The Dynamics of Communication

Jan van Eijck

CWI and ILLC, Amsterdam, Uil-OTS, Utrecht

jve@cwi.nl

Toulouse, October 19, 2004

Abstract

Logics of communication should provide accounts of changes in the state of information of a group of discourse participants, on the basis of message exchanged within the group. We will give an overview of the way this is done in dynamic epistemic logics, focussing on a number of different types of informative actions with their epistemic effects, and indicating the relevance of this work for semantics and pragmatics of natural language. At the end of the talk we will sketch a recent result on translating logics for generic epistemic updating into PDL (propositional dynamic logic).

The framework is that of Baltag, Moss and Solecki, The Logic of Epistemic Actions [1].

Suppose I try saying something.

What way do I have of knowing
that if I say I know something
I don't really not know it?

Or what way do I have of knowing
that if I say I don't know something
I don't really in fact know it?

Chuang Tzu

Four Nasty Notes

Four Nasty Notes

Dear A, Your wife is cheating you. Sincerely, B.

Four Nasty Notes

Dear A, Your wife is cheating you. Sincerely, B.

Dear A, Your wife is cheating you. Sincerely, NN.

Four Nasty Notes

Dear A, Your wife is cheating you. Sincerely, B.

Dear A, Your wife is cheating you. Sincerely, NN.

Dear A, Your wife is cheating you, and you know it. Sincerely, B.

Four Nasty Notes

Dear A, Your wife is cheating you. Sincerely, B.

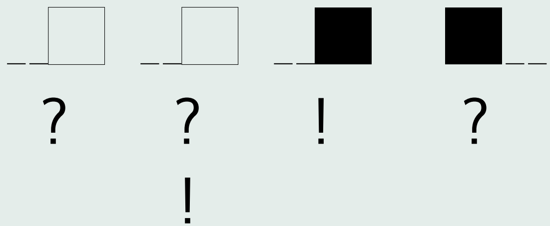
Dear A, Your wife is cheating you. Sincerely, NN.

Dear A, Your wife is cheating you, and you know it. Sincerely, B.

Dear A, Your wife is cheating you, and you know it. Sincerely, NN.

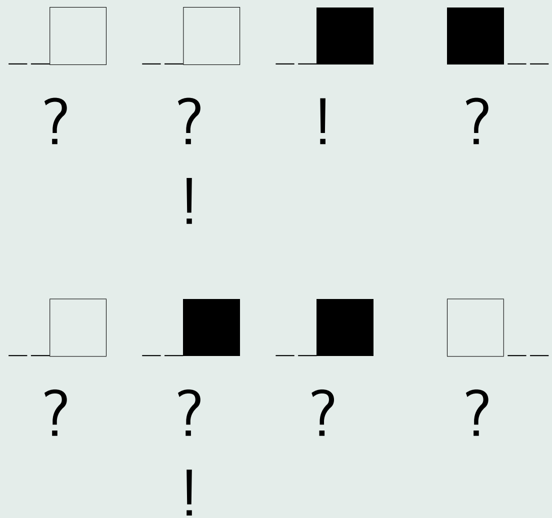
Reasoning about Knowledge and Ignorance

The riddle of the caps:



Reasoning about Knowledge and Ignorance

The riddle of the caps:



Riddle of the Muddy Children

"At least one child is muddy!"

a	b	c
○	○	●
?	?	!
!	!	!

Riddle of the Muddy Children

"At least one child is muddy!"

a	b	c
○	○	●
?	?	!
!	!	!

Reasoning of a and b: Suppose I am muddy. Then c would not have known in the first round that she was. But she did. So I am clean.

"At least one child is muddy!"

a	b	c	d
○	○	●	●
?	?	?	?
?	?	!	!
!	!	!	!

"At least one child is muddy!"

a	b	c	d
○	○	●	●
?	?	?	?
?	?	!	!
!	!	!	!

Reasoning of $c(d)$: Suppose I am clean. Then $d(c)$ would have known in the first round that she was dirty. But she didn't. So I am muddy.

"At least one child is muddy!"

a	b	c	d
○	○	●	●
?	?	?	?
?	?	!	!
!	!	!	!

Reasoning of $c(d)$: Suppose I am clean. Then $d(c)$ would have known in the first round that she was dirty. But she didn't. So I am muddy.

Reasoning of $a(b)$: Suppose I am dirty. Then c and d would not have known in the second round that they were dirty. But they knew. So I am clean.

"At least one child is muddy!"

a	b	c	d
○	●	●	●
?	?	?	?
?	?	?	?
?	!	!	!
!	!	!	!

“At least one child is muddy!”

a	b	c	d
○	●	●	●
?	?	?	?
?	?	?	?
?	!	!	!
!	!	!	!

Reasoning of b: Suppose I am clean. Then c and d would have known in the second round that they are dirty. But they didn’t know. So I am dirty. Similarly for c and d.

"At least one child is muddy!"

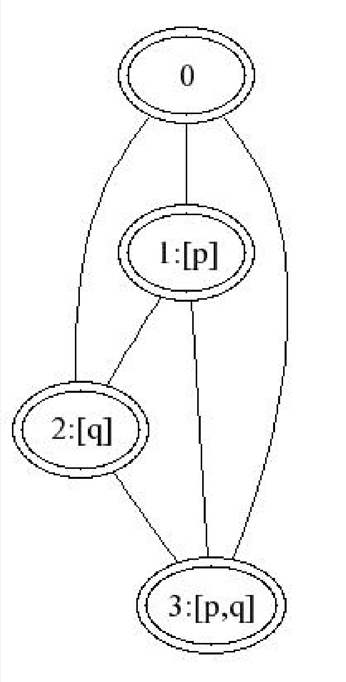
a	b	c	d
○	●	●	●
?	?	?	?
?	?	?	?
?	!	!	!
!	!	!	!

Reasoning of b: Suppose I am clean. Then c and d would have known in the second round that they are dirty. But they didn't know. So I am dirty. Similarly for c and d.

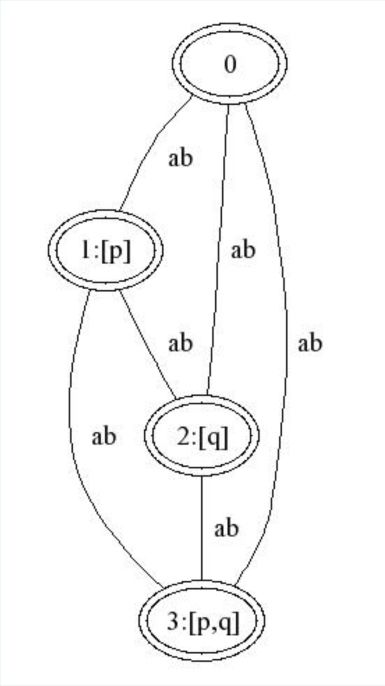
Reasoning of a: Suppose I am dirty. Then b, c and d would not have known their situation in the third round. But they did know. So I am clean.

Representing the knowledge of a set of agents

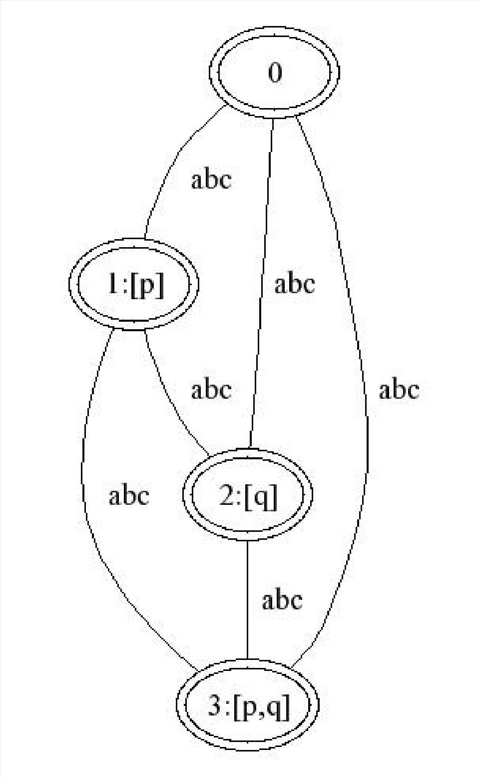
Situation of complete ignorance about p and q, for a single agent:



Situation of complete ignorance about p and q, for two agents a and b:



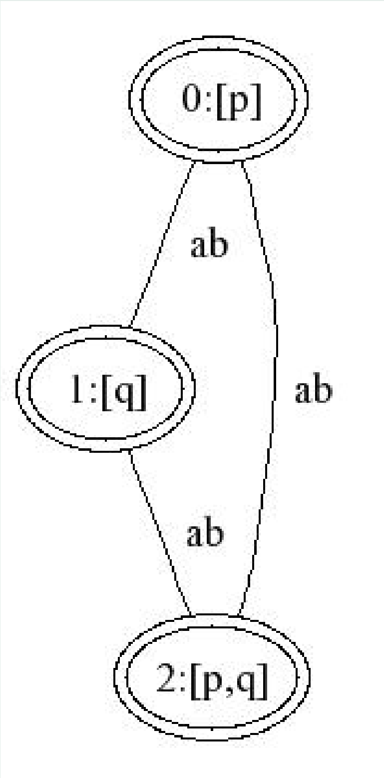
Situation of complete ignorance about p and q, for three agents a, b and c:



What happens if it is publicly announced that $p \vee q$ is the case?

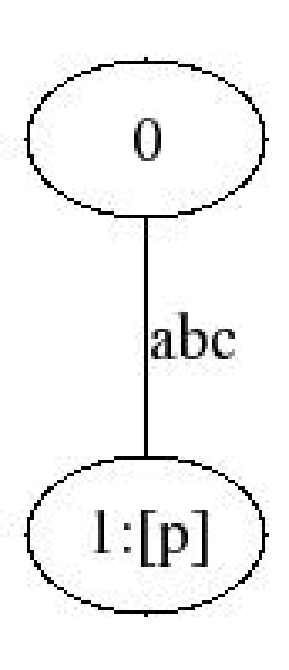
What happens if it is publicly announced that $p \vee q$ is the case?

We should get:



True Nirwana

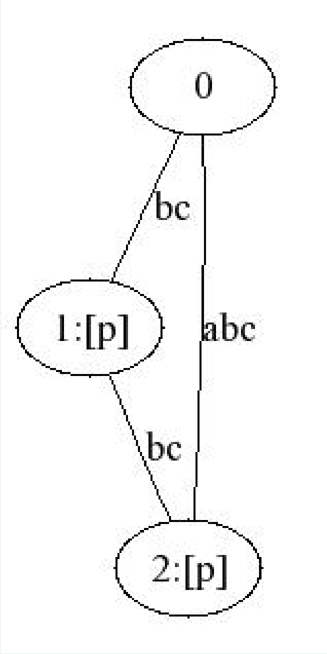
“Everyone knows that noone knows anything.”



False Nirwana

Seekers: “I know nothing, but my guru might know something.”

Guru: “My disciples know nothing, but I might know something.”

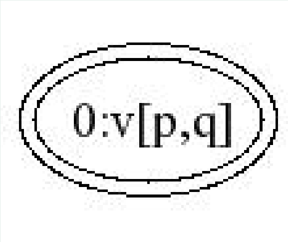


Communicative Actions as Models: Public Announcement

The public announcement that $p \vee q$ is modelled in BMS style as follows:

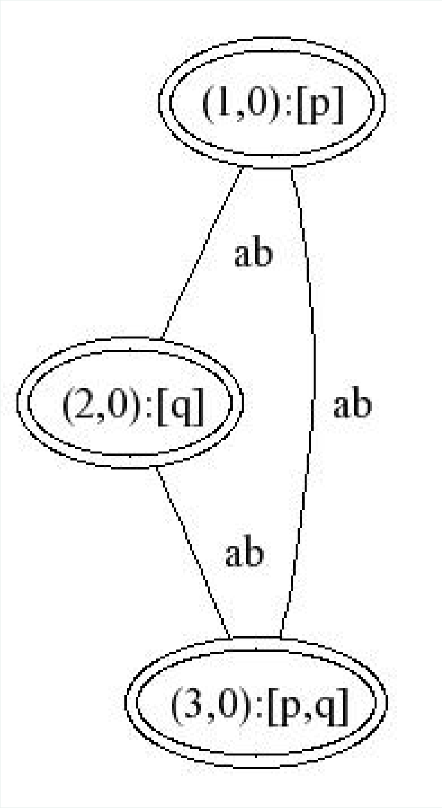
Communicative Actions as Models: Public Announcement

The public announcement that $p \vee q$ is modelled in BMS style as follows:



Worlds after the update are the pairs (w, a) where w is a world before the update, and a is an action, and where (w, a) is such that w satisfies the precondition formula of a.

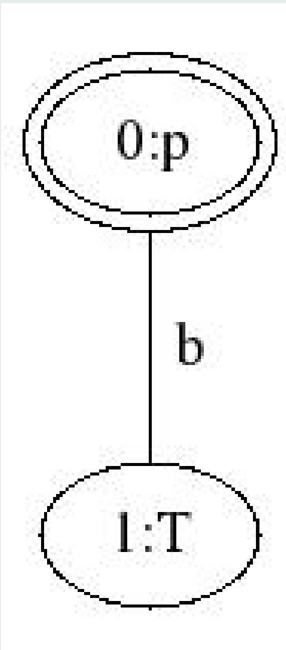
Only those epistemic links are present that are present both in the old epistemic model and in the action model.

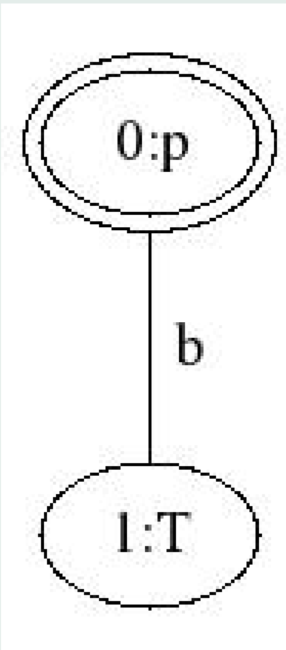


After public announcement $p \vee q$.

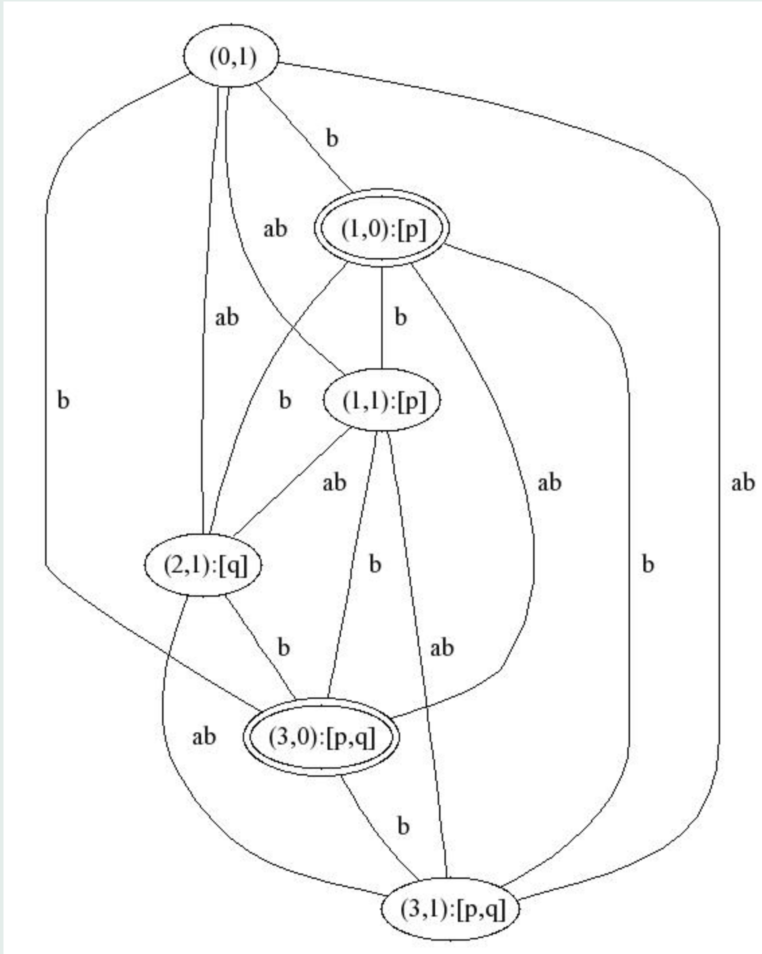
Communicative Actions as Models: Private Messages

Private communication is modelled differently. The private message to a that p has the following model:



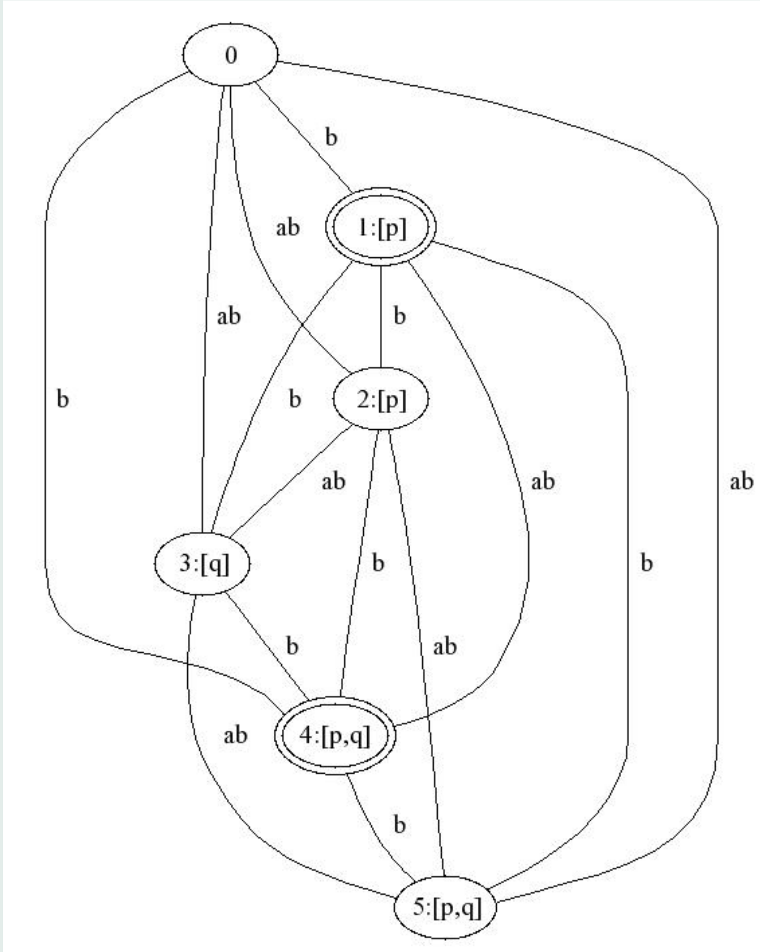


0:p
b
1:T

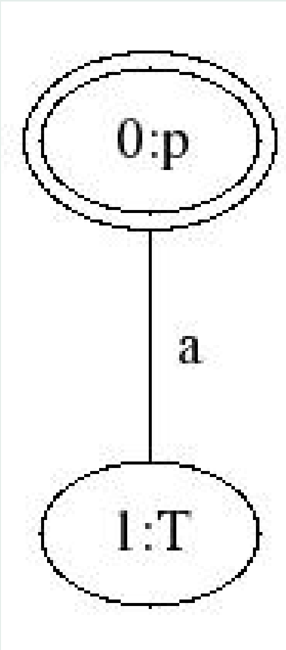


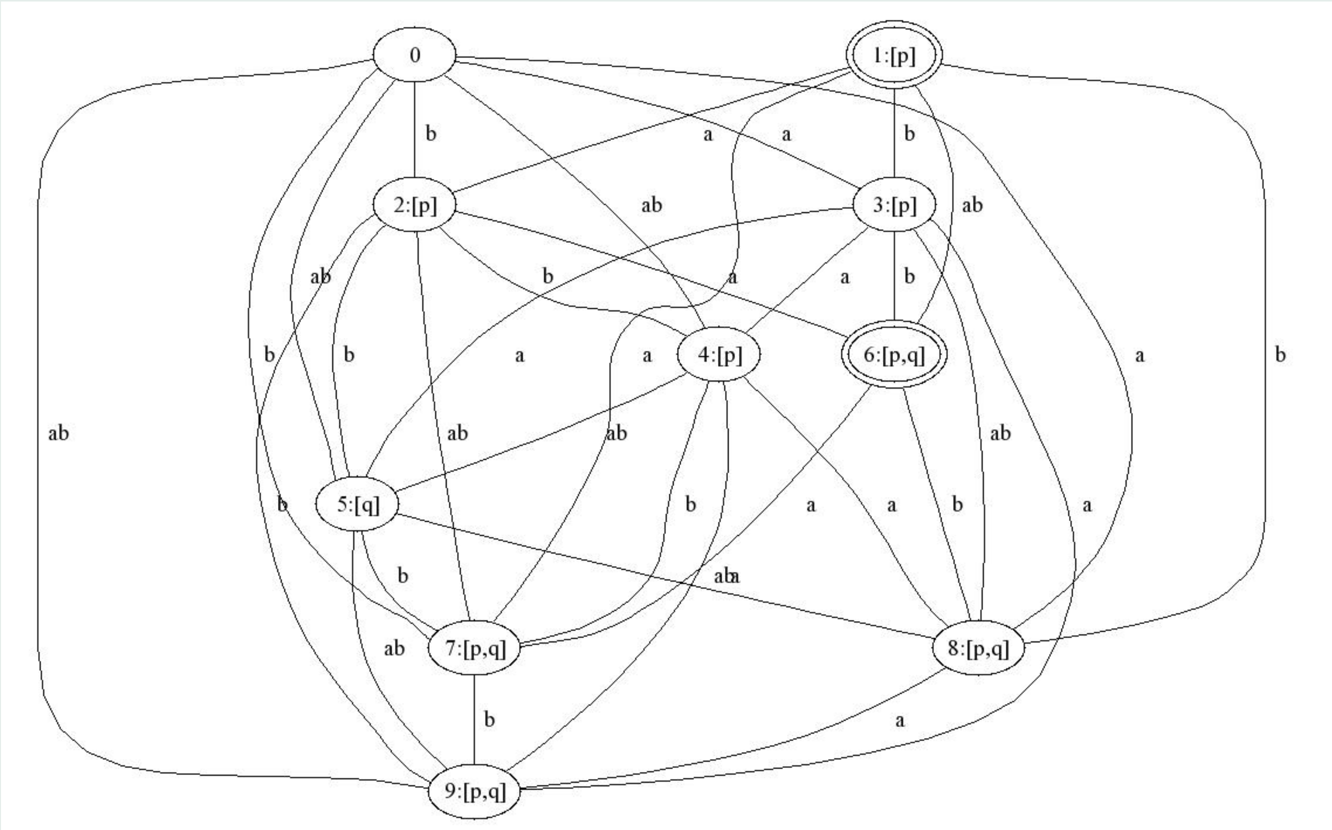
(0,1)
b
ab
(1,0):[p]
ab
b
b
(1,1):[p]
b
ab
ab
ab
(2,1):[q]
b
b
b
ab
ab
(3,0):[p,q]
b
(3,1):[p,q]

After renaming:



Suppose in the new situation b is told in private that p.

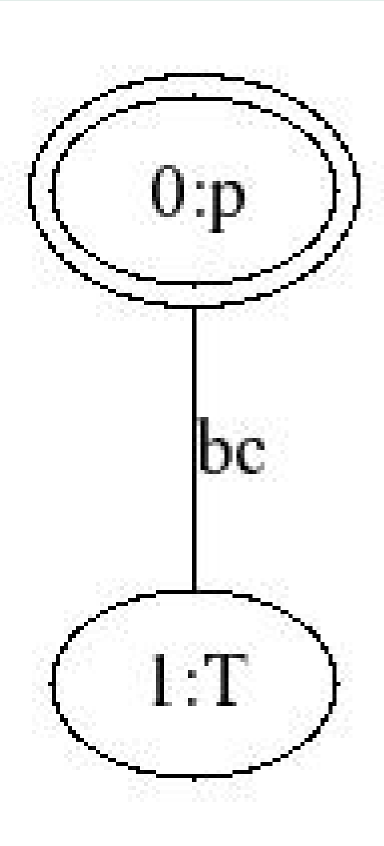



0
1:[p]
b a a b
2:[p]
ab
3:[p]
ab
ab b a a b
b b a a
4:[p]
6:[p,q]
a b
ab ab ab ab
b
5:[q]
b a a b a
b ab
ab
7:[p,q]
8:[p,q]
b a
9:[p,q]

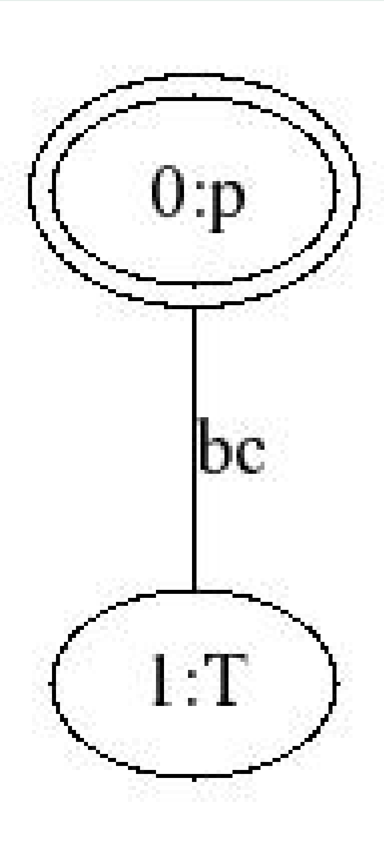
Oscillating between True and False Nirwana

Oscillating between True and False Nirwana

Private insight about p for a:



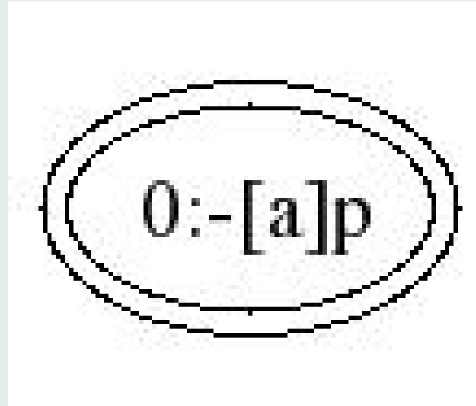
Oscillating between True and False Nirwana

Private insight about p for a: 

Public statement of the guru's ignorance: 

Public statement of the guru's ignorance: 

These two communications map from true nirwana to false nirwana and vice versa (check for yourself).

Public statement of the guru's ignorance: 

These two communications map from true nirwana to false nirwana and vice versa (check for yourself).

Research question: characterize the updates for which this can happen.

Updates that falsify themselves

Updates that falsify themselves

"Your wife is cheating you, and you don't know it!"

Updates that falsify themselves

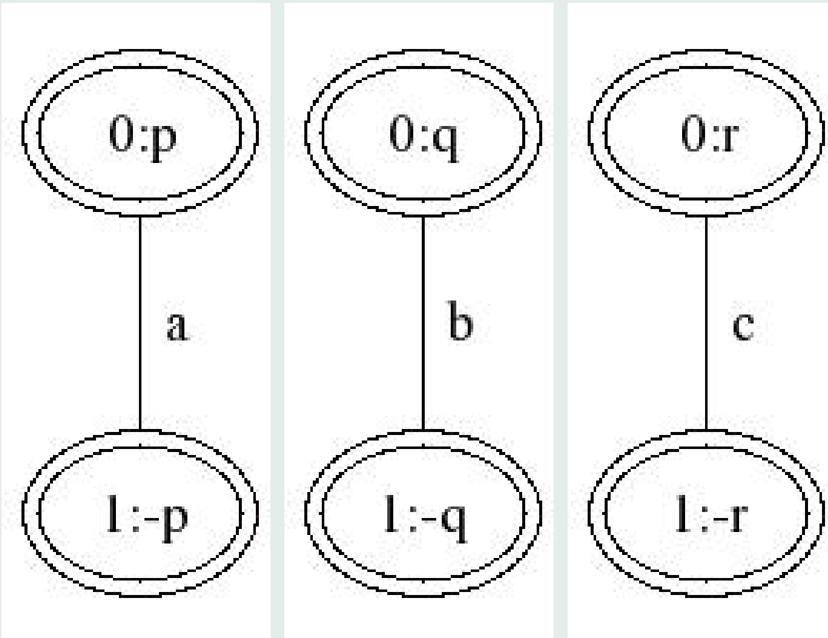
“Your wife is cheating you, and you don’t know it!”

Research question: characterize the updates that falsify themselves.

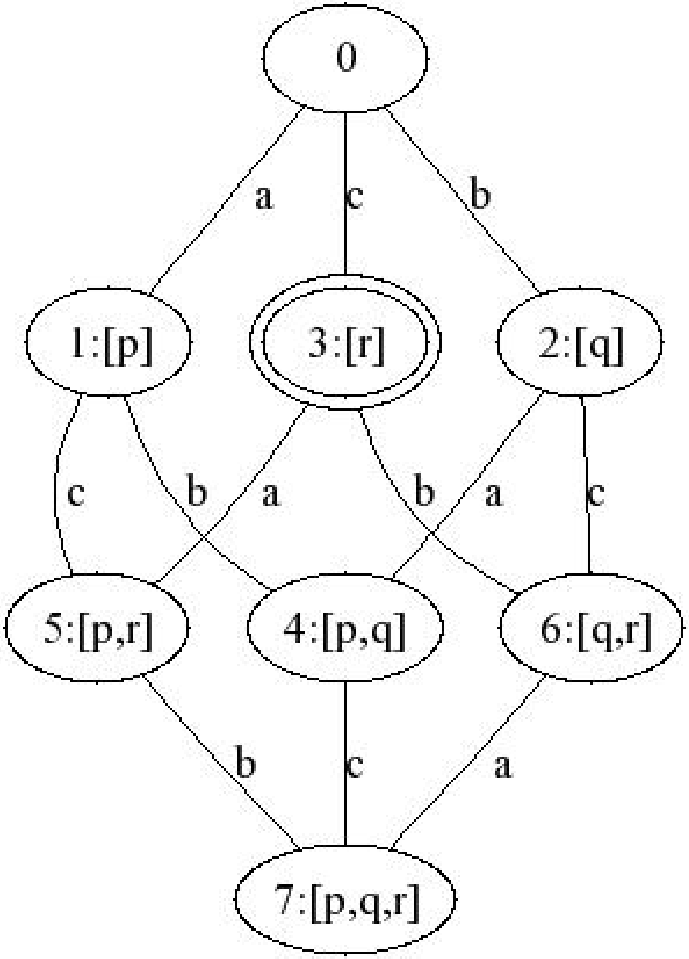
Muddy Children Again

Muddy Children Again

The awareness of the children about the mud on the others:

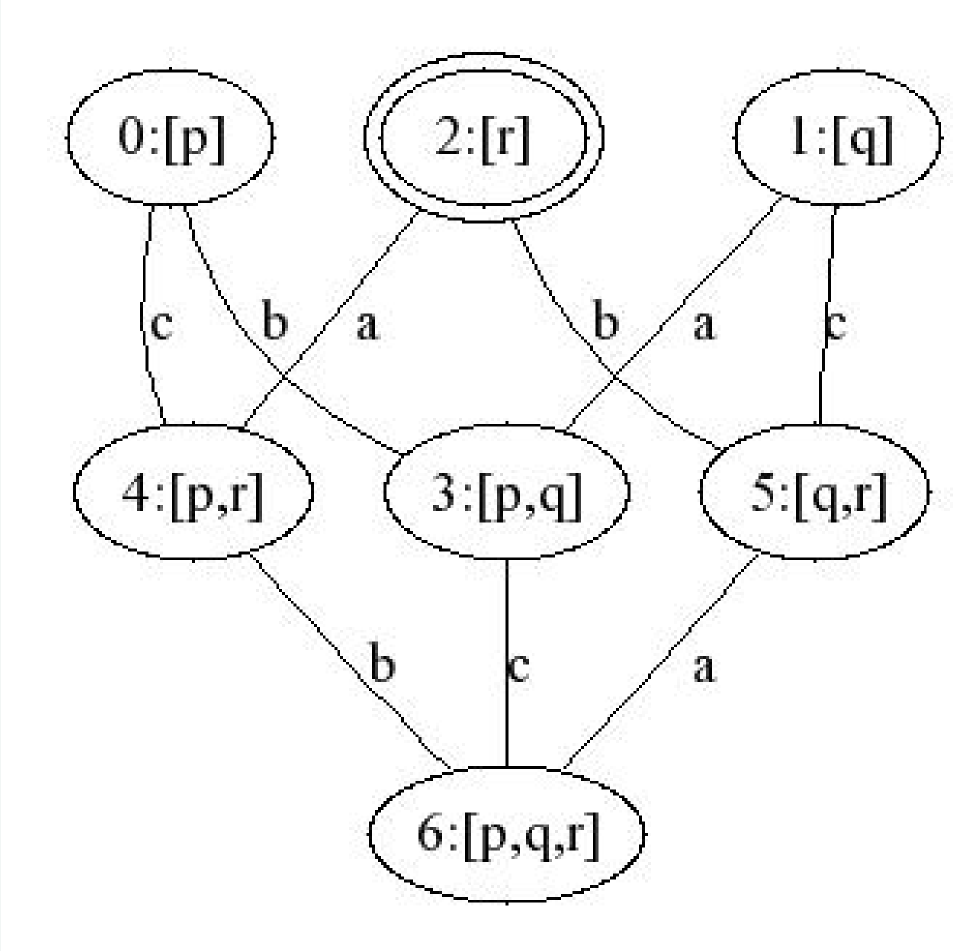


The initial situation:

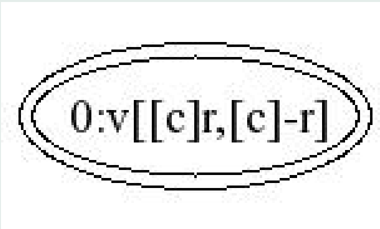

0
a
c
b
1:[p]
3:[r]
2:[q]
c
b
a
b
a
c
5:[p,r]
4:[p,q]
6:[q,r]
b
c
a
7:[p,q,r]

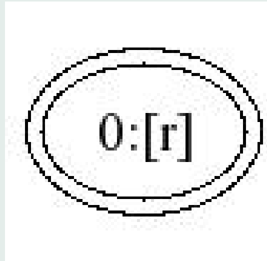
Update with the information that at least one child is muddy:


0:v[p,q,r]


0:[p]
2:[r]
1:[q]
c
b
a
b
a
c
4:[p,r]
3:[p,q]
5:[q,r]
b
c
a
6:[p,q,r]

Update with the information that c knows about her state:





Application to NL: The Meaning of Questions

Three kinds of information updates (there are many others):

- Public announcement
- Public revelation
- Private message

Application to NL: The Meaning of Questions

Three kinds of information updates (there are many others):

- Public announcement
- Public revelation
- Private message

Applying the slogan the meaning of a question is the set of its possible answers yields three kinds of semantics for questions:

PA Question Answered by public announcement

PR Question Answered by public revelation

PrM Question Answered by private message

PDL + Epistemic Updates

$$\varphi \;::=\; \top \mid p \mid \neg\varphi \mid \varphi_1 \wedge \varphi_2 \mid [\pi]\varphi \mid [A, s]\varphi$$

$$\pi \;::=\; a \mid ?\varphi \mid \pi_1 ; \pi_2 \mid \pi_1 \cup \pi_2 \mid \pi^*$$

PDL + Epistemic Updates

$$\varphi \ ::= \ \top \mid p \mid \neg\varphi \mid \varphi_1 \wedge \varphi_2 \mid [\pi]\varphi \mid [A, s]\varphi$$

$$\pi \ ::= \ a \mid ?\varphi \mid \pi_1; \pi_2 \mid \pi_1 \cup \pi_2 \mid \pi^*$$

- the individual knowledge operator $K_a\varphi$ (or $\Box_a\varphi$) takes the shape $[a]\varphi$,

PDL + Epistemic Updates

$$\varphi ::= \top \mid p \mid \neg\varphi \mid \varphi_1 \wedge \varphi_2 \mid [\pi]\varphi \mid [A, s]\varphi$$

$$\pi ::= a \mid ?\varphi \mid \pi_1; \pi_2 \mid \pi_1 \cup \pi_2 \mid \pi^*$$

- the individual knowledge operator $K_a\varphi$ (or $\Box_a\varphi$) takes the shape $[a]\varphi$,
- the general knowledge operator $E_B\varphi$ takes the shape $[B]\varphi$, where B is shorthand for $b_1 \cup b_2 \cup \cdots \cup b_n$.

PDL + Epistemic Updates

$$\varphi ::= \top \mid p \mid \neg\varphi \mid \varphi_1 \wedge \varphi_2 \mid [\pi]\varphi \mid [A, s]\varphi$$

$$\pi ::= a \mid ?\varphi \mid \pi_1 ; \pi_2 \mid \pi_1 \cup \pi_2 \mid \pi^*$$

- the individual knowledge operator $K_a\varphi$ (or $\Box_a\varphi$) takes the shape $[a]\varphi$,
- the general knowledge operator $E_B\varphi$ takes the shape $[B]\varphi$, where B is shorthand for $b_1 \cup b_2 \cup \cdots \cup b_n$.
- the common knowledge operator $C_B\varphi$ takes the shape $[B^*]\varphi$.

$$\begin{aligned}
[\![\top]\!]^{\mathbf{M}} &= W_{\mathbf{M}} \\
[\![p]\!]^{\mathbf{M}} &= \{w \in W_{\mathbf{M}} \mid p \in V_{\mathbf{M}}(w)\} \\
[\![\neg\varphi]\!]^{\mathbf{M}} &= W_{\mathbf{M}} - [\![\varphi]\!]^{\mathbf{M}} \\
[\![\varphi_1 \wedge \varphi_2]\!]^{\mathbf{M}} &= [\![\varphi_1]\!]^{\mathbf{M}} \cap [\![\varphi_2]\!]^{\mathbf{M}} \\
[\![[\pi]\varphi]\!]^{\mathbf{M}} &= \{w \in W_{\mathbf{M}} \mid \forall v(\text{ if } (w,v) \in [\![\pi]\!]^{\mathbf{M}} \text{ then } v \in [\![\varphi]\!]^{\mathbf{M}})\} \\
[\![[A,s]\varphi]\!]^{\mathbf{M}} &= \{w \in W_{\mathbf{M}} \mid \text{if } \mathbf{M} \models_w \mathsf{pre}(s) \text{ then } (w,s) \in [\![\varphi]\!]^{\mathbf{M}\otimes A}\}
\end{aligned}$$

$$\begin{aligned}
[\![a]\!]^{\mathbf{M}} &= \xrightarrow{a}_{\mathbf{M}} \\
[\![?\varphi]\!]^{\mathbf{M}} &= \{(w,w) \in W_{\mathbf{M}} \times W_{\mathbf{M}} \mid w \in [\![\varphi]\!]^{\mathbf{M}}\} \\
[\![\pi_1;\pi_2]\!]^{\mathbf{M}} &= [\![\pi_1]\!]^{\mathbf{M}} \circ [\![\pi_2]\!]^{\mathbf{M}} \\
[\![\pi_1 \cup \pi_2]\!]^{\mathbf{M}} &= [\![\pi_1]\!]^{\mathbf{M}} \cup [\![\pi_2]\!]^{\mathbf{M}} \\
[\![\pi^*]\!]^{\mathbf{M}} &= ([\![\pi]\!]^{\mathbf{M}})^*
\end{aligned}$$

PDL + Epistemic Updates Reduces to PDL

Program transformation [2]:

$$
\begin{aligned}
T^A_{ij}(a) &= \begin{cases} ?\mathsf{pre}(s_i); a & \text{if } s_i \xrightarrow{a} s_j, \\ ?\bot & \text{otherwise} \end{cases} \\
T^A_{ij}(?\varphi) &= \begin{cases} ?\varphi & \text{if } i = j, \\ ?\bot & \text{otherwise} \end{cases} \\
T^A_{ij}(\pi_1; \pi_2) &= \bigcup_{k=0}^{n-1} (T^A_{ik}(\pi_1); T^A_{kj}(\pi_2)) \\
T^A_{ij}(\pi_1 \cup \pi_2) &= T^A_{ij}(\pi_1) \cup T^A_{ij}(\pi_2) \\
T^A_{ij}(\pi^*) &= K^A_{ijn}(\pi)
\end{aligned}
$$

where $K^A_{ijk}(\pi)$ is a (transformed) program for all the π^* paths from s_i to s_j that can be traced through A while avoiding a pass through intermediate states s_k and higher.

In particular:

- $K^A_{ij0}(\pi)$ is a program for all the π^* paths from s_i to s_j that can be traced through A without stopovers at intermediate states, i.e., if $i = j$ it either is the skip action or a direct π loop, and otherwise it is a direct π step.

In particular:

- $K^A_{ij0}(\pi)$ is a program for all the π^* paths from s_i to s_j that can be traced through A without stopovers at intermediate states, i.e., if $i = j$ it either is the skip action or a direct π loop, and otherwise it is a direct π step.
- $K^A_{ijn}(\pi)$ is a program for all the π^* paths from s_i to s_j that can be traced through A, for stopovers at any s_k $(0 \leq k \leq n-1)$ are allowed.

In particular:

- $K^A_{ij0}(\pi)$ is a program for all the π^* paths from s_i to s_j that can be traced through A without stopovers at intermediate states, i.e., if $i = j$ it either is the skip action or a direct π loop, and otherwise it is a direct π step.
- $K^A_{ijn}(\pi)$ is a program for all the π^* paths from s_i to s_j that can be traced through A, for stopovers at any s_k $(0 \leq k \leq n-1)$ are allowed.
- Note that it is immaterial how many times a stopover is made at a particular intermediate state.

$K^A_{ijk}(\pi)$ is defined by recursing on k, as follows:

$$K^A_{ij0}(\pi) \;=\; \begin{cases} ?\top \cup T^A_{ij}(\pi) & \text{if } i = j, \\ T^A_{ij}(\pi) & \text{otherwise} \end{cases}$$

$$K^A_{ij(k+1)}(\pi) \;=\; \begin{cases} (K^A_{kkk}(\pi))^* & \text{if } i = k = j, \\ (K^A_{kkk}(\pi))^*; K^A_{kjk}(\pi) & \text{if } i = k \neq j, \\ K^A_{ikk}(\pi); (K^A_{kkk}(\pi))^* & \text{if } i \neq k = j, \\ K^A_{ijk}(\pi) \cup (K^A_{ikk}(\pi); (K^A_{kkk}(\pi))^*; K^A_{kjk}(\pi)) & \text{otherwise} \\ & (i \neq k \neq j). \end{cases}$$

Lemma 1 (Kleene Path)

Suppose $(w, w') \in [\![T^A_{ij}(\pi)]\!]^{\mathbf{M}}$ *iff there is a* π *path from* (w, s_i) *to* (w', s_j) *in* $\mathbf{M} \otimes A$.

Then $(w, w') \in [\![K^A_{ijn}(\pi)]\!]^{\mathbf{M}}$ *iff there is a* π^* *path from* (w, s_i) *to* (w', s_j) *in* $\mathbf{M} \otimes A$.

Lemma 1 (Kleene Path)
Suppose $(w, w') \in [\![T^A_{ij}(\pi)]\!]^{\mathbf{M}}$ *iff there is a* π *path from* (w, s_i) *to* (w', s_j) *in* $\mathbf{M} \otimes A$.
Then $(w, w') \in [\![K^A_{ijn}(\pi)]\!]^{\mathbf{M}}$ *iff there is a* π^* *path from* (w, s_i) *to* (w', s_j) *in* $\mathbf{M} \otimes A$.

Lemma 2 (Program Transformation)
Assume A *has* n *states* $s_0, \ldots, s_{n-1}$. *Then:*

$$\mathbf{M} \models_w [A, s_i][\pi]\varphi \text{ iff } \mathbf{M} \models_w \bigwedge_{j=0}^{n-1} [T^A_{ij}(\pi)][A, s_j]\varphi.$$

Related Work and Conclusion

- [1] shows in a non-constructive way that logics of finite updates for particular action signatures can be embedded in PDL.

Related Work and Conclusion

- [1] shows in a non-constructive way that logics of finite updates for particular action signatures can be embedded in PDL.
- [4] shows how generic updates with epistemic actions can be axiomatized in automata PDL [3, Chapter 10.3].

Related Work and Conclusion

- [1] shows in a non-constructive way that logics of finite updates for particular action signatures can be embedded in PDL.
- [4] shows how generic updates with epistemic actions can be axiomatized in automata PDL [3, Chapter 10.3].
- [2] shows that the detour through automata is not necessary.

Related Work and Conclusion

- [1] shows in a non-constructive way that logics of finite updates for particular action signatures can be embedded in PDL.
- [4] shows how generic updates with epistemic actions can be axiomatized in automata PDL [3, Chapter 10.3].
- [2] shows that the detour through automata is not necessary.
- If you are looking for a generic logic of communication, PDL + Epistemic Updates might be your choice!

References

[1] A. Baltag, L.S. Moss, and S. Solecki. The logic of public announcements, common knowledge, and private suspicions. Technical report, Dept of Cognitive Science, Indiana University and Dept of Computing, Oxford University, 2003.

[2] Jan van Eijck. Reducing dynamic epistemic logic to PDL by program transformation. CWI, Amsterdam, www.cwi.nl:~/papers/04/delpdl/, 2004.

[3] D. Harel, D. Kozen, and J. Tiuryn. Dynamic Logic. Foundations of Computing. MIT Press, Cambridge, Massachusetts, 2000.

[4] B. Kooi and J. van Benthem. Reduction axioms for epistemic actions. In R. Schmidt, I Pratt-Hartmann, M. Reynolds, and H. Wansing, editors, AiML-2004: Advances in Modal Logic, number UMCS-

04-9-1 in Technical Report Series, pages 197–211. University of Manchester, 2004.

Right is not right, so is not so.
If right were really right, it would differ so clearly from not right that there would be no need for argument.
If so were really so, it would differ so clearly from not so that there would be no need for argument.
Forget the years; forget distinctions. Leap into the boundless and make it your home!

Chuang Tzu