Referee Report: Information and Computation ms C3020

This is a very interesting, original contribution, which clearly deserves acceptance. It proposes simpler axiomatizations for dynamic-epistemic logics with common knowledge, by embedding them (via reduction axioms) into a system called “epistemic PDL”, which is nothing but PDL with an epistemic interpretation of its basic modalities. The proof uses the idea behind Kleene’s Theorem for regular languages. Various intermediary logics are distinguished, via a number of interesting expressivity results. The approach is computationally useful, as shown in Section 5, where it is used to compute valid reduction axioms for (common knowledge after) various important communication types.

I strongly recommend acceptance, while I also give below a list of typos, corrections and conceptual comments for the authors.

List of typos, corrections and comments:

1. One the key definitions of this paper (Definition 4, pg. 4, Section 2.2, defining “relativized common knowledge”) is actually wrong! More precisely (since in Logic a definition cannot in itself be “wrong”...): (a) it is not equivalent (as claimed) with the expression given in the Remark in the center of page 5 (just before Section 2.4); and moreover (b) it doesn’t fit with the axiomatic system on page 5 (in the sense that the system is obviously not complete for this semantics of relativized common knowledge). Indeed, the notion $C_B(\phi, \psi)$, as defined, satisfies a stronger axiom than (Mix), since its underlying relation is reflexive, and so we have

   $C_B(\phi, \psi) \rightarrow \psi$

   (and not just

   $C_B(\phi, \psi) \rightarrow (\phi \rightarrow \psi)$

   as claimed by Mix, in this paper’s version).

I hesitated for a while in deciding where is the error, i.e. which definition corresponds best with author’s intentions and intuitions (Def. 4 in sect. 2.2, or the Remark on page 5 of section 2.3, which agrees with the axiom system in 2.3). I think the answer is (as given above): Definition 4 on page 4 is the wrong one. The intention behind the concept of relativized common knowledge doesn’t seem to be the one
of a simple reflexive modality (which automatically implies truth of the sentence prefixed by the modality). On the contrary, the truth of a sentence prefixed by this modality is to be accepted only \textit{conditionally}. More precisely, the correct clause for defining (in Def. 4 of sect. 2.2) $M, w \models C_B(\phi, \psi)$ should be:

$$M, v \models \psi \text{ for all } v \text{ such that } (w, v) \in (\Delta \cap \|\phi\|^2); (R(B) \cap \|\phi\|^2)^n$$

where $\Delta$ is the diagonal relation and $\mid$ is relational composition. This definition (unlike the one in text) agrees with the Remark on page 5, as well as with the Mix axiom, as stated on page 5.

2. The above correction leads me to another suggestion: a more elaborate intuitive explanation of the notion of relativized common knowledge would be most welcome! The “dynamic” explanation given in text (“if $\phi$ is announced it will become common knowledge among $B$ that $\psi$ was the case before the announcement”) is very useful (and it actually led me to the above solution to the problem mentioned above), but one really feels the need for more clarification and more discussion here! In particular, a \textit{brief} way of reading (in English) the formula $C_B(\phi, \psi)$ could be suggested (e.g. $\psi$ is $\phi$-\textit{conditional common knowledge}, or $\psi$ is \textit{common knowledge} given $\phi$, or $\psi$ is \textit{common knowledge relative to} $\phi$). Maybe a brief mentioning (as a comparison) of the standard notion of \textit{conditional probability} would help. (In fact, this comparison suggests an alternative notation for $C_B(\phi, \psi)$, namely $C_B(\psi|\phi)$.)

3. The two axioms called $Dist$ (distribution) on page 5 are usually called $K$.

4. A typo in the first line of the Remark in the center of page 5: misspelling of the word “dynamic” (as “dunamic”).

5. In Def 6 (“closure”) on page 5, or immediately after (or before) this definition, it might be useful (especially for the modal-logic-literate reader) to mention that this “closure” is simply the (appropriate analogue of the) Fisher-Ladner closure.

6. Theorems 2 and 3 (in section 2.5) are not proved. More importantly, their (more or less obvious) inductive proofs depend essentially on the \textit{existence of the translation function $t$ “defined” in Definition 9} (page 6, Section 2.5). However, the existence of this function is not at all obvious! Definition 9 does \textbf{not} proceed by the standard recursion on
syntactic complexity of formulas: the formulas on the right-hand side of the defining clauses (especially in the last four clauses) are not in any obvious sense simpler than the ones on the left-hand side: so there is no obvious reason for which the process of “translation” would ever terminate (with a formula belonging to EL-RC); or in other words the existence of a function $t$ between the two languages (satisfying all the clauses) must be justified. The way this was done for a similar (simpler..) language in [1], [3] (using this paper’s list of references) was by defining a non-standard notion of complexity and proving termination of the corresponding translation by using results in the theory of rewriting systems. I do suppose the same method could be adapted here (or else some other way to prove termination could be employed), but nevertheless an argument is surely needed! Once this is done, Theorem 2 can be proved by induction on the given (non-standard) notion of logical complexity.

In fact, I think (?) that a similar comment applies to the later proof of Theorem 13 (page 26) and to the translation function in Definition 28 (page 27).

7. The proof of Theorem 4 refers to Lemma 2 (as showing that every formula in $PAL - RC$ is provably equivalent to one in EL-RC). But I think this lemma is missing! The only “Lemma 2” in the paper is in the next section (2.6), and it has no relation to what’s needed here. But the needed lemma (a proof-theoretic analogue of Theorem 2) could indeed be easily stated, and its proof follows from the axioms (but again, I think induction on a non-standard notion of complexity is needed).

8. In the formulation of the EL-RC Game (Definition 10, page 7, section 2.6), there is an ambiguity due to the use of the same variable $(n)$ to denote two (possibly different) numbers: the first $n$ refers to the number of rounds (as in line 2 of this definition, in italics: $n$-round EL-RC game); the second $n$ refers to the length of a $B$-path chosen by the Spoiler in an $RC_B$-move (as written in the definition of an $RC_B$ move: “Spoiler chooses a $B$-path $x_0 \ldots x_n$ ”, then again below “end points $x_n$ ”). These two $n$’s are not (and should not be) the same!

9. In defining the third kind of moves ($[\phi]$-moves) in Definition 12 (The PAL-C Game) on page 8, the notations $\overline{S}$ and $\overline{F}$ are used, without explanation. I think it’s worth mentioning here that these simply denote the complement sets...
10. No intuition for "epistemic PDL" (Section 4.2, Definition 21) is provided. No way to read (in English), or at least to understand (in epistemic terms), an epistemic-PDL formula is given. This is important, since the authors are looking for the "right" language for dynamic-epistemic logic; epistemic PDL is apparently proposed here for purely technical reasons, without any independent justification or explanation. However, I think the language can be justified as an epistemic language, by referring to Rohit Parikh's notion of "levels of knowledge". In one of his papers, Parikh generalizes the notions of common knowledge and individual knowledge, by introducing "regular levels of knowledge", i.e. many intermediary epistemic levels of inter-agent knowledge, one for each regular language over the basic alphabet of "agents"; common knowledge is the top of this hierarchy of levels (while each agent's individual knowledge is a bottom level). His discussion concerns only "truthful, introspective knowledge", so he assumes S5. But the analogue of this notion in this paper's more general context (of knowledge/belief, without any assumptions of truthfulness or introspection, and thus without assuming any of the S5 axioms) corresponds precisely to the modalities ("programs") of test-free epistemic PDL. By considering instead full epistemic PDL (with test), the authors could thus claim to capture a notion of "levels of conditional (or relativized) knowledge" (or rather, belief). This would give some epistemic flavor to "epistemic PDL".

11. On page 31, line -2 (i.e. second from below): the reduction formula for distributed knowledge after an update contains an obvious mistake. The right-hand side of the equivalence should be \( A \rightarrow D_B[A]\phi \). In fact, as a stylistic observation, it would be more consistent with the conventions of this paper to replace \( A \) by \( \psi \), since variables \( A, B \ldots \) are everywhere else used only to denote agents.