF we use SDF to describe the signature of terms. An SDF production is an ASF term constructor. A non-terminal in SDF is an algebraic sort in ASF. ASF specification are collections of equations. The algebraic equations define which terms are equal to which other terms. This is where the algebra stops, and term rewriting begins. In fact the equations are interpreted as rewrite rules. Each pattern on the left-hand side is searched in the input term, and replaced by the right-hand side.

The parse trees of SDF contain the full input program text and all SDF productions that have been used to parse it. ASF provides full access to all details of the parse tree, such that any analysis or transformation is possible.

Parse trees and abstract syntax trees are very complicated structures. In ASF you do not have to deal with this complexity. Parse tree patterns are expressed in the programming language that is analyzed or transformed. This is called "concrete syntax". ASF rewrites parse trees to other parse trees under the hood, but the programmer types in source code of the language analyzed or transformed. This manual contains many examples of this.

ASF+SDF uses the module mechanism provided by SDF. Each module M in an SDF specification is stored in a file $M.sdf$. To each module M one can simply add equations as needed by providing them in a file $M.asf$. The result is an ASF+SDF specification. We call $M.sdf$ the *SDF part* of module M and $M.asf$ the *ASF part* of M .

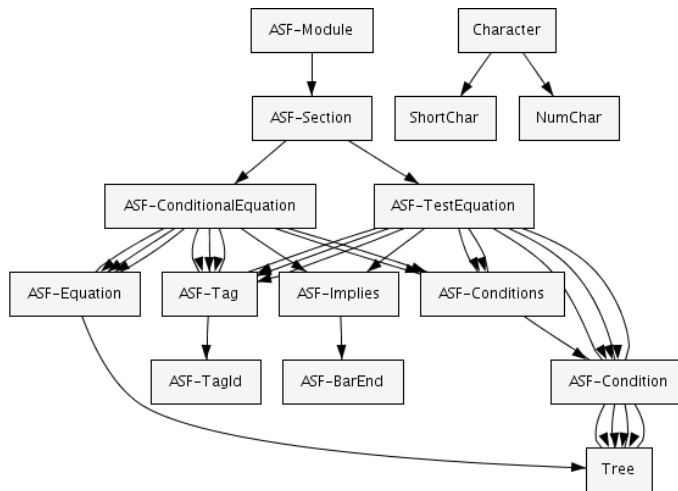
Note that some of the semantics of ASF is triggered by the attributes of productions in SDF. The `{bracket}` attribute is one example, another important example is the `{traversal(...)}` attribute.

The `variables` and `lexical variables` sections of SDF are important. In these sections you describe the syntax of meta variables. These are exactly the variables used in the concrete syntax patterns used in ASF to construct and deconstruct source code.

Overview

This document describes the syntax of ASF and its semantics. The syntax of ASF is structured as follows:

Figure 1.1. Syntactic structure of ASF



We will now describe the ingredients of the ASF part of modules. Each module contains any number of sections. There are equations sections and tests sections. The equations sections contain normal equations and conditional equations. The tests sections contain normal tests and conditional tests. The document is split into three parts. First we discuss the basic features of ASF+SDF, then we discuss the advanced features of ASF+SDF. Note that both the syntax and the name of the `Tree` non-terminal from the above picture is defined by the programmer in SDF. It will be different for every ASF+SDF program.

- Basic ASF+SDF: unconditional and conditional equations, default equations and test
- Advanced ASF+SDF: list matching, memo functions, lexical constructor functions, traversal functions, and dealing with layout and source code comments

We conclude the description of ASF+SDF with Historical notes and bibliographic background on ASF+SDF

Simple ASF+SDF

Equations are used to define the semantics of functions. Each equation deals with a particular case for the function. The case is identified by the pattern on the left-hand side of the equation. The syntax of a function is defined earlier in SDF, in ASF we provide a semantics for it by defining rewrite rules that are called equations.

- The equations section.
- Unconditional equations.
- Conditional equations.
- Executing equations.
- Examples.

The Equations Section

Equations are always contained in an equations section that has the following structure:

```
equations
  <Equation>*
```

In other words, the keyword `equations` followed by zero or more equations. These may be unconditional or conditional equations as described below.

Unconditional Equations

An unconditional equation has the form

```
[<TagId>] <Term> = <Term>
```

where `<TagId>` is the name of the equation. It is a sequence of letters, digits, and/or minus signs (-) starting with a letter or a digit. Next follows an equality between two `<Term>`s. A `<Term>` is any string described by the SDF part of the module in which the equation occurs. There are some restrictions on the terms in an equation:

- The terms on both sides of the equal sign are of the same sort.
- The term on the left-hand side is not a single variable.
- The variables that occur in the term on the right-hand side also occur in the term on the left-hand side of the equal sign.

It is assumed that the variables occurring in the equation are universally quantified. In other words, the equality holds for all possible values of the variables.

Conditional Equations

An unconditional equation is a special case of a *conditional equation*, i.e., an equality with one or more associated conditions (premises). The equality is sometimes called the *conclusion* of the conditional equation. In ASF+SDF a conditional equation can be written in three (syntactically different, but semantically equivalent) ways. Using `when`

```
[<TagId>] <Term1> = <Term2> when <Condition1>, <Condition2>, ...
```

or using an implication arrow `===>`

```
[<TagId>] <Condition1>, <Condition2>, ... ===> <Term1> = <Term2>
```

or using a horizontal bar

```
[<TagId>] <Condition1>, <Condition2>, ...
          =====
          <Term1> = <Term2>
```

where `<Condition1>`, `<Condition2>`, ... are conditions which may be of one of the following forms:

- `<Term1> := <Term2>`, a *match condition* that succeeds when the reduction of `<Term2>` matches `<Term1>`; variables in `<Term1>` get new values.
- `<Term1> !:= <Term2>`, a *non-match condition* that succeeds when the reduction of `<Term2>` does not match `<Term1>`.
- `<Term1> == <Term2>`, an *equality condition* (also known as *positive condition*) that succeeds when the reductions of `<Term1>` and `<Term2>` are syntactically identical.

- $\langle \text{Term}_1 \rangle \neq \langle \text{Term}_2 \rangle$, an *inequality condition* (also known as *negative condition*) that succeeds when the reductions of $\langle \text{Term}_1 \rangle$ and $\langle \text{Term}_2 \rangle$ are syntactically unequal.

The conditions of an equation are evaluated from left to right. Let, initially, Vars be the set of variables occurring in the left-hand side of the conclusion of the equation.

Match conditions are evaluated as follows. The left-hand side of a match condition must contain at least one *new* variable not in Vars . Reduce the right-hand side of the match condition to a normal form. The match condition succeeds if this normal form and the left-hand side of the condition match. The new variables resulting from this match are added to Vars and bound to the corresponding parts of the right-hand side of the condition.

Important

If a variable V occurs both in Vars and in the left-hand side of a condition, then it must match a subterm in the right hand side of the condition that is syntactically identical to the current value of V .

For the evaluation of each equality condition we require that the condition contains only variables in Vars . Reduce both sides of the condition to normal form and the condition succeeds if both normal forms are identical. Technically, this is called a *join* condition. The evaluation of negative conditions is described by replacing in the above description "identical" and "match" by "not identical" and "do not match", respectively.

Important

It is not allowed to introduce new variables in a negative condition.

After the successful evaluation of the conditions, all variables occurring in the right-hand side of the conclusion of the equation should be in Vars . New variables (see above) should therefore *not* occur on *both* sides of a positive condition, in a negative condition, or in the right-hand side of the conclusion.

Executing Equations

In the ASF+SDF Meta-Environment, equations can be executed as *rewrite rules*. This can be used to reduce some initial closed term (i.e., not containing variables) to a *normal form* (i.e., a term that is not reducible any further) by repeatedly applying rules from the specification. A term is always reduced in the context of a certain module, say M . The rewrite rules that may be used for the reduction of the term are the rules declared in M itself and in the modules that are (directly or indirectly) imported by M . The search for an applicable rule is determined by the reduction strategy, that is, the procedure used to select a subterm for possible reduction. In our case the *leftmost-innermost* reduction strategy is used. This means that a left-to-right, depth-first traversal of the term is performed and that for each subterm encountered an attempt is made to reduce it. Next, the rules are traversed one after the other. The textual order of the rules is irrelevant, but default equations come last.

If the selected subterm and the left-hand side of a rule (more precisely: of the left-hand side of its conclusion) match, we say that a *redex* has been found and the following happens. The conditions of the rule are evaluated and if the evaluation of a condition fails, other rules (if any) with matching left-hand sides are tried. If the evaluation of all conditions succeeds, the selected subterm is replaced by the right-hand side of the rule (more precisely: the right-hand side of the conclusion of the rule) after performing proper *substitutions*. Substitutions come into existence by the initial matching of the rule and by the evaluation of its conditions. For the resulting term the above process is repeated until no further reductions are possible and a normal form is reached (if any).

Important

A specification should always be *confluent* and *terminating*. Confluent means that the order in which the rules are applied has no effect on the outcome. Terminating means that the application of rules cannot go on indefinitely. We do not check for these two properties. Also see Common Errors when Executing Specifications.

Examples

We give here a number of elementary examples with the sole purpose to illustrate a range of features of ASF+SDF:

- Addition and Multiplication on Numerals.
- Booleans: simple truth values and operations.
- FriendlyBooleans: truth values with user-defined syntax.

For larger and more *interesting* examples, we refer to ASF+SDF by Example. [Add link](#).

Addition and Multiplication on Numerals

The natural numbers 0, 1, 2, ... can be represented in a form that is more convenient for formal reasoning:

- 0 is represented by $\mathbf{0}$.
- 1 is represented by $\mathbf{succ(0)}$.
- 2 is represented by $\mathbf{succ(succ(0))}$.
- ...
- The number N is represented by $\mathbf{succ^N(0)}$, i.e., N applications of $\mathbf{succ}$ to $\mathbf{0}$.

Let's first formalize the grammar of these numerals in an SDF definition `Numerals.sdf`.

Example 1.1. Numerals.sdf

```
module Numerals
exports
  sorts NUM                                ❶
  imports basic/whitespace                 ❷
  context-free syntax
    "0" -> NUM                             ❸
    succ(NUM) -> NUM                       ❹
  context-free start-symbols
    NUM                                    ❺
```

Notes:

- ❶ Declares the sort `NUM` that will represent all numerals.
- ❷ Import the library module `basic/whitespace` in order to define layout and comments.
- ❸ Define the constant `0`.
- ❹ Define the successor function `succ`. Note that we use a prefix function here (see *XXX*); the unabbreviated definition would be

```
"succ" "(" NUM ")" -> NUM
```
- ❺ Declare `NUM` as start symbol to enable the parsing of `NUM`s.

Having defined numerals, we can parse texts like `0`, `succ(0)`, etc. as syntactically correct `NUM`s.

The next step is to define addition and multiplication on numerals. Let's start with addition. We do this by introducing a module `Adder.sdf` that defines syntax for the `add` function.

Example 1.2. Adder.sdf

```

module Adder
exports
  imports Numerals                                ❶
  context-free syntax
    add(NUM, NUM) -> NUM                          ❷
  variables
    "X"          -> NUM                          ❸
    "Y"          -> NUM

```

Notes:

- ❶ Import the previously defined module Numerals.
- ❷ Define the syntax of the `add` function.
- ❸ Define the variables `X` and `Y` of sort `NUM`. They will be used in the equations.

Now we are ready for `Adder.asf` that defines the equations for the `add` function.

Example 1.3. Adder.asf: the ASF part of the Adder module

```

equations                                ❶
[1] add(0, X) = X                        ❷
[2] add(succ(X), Y) = succ(add(X, Y))    ❸

```

Notes:

- ❶ The ASF part starts with the keyword `equations`.
- ❷ First equation: adding `0` to an arbitrary numeral `X` yields that same numeral.
- ❸ Adding `Y` to the successor of `X` is the same as applying the successor to the addition of `X` and `Y`.

After these preparations we can parse and reduce terms using the module `Adder`:

- The term `0` reduces to `0` in zero steps; no simplifications are possible.
- The term `SUCC(0)` reduces to `SUCC(0)` in zero steps; no simplifications are possible.
- The term `add(succ(succ(0)), succ(succ(0)))` reduces to `succ(succ(succ(succ(0))))` in 3 steps; this corresponds to $2 + 2 = 4$.

We complete this example with `Multiplier.sdf` and `Multiplier.asf` that define a multiplication operator on numerals as well.

Example 1.4. Multiplier.sdf

```

module Multiplier
exports
  imports Adder                                ❶
  context-free syntax
    mul(NUM, NUM) -> NUM                      ❷

```

Notes:

- ❶ Import the module `Adder` that have just defined above.
- ❷ Define the syntax of the function `mul`.

The equations for `mul` are defined in `Multiplier.asf`.

Example 1.5. `Multiplier.asf`

equations

```
[1] mul(0, X) = 0 ❶
[2] mul(succ(X), Y) = add(mul(X, Y), Y) ❷
```

Notes:

- ❶ Multiplying any numeral by zero yields zero.
- ❷ Reduce multiplication to addition: multiplying Y by $X+1$ is the same as multiplying Y by X and then adding Y .

We can now parse and reduce terms using the module `Multiplier`:

- The term `mul(succ(succ(succ(0))), succ(succ(0)))` reduces to `succ(succ(succ(succ(succ(succ(0))))))`; this corresponds to $3 * 2 = 6$.

Booleans

The Boolean constants `true` and `false` and the Boolean functions `and`, `or` and `not` are also completely elementary and therefore well-suited for illustrating some more features of ASF+SDF. The syntax of Booleans is given in `Booleans.sdf`.

Example 1.6. Booleans.sdf

```

module Booleans
exports
  sorts BoolCon                                ❶
  context-free syntax
    "true"  -> BoolCon                         ❷
    "false" -> BoolCon

  sorts Boolean                                ❸
  context-free start-symbols                   ❹
    Boolean
  context-free syntax
    BoolCon          -> Boolean                ❺
    and(Boolean, Boolean) -> Boolean           ❻
    or(Boolean, Boolean)  -> Boolean
    not(Boolean)          -> Boolean

hiddens                                       ❼
  imports basic/Comments                      ❽
  variables
    "B" -> Boolean                            ❾

```

Notes:

- ❶ Introduce the sort `BoolCon` that will represent the constants `true` and `false`. It is good practice to define constants as a separate sort.
- ❷ Here are the definitions of the constants themselves.
- ❸ Introduce our sort of interest: `Boolean`.
- ❹ Since we will be dealing with Boolean terms, it is mandatory that they can be parsed. Hence we need to define a start symbol for them.
- ❺ Every Boolean constant `BoolCon` is a `Boolean`.
- ❻ Definition of the syntax of `and`, `or` and `not`.
- ❼ It is good practice to define comments (as needed in the equations) and variables as hidden. This has the advantage that comment conventions and variable declaration do propagate to the modules that import the current module.
- ❽ Import standard comments.
- ❾ Define the single variable `B` for later use in the equations.

After these preparations we are ready to parse Boolean terms like:

- `true`
- `and(true, false)`
- `and(or(true, not(false)), true)`
- `or(true, B)`
- and so on and so forth.

The definition of the functions on Booleans now simply requires writing down the truth tables in the form of equations and is given in `Booleans.asf`.

Example 1.7. Booleans.asf

```
equations
[or1] or(true, true)      = true
[or2] or(true, false)    = true
[or3] or(false, true)    = true
[or4] or(false, false)   = false

[and1] and(true, true)   = true
[and2] and(true, false)  = false
[and3] and(false, true)  = false
[and4] and(false, false) = false

[not1] not(true) = false
[not2] not(false) = true
```

We can now parse and reduce terms using Booleans:

- The term `true` reduces to `true`.
- The term `and(true, false)` reduces to `false`.
- The term `and(or(true, not(false)), true)` reduces to `true`.

As a final touch, a similar but shorter definition of the Boolean functions is possible. By using variable *B*, which we did declare but have not used so far, a shorter definition is possible see, for instance, the definition for `or` below.

Example 1.8. A shorter definition for `or`.

```
[or1'] or(true, B) = true
[or2'] or(false, B) = B
```

Booleans with user-defined syntax

The Booleans we have seen in the previous example are fine, but the strict prefix notation makes Boolean terms less readable. Would it be possible to use more friendly notation like `true & false` instead of `and(true, false)` or `true | false` instead of `or(true, false)`? Can this be defined in ASF+SDF? The answers are yes and yes. In fact user-defined syntax is one of the unique features of ASF+SDF.

In `FriendlyBooleans.sdf` we show a version of the Booleans with user-defined syntax.

Example 1.9. FriendlyBooleans.sdf

```

module FriendlyBooleans
exports
  sorts BoolCon
  context-free syntax
    "true"  -> BoolCon
    "false" -> BoolCon

  sorts Boolean
  context-free start-symbols
    Boolean
  context-free syntax
    BoolCon          -> Boolean
    Boolean "&" Boolean -> Boolean {left}      ❶
    Boolean "|" Boolean -> Boolean {left}
    not(Boolean)      -> Boolean              ❷
    "(" Boolean ")"    -> Boolean {bracket}    ❸
  context-free priorities
    Boolean "&" Boolean -> Boolean >
    Boolean "|" Boolean -> Boolean
  hiddens
    imports basic/Comments
    variables
      "B" -> Boolean

```

Notes:

- ❶ Here the infix syntax for the and and or function is defined.
- ❸ Since we have infix functions, parentheses are needed for grouping.
- ❷ We keep the prefix version of the not function to illustrate the mixture of prefix and infix notation.
- ❹ A priority rules defines that & binds stronger than |.

After these preparations, the equations are shown in FriendlyBooleans.asf.

Example 1.10. FriendlyBooleans.asf

```

equations
[or1] true | B    = true
[or2] false | B   = B

[and1] true & B    = B
[and2] false & B   = false

[not1] not(true)   = false
[not2] not(false)  = true

```

Important

Something interesting is going on here: we defined syntax rules in FriendlyBooleans.sdf and use them here in FriendlyBooleans.asf. This means that we use syntax in the equations that we have defined ourselves! Think about the implications of this: if we have an SDF definition for a programming language, we can easily write equations that contain programming language

fragments. This unique feature makes ASF+SDF the ultimate language for writing program transformations.

Tests

Specification writers are supposed to make no errors, but we are all human. It is therefore convenient to explicitly state your expectations about the normal forms for some typical input terms. The tests in ASF+SDF provide a convenient way to document this and to run unit test for a module.

Tests are always contained in a tests section that has the following structure:

```
tests
  <Test>*
```

The global structure of the ASF part of a module then becomes:

```
equations
  <Equation>*
tests
  <Test>*
```

In fact, a module may contain an *several* test and equations sections in arbitrary order.

Each test has the form of a condition preceded by a label:

```
[<TagId>] <Condition>
```

A test may not contain variables and only the operators == and != can be used.

The tests for each module can be run via the check menu item in the user-interface of The Meta-Environment.

Example

Reconsider the functions on lists given earlier. We can add tests to that definition as follows.

Example 1.11. Lists with tests

```
equations
  ... see List.asf ...

tests
  [sanity]          []                != [1]
  [append1]         [1,2,3] ++ 4      == [1,2,3,4]
  [append2]         1 ++ [2,3,4]      == [1,2,3,4]
  [length1]         length([1,2,3])   == 3
  [is-element1]     is-element(2, [1,2,3]) == true
  [is-element2]     is-element(5, {1,2,3})) != true
```

Advanced Equations

The simple equations described in the previous section are powerful enough to formulate the solution of any computational problem. However, ASF+SDF provides some more advanced features that can make the life of the specification writer a lot simpler:

- Default equations: equations that apply only when no other equations are applicable.

- List matching: decompose and compose arbitrary lists.
- Memo functions: memorize the values of previous function invocations.
- Constructor functions: define which functions are irreducible.
- Lexical constructor functions: get access to and modify the lexical (string) representation of programs.
- Traversal functions: traverse complex structures with a minimal specification effort; this is important for solving real-life analysis and transformation problems.

Default Equations

As we have seen in Executing Equations, the evaluation strategy for normalizing terms given the equations is based on innermost rewriting. All equations have the same priority. Given the outermost function symbol of a redex the set of equations with this outermost function symbol in the left-hand side is selected and all these rules will be tried. However, sometimes a specification writer would like to write down a rule with a special status: *try this rule if all other rules fail*. A kind of default behaviour is needed. ASF+SDF offers functionality in order to obtain this behaviour. If the *TagId* of an equation starts with `default` - this equation is considered to be a special equation which will only be applied if no other rule matches.

Example: Comparing Types

Suppose we are solving a typechecking problem and have a sort `Type` that represents the possible types. It is likely that we will need a function `compatible` that checks whether two types are compatible, for instance, when they appear on the left-hand and right-hand side of an assignment statement or when the actual/formal correspondence of procedure parameters has to be checked. Potentially, `Type` may contain a lot of different type values and comparing them all is a combinatorial problem.

The modules `Types.sdf` and `Types.asf` show how to solve this problem using a default equation. In `Types.sdf`, we define a sort `Type` that can have values `natural`, `string` and `nil-type`.

Example 1.12. `Types.sdf`

```
module Types

imports basic/Whitespace
imports basic/Booleans

exports
  context-free start-symbols Type
  sorts Type

  context-free syntax
    "natural"      -> Type
    "string"       -> Type
    "nil-type"     -> Type
    compatible(Type, Type) -> Boolean

hiddens
  variables
    "Type"[0-9]* -> Type
```

In `Types.asf`, we define three equations: two for checking the cases that the arguments of `compatible` are equal and one default equation for checking the remaining cases.

Example 1.13. Types.asf

```
equations
[Type-1] compatible(natural, natural) = true
[Type-2] compatible(string, string)   = true
[default-Type]
        compatible(Type1, Type2)      = false
```

An alternative definition is given in Types2.asf where equation [Type-1] has a left-hand side that contains the same variable (*Type*) twice. This has as effect that the left-hand side only matches if the two arguments of *compatible* are identical.

Example 1.14. Types2.asf

```
equations
[Type-1] compatible(Type, Type) = true
[default-Type]
        compatible(Type1, Type2) = false
```

To complete this story, yet another specification style for this problem exists that uses a negative condition instead of a default equation. This is shown in Types3.asf.

Example 1.15. Types3.asf

```
equations
[Type-1] compatible(Type, Type) = true
[Type-2]      Type1 != Type2
=====
        compatible(Type1, Type2) = false
```

You may not (yet) be impressed by the savings that we get in this tiny example. You will, however, be pleasantly surprised when you use the above techniques and see how short specification become when dealing with real-life cases.

List Matching

List matching, also known as *associative matching*, is a powerful mechanism to describe complex functionality in a compact way. Unlike the matching of ordinary (non-list) variables, the matching of a list variable may have more than one solution since the variable can match lists of arbitrary length. As a result, backtracking is needed. For instance, to match $X \ Y$ (a list expression containing the two list variables *X* and *Y* indicating the division of a list into two sublists) with the list *ab* (a list containing two elements) the following three alternatives have to be considered:

- $X = (\text{empty}), Y = ab$
- $X = a, Y = b$
- $X = ab, Y = (\text{empty})$.

In the unconditional case, backtracking occurs only during matching. When conditions are present, the failure of a condition following the match of a list variable leads to the trial of the next possible match of the list variable and the repeated evaluation of following conditions.

Example: Sets

Let's consider the problem of removing double elements from a list. This shown is in Sets.sdf and Sets.asf.

Example 1.16. Sets.sdf

```

module Sets
exports
  imports basic/Whitespace
  context-free start-symbols Set
  sorts Elem Set

  lexical syntax
    [a-z]+ -> Elem ❶

  context-free syntax
    "{" {Elem "",""}* "}" -> Set ❷

hiddens
  variables
    "Elem"[0-9]* -> Elem ❸
    "Elem*" [0-9]* -> {Elem "",""}* ❹

```

Notes:

- ❶ This defines constants of sort `Elem` as a sequence of one or more lowercase letters.
- ❷ This defines the syntax of a `Set`: an opening curly bracket, a list of zero or more `Elem`s separated by comma's, followed by a closing curly bracket. This is a typical syntax pattern; don't get confused by the different roles that the curly brackets play: the `"{"` and `"}"` are literal string that are part of the syntax of `Sets`, while `{Elem "",""}*` describes a syntactic list of `Elem`s separated by commas.
- ❸ This defines variables `Elem1`, `Elem2` and so on of sort `Elem`.
- ❹ This defines variables `Elem*1`, `Elem*2` and so on of sort `{Elem "",""}*`. Observe the funny variable name containing the non-alphanumeric character `*`. Some ASF+SDF specification writers use the convention that variables that range over list sorts end on either `*` or `+` and maybe followed by digits. See [How to name Variables](#).

We can now parse sets like:

- `{}`
- `{a}`
- `{a, b}`
- `{a, b, c, d, e, f, g, h, i, j, k, l, m, n, o, p, q, r, s, t, u, v, w, x, y, z}`
- and so on.

The actual solution of our problem, removing duplicates from a list, is shown in `Sets.asf`. Observe that the single equation has a left-hand side with two occurrences of variable `Elem`, so it will match on a list that contains two identical elements.

In the right-hand side, one of these occurrences is removed. However, this same equation remains applicable as long as the list contains duplicate elements.

Example 1.17. Sets.asf

```

equations
[set] {Elem*1, Elem, Elem*2, Elem, Elem*3} =
      {Elem*1, Elem, Elem*2, Elem*3}

```

Important

This specification of sets is very elegant but may become very expensive to execute when applied to large sets. There are several strategies to solve this:

- Use ordered lists and ensure that the insert operation checks for duplicates.
- Use a more sophisticated representation like, for instance, a balanced tree.

Example: Lists

A sample of operations on lists of integers is shown in Lists.sdf and Lists.asf.

Example 1.18. Lists.sdf

```

module Lists

imports basic/Booleans
imports basic/Integers
imports basic/Whitespace

exports
  context-free start-symbols
    Boolean Integer List           ❶
  sorts List

  context-free syntax
    "[" {Integer ","}* "]" -> List ❷
    List "++" Integer      -> List ❸
    Integer "++" List      -> List ❹
    is-element(Integer, List) -> Boolean ❺
    length(List)           -> Integer ❻
    reverse(List)          -> List ❼
    sort(List)             -> List ❽

  hiddens
    variables
      "Int"[0-9]* -> Integer
      "Int"*[0-9]* -> {Integer ","}*

```

Notes:

- ❶ In the context-free syntax below, we define functions with result sorts **Boolean**, **Integer** and **List** and all these sorts will probably occur in input terms. It is therefore a good idea to declare all these sort as start symbol.
- ❷ Define the syntax of lists of integers. Examples are:
 - []
 - [1]
 - [1, 3, 5, 7, 11]
- ❸ Append an integer to a list.
- ❹ Prepend an integer to a list.
- ❺ Check for element in list.
- ❻ Determine length of list.
- ❼ Reverse a list.
- ❽ Sort a list.

Given these syntax definitions, we can define the meaning of the various functions in Lists.asf.

Example 1.19. Lists.asf

```

equations
  [app-1] [Int*] ++ Int = [Int*, Int]                                ❶
  [pre-1] Int ++ [Int*] = [Int, Int*]

  [len-1] length([]) = 0                                           ❷
  [len-2] length([Int, Int*]) = 1 + length([Int*])

  [is-1] is-element(Int, [Int*1, Int, Int*2]) = true              ❸
  [default-is]
    is-element(Int, [Int*]) = false

  [rev-1] reverse([]) = []                                         ❹
  [rev-2] reverse([Int, Int*]) = reverse([Int*]) ++ Int

  [srt-1]
    Int1 > Int2 == true                                           ❺
    =====
    sort([Int*1, Int1, Int*2, Int2, Int*3]) =
    sort([Int*1, Int2, Int*2, Int1, Int*3])
  [default-srt]
    sort([Int*]) = [Int*]
    
```

Notes:

- ❶ The definition of the append and prepend operators ++ illustrates how list variables like `Int*` can be used to first extract elements from a list (on the left-hand side) and later insert them in a new list (on the right-hand side).
- ❷ The `length` function is defined by a simple induction on lists.
- ❸ The two occurrences of the variable `Int` in equation [is-1] illustrates the use of list matching for a search for a given element in a list.
- ❹ The `reverse` function is defined by recurring over the elements of the list. Note how the append operator ++ is used in equation [rev-2]. One can avoid using an auxiliary operator at the expense of introducing a condition and an extra variable:

```

[rev-2'] [Int*1] := reverse([Int*])
=====
reverse([Int, Int*]) = [Int*1, Int]
    
```

- ❺ The equations for the function `sort` show, once more, the expressive power of list matching: in the left-hand side of [srt-1] two elements `Int1` and `Int2` are picked and if `Int1` is greater than `Int2`, the elements are swapped and `sort` is applied again.

Memo Functions

Computations may contain unnecessary repetitions. This is the case when a function with the same argument values is computed more than once. *Memo functions* exploit this behaviour and can improve the efficiency of ASF+SDF specifications considerably. They are defined by adding a `memo` attribute to a function definition

Memo functions are executed in a special manner by storing, on each invocation, the set of argument values and the derived normal form in a *memo table*. On a subsequent invocation with given arguments, it is first checked whether the function has been computed before with those arguments. If this is the case, the normal form stored in the memo table is returned as value. If not, the function is normalized as usual, the combination of arguments and computed normal form is stored in the memo table, and the normal form is returned as value.

Adding a memo attribute does not affect the meaning of a function. There is, however, some overhead involved in accessing the memo table and it is therefore not a good idea to add the memo attribute to each function.

Important

There are currently no good tools to determine which functions should become memo functions. This can only be determined by experimentation and measurement.

Example: Fibonacci

The Fibonacci function shown in Fib.sdf and Fib.asf below illustrates the use of memoization.

Example 1.20. Fib.sdf

```
module Fib

imports basic/whitespace
imports Adder ❶

exports
  context-free syntax
  fib(NUM) -> NUM {memo} ❷
```

Notes:

- ❶ Import the module Adder we have seen before in the section Addition and Multiplication of Numerals.
- ❷ Define the syntax of the fib function. The memo attribute indicates that memoization should be used when executing this function.

The definition of the Fibonacci function fib is given in Fib.asf.

Example 1.21. Fib.asf

```
equations
[fib-0] fib(0) = succ(0)
[fib-1] fib(succ(0)) = succ(0)
[fib-n] fib(succ(succ(X))) = add(fib(succ(X)), fib(X))
```

The resulting improvement in performance as follows:

Table 1.1. Execution times for the evaluation of fib(n)

fib(n)	Execution time without memo (sec)	Execution time with memo (sec)
fib(16)	2.0	0.7
fib(17)	3.5	1.1
fib(18)	5.9	1.8
fib(19)	10.4	3.3

Constructor Functions

Some functions symbols, like succ in the definition of Numerals cannot be reduced; they are constructors that are used to build the datastructures that are being defined. Other function symbols, like add and mul, are reducible; the equations define how these symbols can be removed from a term. There are currently, two ways to indicate that a function is a constructor:

- The function has an attribute named `cons` with a string as argument. This is used by external tools to give names to constructors in the syntax tree that is built. An example is the tool **ApiGen** that generates C or Java interfacing code given an SDF definition.
- The function has an attribute named `constructor` without any arguments. This attribute is used by the ASF+SDF implementation to check the use of constructors functions.

This is a confusing state of affairs that should be repaired.

Lexical Constructor Functions

This section has still to be converted to V2.0: (a) Describe structured lexicals and give examples. (b) Split examples below in SDF and ASF part.

A context-free syntax rule describes the underlying tree structure. A lexical syntax rule defines in fact the underlying structure of the lexical tokens. Sometimes it is necessary to create or manipulate the characters of lexical tokens, for instance when converting one lexical entity into another. ASF+SDF provide the so-called *lexical constructor functions* to manipulate the characters of the lexical entities. Furthermore, ensure the lexical constructor functions that the underlying structure is not violated or that illegal lexical entities are created.

The only way to access the actual characters of a lexical token is by means of *lexical constructor functions*. For each lexical sort `LEX` a lexical constructor function is automatically derived as follows:

```
lex( LEX ) -> LEX
```

Example: Removing Leading Zeros

In the example below the lexical constructor function `nat-con` is used to remove the leading zeros from a number.

Example 1.22. Use of lexical constructor

```
module Nats
imports basic/Whitespace
exports
  context-free start-symbols Nat-con
  sorts Nat-con

  lexical syntax
    [0-9]+ -> Nat-con

hiddens
  lexical variables
    "Char+"[0-9]* -> [0-9]+
equations

[1] nat-con(0 Char+) = nat-con(Char+)
```

Important

The argument of a lexical constructor may only be character that fit in the underlying lexical definition. There is *an explicit check that the characters and sorts match the lexical definition of the corresponding sort*. This means that when writing a specification it is impossible to construct illegal lexical entities, for instance, by inserting letters in an integer. In the example below via the lexical constructor function `nat-con` a natural number containing the letter `a` is constructed. But this will result in a syntax error.

Example: Illegal Use of Lexical Constructor Function

Example 1.23. Illegal use of lexical constructor functions

```

module Nats

imports basic/Whitespace

exports
  context-free start-symbols Nat-con
  sorts Nat-con

  lexical syntax
    [0-9]+ -> Nat-con

hiddens
  lexical variables
    "Char+"[0-9]* -> [0-9]+

equations

  [1] nat-con(Char+) = nat-con(Char+ a)

```

Traversal Functions

Program analysis and program transformation usually take the syntax tree of a program as starting point. One common problem that one encounters is how to express the *traversal* of the tree: visit all the nodes of the tree and extract information from some nodes or make changes to certain other nodes. The kinds of nodes that may appear in a program's syntax tree are determined by the grammar of the language the program is written in. Typically, each rule in the grammar corresponds to a node category in the syntax tree. Real-life languages are described by grammars which can easily contain several hundreds, if not thousands, of grammar rules. This immediately reveals a hurdle for writing tree traversals: a naive recursive traversal function should consider many node categories and the size of its definition will grow accordingly. This becomes even more dramatic if we realize that the traversal function will only do some real work (apart from traversing) for very few node categories. Traversal functions in ASF+SDF solve this problem.

Definition

We distinguish three kinds of traversal functions, defined as follows.

Transformer. A *transformer* is a sort-preserving transformation that will traverse its first argument. Possible extra arguments may contain additional data that can be used (but not modified) during the traversal. A transformer is declared as follows:

$$f(S_1, \dots, S_n) \rightarrow S_1 \{ \text{traversal}(\text{trafo}, \dots) \}$$

Because a transformer always returns the same sort, it is type-safe. A transformer is used to transform a tree.

Accumulator. An *accumulator* is a mapping of all node types to a single type. It will traverse its first argument, while the second argument keeps the accumulated value. An accumulator is declared as follows:

$$f(S_1, S_2, \dots, S_n) \rightarrow S_2 \{ \text{traversal}(\text{accu}, \dots) \}$$

After each application of an accumulator, the accumulated argument is updated. The next application of the accumulator, possibly somewhere else in the term, will use the *new* value of the accumulated

argument. In other words, the accumulator acts as a global, modifiable, state during the traversal. An accumulator function never changes the tree, only its accumulated argument. Furthermore, the type of the second argument has to be equal to the result type. The end-result of an accumulator is the value of the accumulated argument. By these restrictions, an accumulator is also type-safe for every instantiation. An accumulator is meant to be used to extract information from a tree.

Accumulating transformer. An *accumulating transformer* is a sort preserving transformation that accumulates information while traversing its first argument. The second argument maintains the accumulated value. The return value of an accumulating transformer is a tuple consisting of the transformed first argument and accumulated value. An accumulating transformer is declared as follows:

```
 $f(S_1, S_2, \dots, S_n) \rightarrow \langle S_1, S_2 \rangle \{ \text{traversal}(\text{accu}, \text{trafo}, \dots) \}$ 
```

An accumulating transformer is used to simultaneously extract information from a tree and transform it.

Visiting Orders. Having these three types of traversals, they must be completed with visiting orders. Visiting orders determine the order of traversal and the *depth* of the traversal. We provide the following two strategies for each type of traversal:

- Bottom-up: the traversal visits *all* the subtrees of a node where the visiting function applies in an *bottom-up* fashion. The annotation `bottom-up` selects this behavior. A traversal function without an explicit indication of a visiting strategy also uses the bottom-up strategy.
- Top-down: the traversal visits the subtrees of a node in an top-down fashion and stops recurring at the first node where the visiting function applies and does not visit the subtrees of that node. The annotation `top-down` selects this behavior.

Beside the three types of traversals and the order of visiting, we can also influence whether we want to stop or continue at the matching occurrences:

- Break: the traversal stops at matching occurrences.
- Continue: the traversal continues at matching occurrences.

Examples

We give two simple examples of traversal functions that are both based on a simple tree language that describes binary prefix expressions with natural numbers as leaves. Examples are `f(0, 1)` and `f(g(1, 2), h(3, 4))`.

Example 1.24. `Tree-syntax.sdf`: a simple tree language

```
module Tree-syntax
imports basic/Integer
imports basic/Whitespace

exports
  context-free start-symbols Tree
  sorts Tree

  context-free syntax
    Integer      -> Tree
    f(Tree, Tree) -> Tree
    g(Tree, Tree) -> Tree
    h(Tree, Tree) -> Tree
```

Our first example in `Tree-inc.sdf` and `Tree-inc.asf` transforms a given tree into a new tree in which all numbers have been incremented.

Example 1.25. Tree-inc.sdf: increment all numbers in a tree

```
module Tree-inc
imports Tree-syntax

exports
  context-free syntax
    inc(Tree) -> Tree {traversal(trafo, top-down, continue)}

hiddens
  variables
    "N"[0-9]* -> Integer
```

Example 1.26. Tree-inc.asf: increment all numbers in a tree

```
equations
[1] inc(N) = N + 1
```

Add explanation to the above example.

Our second example in Tree-sum.sdf and Tree-sum.asf computes the sum of all numbers in a tree.

Example 1.27. Tree-sum.sdf: sum all numbers in a tree

```
module Tree-sum
imports Tree-syntax
exports
  context-free syntax
    sum(Tree, Integer) -> Integer {traversal(accu, top-down,
                                             continue)}

hiddens
  variables
    "N"[0-9]* -> Integer

equations
[1] sum(N1, N2) = N1 + N2
```

Example 1.28. Tree-sum.asf: sum all numbers in a tree

```
equations
[1] sum(N1, N2) = N1 + N2
```

Add explanations to the above example.

Requirements

The ASF+SDF definition of a traversal function has to fulfill a number of requirements:

- Traversal functions can only be defined in the context-free syntax section.
- Traversal functions must be prefix functions, see XXX.
- The first argument of the prefix function is always a sort of a node of the tree that is traversed.
- In case of a transformer, the result sort *Tree* should always be same as the sort of the first argument:

```
tf(Tree, A1, ..., An) -> Tree {traversal(trafo, ...)}
```

- In case of an accumulator, the second argument *Accu* represents the accumulated value and the result sort should be of the same sort:

$$tf(Tree, Accu, A_1, \dots, A_n) \rightarrow Accu \{traversal(accu, \dots)\}$$

- In case of an accumulating transformer, the first argument represents the tree node *Tree*, the second the accumulator *Accu*, and the result sort should be a tuple consisting of the tree node sort (first element of the tuple) and the accumulator (second element of the tuple):

$$tf(Tree, Accu, A_1, \dots, A_n) \rightarrow \langle Tree, Accu \rangle \{traversal(accu, trafo, \dots)\}$$

- The traversal functions may have more arguments, the only restriction is that they should be consistent over the various occurrences of the same traversal function.

$$\begin{aligned} tf(Tree_1, Accu, A_1, \dots, A_n) &\rightarrow Tree_1 \{traversal(trafo, continue, top-down)\} \\ tf(Tree_2, Accu, A_1, \dots, A_n) &\rightarrow Tree_2 \{traversal(trafo, continue, top-down)\} \end{aligned}$$

- The order of the traversal attributes is free, but should be used consistently, for instance the following definition is not allowed:

$$\begin{aligned} tf(Tree_1, Accu, A_1, \dots, A_n) &\rightarrow Tree_1 \{traversal(trafo, top-down, continue)\} \\ tf(Tree_2, Accu, A_1, \dots, A_n) &\rightarrow Tree_2 \{traversal(trafo, continue, top-down)\} \end{aligned}$$

- If the number of arguments of the traversal function changes, you should introduce a new function name. The following definitions are not correct:

$$\begin{aligned} tf(Tree_1, Accu, A_1, A_2) &\rightarrow Tree_1 \{traversal(trafo, top-down, continue)\} \\ tf(Tree_2, Accu, A_1, A_2, A_3) &\rightarrow Tree_2 \{traversal(trafo, continue, top-down)\} \end{aligned}$$

but should be:

$$\begin{aligned} tf1(Tree_1, Accu, A_1, A_2) &\rightarrow Tree_1 \{traversal(trafo, top-down, continue)\} \\ tf2(Tree_2, Accu, A_1, A_2, A_3) &\rightarrow Tree_2 \{traversal(trafo, continue, top-down)\} \end{aligned}$$

In the SDF part of a module it is needed to define traversal functions for all sorts which are needed in the equations.

Well-formedness of ASF+SDF

In order to improve the quality of the written specifications, a number of checks are performed before an ASF+SDF specification can be executed. The checks are performed on two levels: the first level are SDF specific checks; these are further discussed in The Syntax Definition Formalism SDF (XXX). The second level are ASF+SDF specific checks (leading to warnings or errors) that we discuss here.

There also some issues of writing style for ASF+SDF specifications that we discuss here. Most messages are self-explanatory, for others we add some additional explanation.

A Matter of Style

We can give you some advice on the writing style for ASF+SDF specifications:

- Indentation style of equations: use the most readable layout for equations.
- Naming conventions for variables: use names that enhance readability.
- Importing layout and comment definitions: understand the best way to introduce layout and comments.
- Using the ASF+SDF library: exploit predefined library modules.

Consider these advices as current best practices and apply them as much as possible.

How to indent your Equations

The preferred style for writing equations is as follows:

- Place the tag of the equation on a separate line.
- Place the conditions of the equation on separate lines.
- Vertically align the left-hand sides of the conditions, the implication sign, and the conclusion.

Here is an example:

```
[check-tuple-exp1]
<$Etype1, $Tenv'> := check($Exp1, $Tenv),
<<$Etype+>, $Tenv''> := check(<$Exp2, $Exp+>, $Tenv')
=====
check(<$Exp1, $Exp2, $Exp+>, $Tenv) =
<<$Etype1, $Etype+>, $Tenv''>

[check-tuple-exp2]
<$Etype1, $Tenv'> := check($Exp1, $Tenv),
<$Etype2, $Tenv''> := check($Exp2, $Tenv')
=====
check(<$Exp1, $Exp2>, $Tenv) = <<$Etype1, $Etype2>, $Tenv''>
```

Some authors prefer to make the implication sign as wide as the conclusion (as shown above). This looks nice but requires some maintenance when you change the conclusion. For that reason, other authors always use one implication sign of a fixed length (3-4 equals characters): ===== or even ==>

How to name Variables

ASF+SDF provides a large freedom in the way you can name variables; they are not limited to alphanumeric strings as in most languages, but you can define arbitrary syntax for them. This freedom has, unfortunately, also a dark side: using too much of this freedom leads to ununderstandable specifications. Here is an example in the context where the library module `Integers` has been imported:

```
variables
"2 + 3" -> Integer
```

Given this definition, nobody will understand what the meaning of `"1 + 2 + 3"` will be. [Answer: an addition with two operands, the constant 1 and the Integer variable `2 + 3`.]

Although there are some rare cases where this can be used to your advantage, we strongly advise against this and suggest the following conventions for defining variables:

- Variables start with an uppercase letter, followed by letters, underscores or hyphens. Optionally they may be followed by digits or single quotes.
- If syntactic constraints make this mandatory, start variables with a distinctive character. Preferred is a dollar sign (\$), but others like a number sign (#) can be considered.
- In case a variable is of a list sort, plus (+) or star (*) characters may be used.

These rules are followed in the following variable declarations:

```
variables
"Integer" [0-9]* -> Integer 1
```

"Int" [0-9']*	-> Integer	❷
"\$int" [0-9']*	-> Integer	❸
"Int*" [0-9']*	-> {Integer " ", "}"	❹

Notes:

- ❶ A plain variable declarations that declares *Integer*, *Integer1*, *Integer2*, *Integer123*,
- ❷ A similar declaration but using digits and quote as suffix for the variable name: *Int*, *Int'*, *Int''*, *Int'''*, *Int1*, *Int123*,
- ❸ Using \$ as prefix: this declares *\$int*, *\$int1*, *\$int123*,
- ❹ Using * in a list variable: *Int**, *Int*1*, *Int*123*,

Important

Restrain yourself in the choice of variable names and follow the above naming conventions.

How to import Layout and Comment Definitions

As we have explained before, there is no built-in definition for the layout or comments in the ASF part of a module *M* and you need to import definitions for layout or comments yourself. If you forget to do this, you get a parse error when trying to parse the equations. There are, in principle, two standard modules that do the job: `basic/Whitespace` or `basic/Comments` (that imports `basic/Whitespace`).

But what is the best way to do this? There are two options:

- Add the import of `basic/Comments` in the exports section of module *M*. This is the simplest method since it exports the definitions in `basic/Comments` to all other modules that import module *M*. However, there are cases where the definitions in `basic/Comments` interferes with the syntax definitions in the modules that import *M*. In those cases, the second method is applicable.
- Add the import of `basic/Comments` to the hiddens section of module *M*. In this way, the definitions in `basic/Comments` remain localized to *M*. The disadvantage of this approach is that the import of `basic/Comments` has to be repeated in every module.

It is common practice to start with the first approach and switch to the second approach when syntactic problems occur.

Important

Be aware of the way in which you import the layout and comment conventions for your equations.

Using the ASF+SDF Module Library

ASF+SDF comes with a library of predefined grammars and datatypes. Use them to save yourself work. See XXX for an overview of the library.

Warning

Add reference.

Further Reading

The main publications on ASF+SDF are [BHK89] and [DHK96].

The main publications on implementation techniques related to ASF+SDF are [Hen91], [Wal91], [Kli93], [BJKO00], [Oli00], [BHKO02], [BKV03], and [Vin05]

Historical notes for SDF are given in the companion article ????

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