Optimal threshold selection for tomogram segmentation by reprojection of the reconstructed image

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Abstract. Grey value thresholding is a segmentation technique commonly applied to tomographic image reconstructions. Many procedures have been proposed to optimally select the grey value thresholds based on the image histogram. In this paper, a new method is presented that uses the tomographic projection data to determine optimal thresholds. The experimental results for phantom images show that our method obtains superior results compared to established histogram-based methods.

1 Introduction

Segmentation of tomographically reconstructed images (also called *tomograms*) is a well known problem in computer vision. It refers to the classification of image pixels into distinct classes that are characterized by a discrete set of grey values. Amongst all segmentation techniques, image thresholding is the simplest, yet often most effective segmentation method. Many algorithms have been proposed for selecting "optimal" thresholds with respect to various optimality measures [1]. Thresholds are typically selected from the histogram of the tomogram, such that the distance between the tomogram and the segmented image is minimized.

To the authors' knowledge, current segmentation techniques for tomographic images are applied to tomograms only and do not exploit the available projection data but are rather based on the image histogram [2–4]. Specifically, the tomogram histogram is often used as a basis for determining global thresholds by, for example, fitting multiple gaussian distributions to the histogram [5] or applying a k-means clustering algorithm to it [6]. More recent thresholding techniques are based on a minimum variance criterion [7] or employ the variance and intensity contrast [8]. In practice however, the image histogram often lacks clear modes that would allow an intuitive selection of threshold values. Indeed, tomograms may be polluted by artifacts that tend to smear out the image histogram. Typical artifacts are streaking artifacts caused by highly absorbing object parts, blurring caused by object motion during scanning (or equivalently caused by small shifts of the detector during acquisition), bias fields, or artifacts caused by a limited field of view and/or a missing wedge. Hence, although threshold selection based on the image histogram can be made fully automatic, it lacks robustness in case clear grey value modes are absent.

This paper presents a new approach to segmentation, which uses the information in the projection data instead of the histogram. Forward projections of the tomogram are computed and compared to the measured projection data. In the search for optimal thresholds, many computationally expensive forward projections must be performed. An important contribution of the current paper is the efficient implementation of the forward projection method, which makes using the original projection data as a segmentation criterion feasible. The proposed method does not suffer from subjectiveness of the user. Our simulation experiments demonstrate that our method is robust and clearly outperforms established histogram-based methods.

2 Grey level estimation

We restrict ourselves to the segmentation of a 2-dimensional image, which is represented on a rectangular grid of width w and height h. Hence, the total number of pixels is given by $n = wh$. The grey-scale image $\boldsymbol{x} \in \mathbb{R}^n$ that we want to segment is a tomographic reconstruction of some physical object, of which projections were acquired using a tomographic scanner. Our method can be used for any scanning geometry, e.g., parallel beam, fan beam and, in 3D, cone beam. Projections are measured as sets of detector values for various angles, rotating around the object. Let m denote the total number of measured detector values (for all angles) and let $\boldsymbol{p} \in \mathbb{R}^m$ denote the measured data. The physical projection process in tomography can be modeled as a linear operator $\boldsymbol{W}$ that maps the image $\boldsymbol{x}$ (representing the object) to the vector $\boldsymbol{p}$ of measured data:

$$\boldsymbol{W}\boldsymbol{x} = \boldsymbol{p}. \tag{1}$$

For parallel projection data, the operator $\boldsymbol{W}$ is a discretized version of the well-known *Radon-transform*. We represent $\boldsymbol{W}$ by an $m \times n$ matrix (w_{ij}). Note that for each projection angle, every pixel i will only project onto a few detector pixels, so the matrix $\boldsymbol{W}$ is very sparse. Exploiting this sparsity forms an essential part of our method. The matrix representation of the projection operator is commonly used in *algebraic reconstruction algorithms*. We refer to Chapter 7 of [9] for details.

From this point on, we assume that an image $\boldsymbol{x}$ has been computed that approximately satisfies Eq. (1). Our approach does not depend on the reconstruction algorithm that was used to compute $\boldsymbol{x}$, e.g., Filtered Backprojection or ART. The image $\boldsymbol{x}$ now has to be segmented using global thresholding of the grey levels. The main motivation of using thresholding in general, is that pixels representing the same "material" in the scanned object should have approximately the same grey values in the tomogram. We rely strongly on the assumption that the scanned object consists of only a few different materials, which is true for all segmentation methods based on thresholding. Our segmentation approach first assigns a real-valued grey level to each of the segmentation classes. Using these grey levels, the projections of the segmented image are then computed. The computed forward projections are compared to the measured projection data, which provides a measure for the quality of the segmentation (along with the chosen grey levels). This quality measure can also be used for other segmentation techniques than thresholding.

We first consider the problem of determining grey levels for each of the segmentation classes of a segmented image. We can consider a segmentation of an image into ℓ classes as a *partition* of the set of pixels, consisting of ℓ subsets. Let $\mathcal{S} = \{S_1, \ldots, S_\ell\}$ be a partition of $\{1, \ldots, n\}$. We label each set by its index t: S_t. Each pixel j is contained in exactly one set $S_t \subset \mathcal{S}$, denoted by $s(j) \in \{1, \ldots, \ell\}$. To each set S_t, a grey level $\rho_t \in \mathbb{R}$ is assigned, which induces an assignment of grey levels to the pixels $1 \leq j \leq n$, where pixel j is assigned the grey level $\rho_{s(j)}$. Define $\boldsymbol{r}_{\mathcal{S}}(\boldsymbol{\rho}) = \left(\rho_{s(1)} \ldots \rho_{s(n)}\right)^T$. The vector $\boldsymbol{r}_{\mathcal{S}}(\boldsymbol{\rho})$ contains, for each pixel j, the corresponding grey level of that pixel.

Our goal is to determine "optimal" grey values $\boldsymbol{\rho}$ for the given partition $\mathcal{S}$. The quality of a vector $\boldsymbol{\rho}$ is determined by computing the projections of the segmented image, using the grey levels from $\boldsymbol{\rho}$, and comparing the computed projections to the measured projections $\boldsymbol{p}$. More formally, we define the problem of finding optimal grey values for a given partition as follows:

Problem 1. Let $\boldsymbol{W} \in \mathbb{R}^{m \times n}$ be a given projection matrix, let $\mathcal{S} = \{S_1, \ldots, S_\ell\}$ be a partition of $\{1, \ldots, n\}$ and let $\boldsymbol{p} \in \mathbb{R}^m$ be a vector of measured projection data. Find $\boldsymbol{\rho} \in \mathbb{R}^\ell$ such that $|\boldsymbol{W}\boldsymbol{r}_{\mathcal{S}}(\boldsymbol{\rho}) - \boldsymbol{p}|_2$ is minimal.

We will start by deriving the equations for solving Problem 1 for a fixed partition $\mathcal{S}$. Subsequently, we describe how the optimal set of grey values can be re-computed efficiently, each time the partition $\mathcal{S}$ has been modified by moving a single pixel from one class to another class. This fast update computation allows us to design efficient algorithms that combine the search for a segmentation $\mathcal{S}$ and the corresponding grey values $\boldsymbol{\rho}$, such that the projection distance is minimal.

Define $\boldsymbol{A} = (a_{it}) \in \mathbb{R}^{m \times \ell}$ by

$$a_{it} = \sum_{j:\ s(j)=t} w_{ij} \quad . \tag{2}$$

The value a_{it} equals the total area of pixels from the set S_t that contribute to detector value i. We denote the row vectors of $\boldsymbol{A}$ by $\boldsymbol{a_i} = \boldsymbol{A}_{\cdot t}$. Clearly, we have

$$[\boldsymbol{W}\boldsymbol{r}_{\mathcal{S}}(\boldsymbol{\rho})]_i = \sum_{t=1}^{\ell} a_{it}\rho_t. \tag{3}$$

Define the *projection difference* $\boldsymbol{d} \in \mathbb{R}^m$ by $\boldsymbol{d} = \boldsymbol{W}\boldsymbol{r}_{\mathcal{S}}(\boldsymbol{\rho}) - \boldsymbol{p} = \boldsymbol{A}\boldsymbol{\rho} - \boldsymbol{p}$. Put $\boldsymbol{c_i} = -2p_i\boldsymbol{a_i}$, $\boldsymbol{Q_i} = \boldsymbol{a_i}\boldsymbol{a_i}^T$. Define the *squared total projection difference* by

$$|\boldsymbol{d}|^2 = |\boldsymbol{A}\boldsymbol{\rho} - \boldsymbol{p}|^2 = \sum_{i=1}^{m}(\boldsymbol{a_i}^T\boldsymbol{\rho} - p_i)^2 = \sum_{i=1}^{m}(\boldsymbol{c_i}^T\boldsymbol{\rho} + \boldsymbol{\rho}^T\boldsymbol{Q_i}\boldsymbol{\rho} + p_i^2). \tag{4}$$

Define $\bar{\boldsymbol{c}} = \sum_{i=1}^{m} \boldsymbol{c_i}$, $\bar{\boldsymbol{Q}} = \sum_{i=1}^{m} \boldsymbol{Q_i}$. Thus $|\boldsymbol{d}|^2 = |\boldsymbol{p}|^2 + \bar{\boldsymbol{c}}^T \boldsymbol{\rho} + \boldsymbol{\rho}^T \bar{\boldsymbol{Q}} \boldsymbol{\rho}$.

Note that each of the terms $\boldsymbol{c_i}$ and $\boldsymbol{Q_i}$ only depend on $\boldsymbol{a_i}$ and p_i, not on $\boldsymbol{\rho}$. A vector $\boldsymbol{\rho}$ that minimizes the projection difference $|\boldsymbol{d}|$ can now be computed by setting the derivatives of $|\boldsymbol{d}|^2$ with respect to $\rho_1, \ldots, \rho_\ell$ to 0, obtaining the system $2\bar{\boldsymbol{Q}}\boldsymbol{\rho} = -\bar{\boldsymbol{c}}$, and solving for $\boldsymbol{\rho}$.

So far, we have assumed that the partition $\mathcal{S}$ was fixed. Suppose that we have computed $\bar{\boldsymbol{c}}$ and $\bar{\boldsymbol{Q}}$ for the partition $\mathcal{S}$. We now change $\mathcal{S}$ into a new partition $\mathcal{S}'$, by changing $s(j)$ for a single pixel j.

The only rows of $\boldsymbol{A}$ that are affected by this transition are the rows i for which $w_{ij} \neq 0$. This means that the new vector $\bar{\boldsymbol{c}}'$ and matrix $\bar{\boldsymbol{Q}}'$ can be computed by the following updates:

$$\bar{\boldsymbol{c}}' = \bar{\boldsymbol{c}} + \sum_{i:w_{ij} \neq 0} (\boldsymbol{c_i}' - \boldsymbol{c_i}) \tag{5}$$

and

$$\bar{\boldsymbol{Q}}' = \bar{\boldsymbol{Q}} + \sum_{i:w_{ij} \neq 0} (\boldsymbol{Q}_{\boldsymbol{i}}' - \boldsymbol{Q_i}). \tag{6}$$

The fact that $\bar{\boldsymbol{c}}'$ and $\bar{\boldsymbol{Q}}'$ can be computed by applying updates for only a few of the terms $\boldsymbol{c_i}$ and $\boldsymbol{Q_i}$ respectively, means that the optimal grey values for the entire image can be recomputed efficiently. This property allows us to develop algorithms for simultaneous segmentation and grey level estimation. Any segmentation algorithm that moves pixels from one class into another class, one at a time, can efficiently keep track of the optimal grey levels. In the next section we demonstrate this approach for a simple thresholding algorithm, finding both the optimal thresholds and grey values in a single pass.

3 Thresholding with simultaneous grey level estimation

The concepts in the previous section apply to any partition $\mathcal{S}$ of the pixels. We will now restrict ourselves to a specific type of partitions, that are induced by a thresholding scheme. Our starting point is a grey level image $\boldsymbol{x} \in \mathbb{R}^n$, that has been computed by any continuous tomographic reconstruction algorithm, e.g., Filtered Back Projection. The image $\boldsymbol{x}$ now needs to be segmented by means of thresholding, using a fixed number of ℓ classes for the pixels.

Every pixel i is assigned a class according to a thresholding scheme using thresholds $\tau_1 < \tau_2 < \ldots < \tau_{\ell-1}$. Put $\boldsymbol{\tau} = (\tau_1 \ \ldots \ \tau_{\ell-1})^T$. Define the *threshold function* by

$$s(i, \boldsymbol{\tau}) = \begin{cases} 1 & (x_i < \tau_1) \\ 2 & (\tau_1 \leq x_i < \tau_2) \\ \ldots & \\ \ell & (\tau_{\ell-1} \leq x_i) \end{cases} . \tag{7}$$

The threshold function induces a partition $\mathcal{S}_{\boldsymbol{\tau}}$ of the set $\{1, \ldots, n\}$. Define

$$\boldsymbol{r}(\boldsymbol{\tau}, \boldsymbol{\rho}) = \left(\rho_{s(1,\boldsymbol{\tau})} \ \ldots \ \rho_{s(n,\boldsymbol{\tau})}\right)^T .$$

Similar to Section 2, we define the quality of a set of thresholds $\boldsymbol{\tau}$ and grey values $\boldsymbol{\rho}$ as the total squared projection distance for the corresponding segmentation. We now consider the problem of simultaneously finding thresholds and grey values, such that the total squared projection difference is minimal:

Problem 2. Let $\boldsymbol{W} \in \mathbb{R}^{m\times n}$ be a given projection matrix, let $x \in \mathbb{R}^n$ be a grey scale image and let $\boldsymbol{p} \in \mathbb{R}^m$ be a vector of measured projection data. Find $\boldsymbol{\tau} \in \mathbb{R}^{\ell-1}$ with $\tau_1 < \ldots < \tau_{\ell-1}$ and $\boldsymbol{\rho} \in \mathbb{R}^\ell$, such that $|\boldsymbol{W}\boldsymbol{r}(\boldsymbol{\tau},\boldsymbol{\rho}) - \boldsymbol{p}|_2$ is minimal.

The simplest case of Problem 2 occurs when the image $\boldsymbol{x}$ is segmented into two classes, using a single threshold τ. In that case, an exhaustive search over all possible thresholds can be performed efficiently: first, a list of all pixels is computed, sorted in ascending order of grey level in the continuous reconstruction $\boldsymbol{x}$. The start segmentation is formed by setting the threshold τ at infinity, so all pixels will be in the segmentation class S_1. The threshold τ is now gradually decreased, each time moving pixels from S_1 to S_2. By using the update operation from Section 2, it is possible to keep track of the optimal grey values and the projection difference by applying only small update steps. Figure 1 shows the steps of the segmentation algorithm. For each pixel j, the time complexity of the update computation is $O(u(j)\ell^2)$, where $u(j) = \#\{i : w_{ij} \neq 0\}$. Each pixel is moved from S_1 to S_2 only once. Therefore, the total time complexity of the algorithm is $O(U\ell^2)$, where U denotes the total number of nonzero elements in the projection matrix $\boldsymbol{W}$.

Make a list L containing all elements $j \in \{1,\ldots,n\}$, sorted in ascending order of x_j;

$\tau := \infty$; $S_1 := \{1,\ldots,n\}$; $S_2 := \emptyset$;

For $i = 1,\ldots,m$: $a_{i1} := \sum_{j=1}^{m} w_{ij}$; $a_{i2} := 0$; compute $\boldsymbol{c_i}$ and $\boldsymbol{Q_i}$;

Compute $\bar{\boldsymbol{c}}$ and $\bar{\boldsymbol{Q}}$;

$S_1 := \emptyset$; $S_2 = \{1,\ldots,n\}$; $k := n+1$;

while $k > 1$ **do**
begin
  $k := k-1$; $\tau := x_{L(k)}$;
  while $(k \geq 1)$ **and** $(x_{L(k)} = \tau)$ **do**
  begin
    $k := k-1$; $j := L(k)$; $S_1 := S_1 - \{j\}$; $S_2 := S_2 \cup \{j\}$;
    for each i such that $w_{ij} \neq 0$: update $\boldsymbol{c_i}$, $\boldsymbol{Q_i}$, $\bar{\boldsymbol{c}}$ and $\bar{\boldsymbol{Q}}$;
  end
  Compute the minimizer $\boldsymbol{\rho}$ of $|d|^2 = |\boldsymbol{p}|^2 + \bar{\boldsymbol{c}}^T\boldsymbol{\rho} + \boldsymbol{\rho}^T\bar{\boldsymbol{Q}}\boldsymbol{\rho}$;
  if $(d < d_{\text{opt}})$ **then**
    $d_{\text{opt}} := d$; $\boldsymbol{\rho}_{\text{opt}} := \boldsymbol{\rho}$; $\tau_{\text{opt}} := \tau$;
end

Fig. 1. basic steps of the algorithm for solving Problem 2 in the case $\ell = 2$.

3.1 More than two grey levels

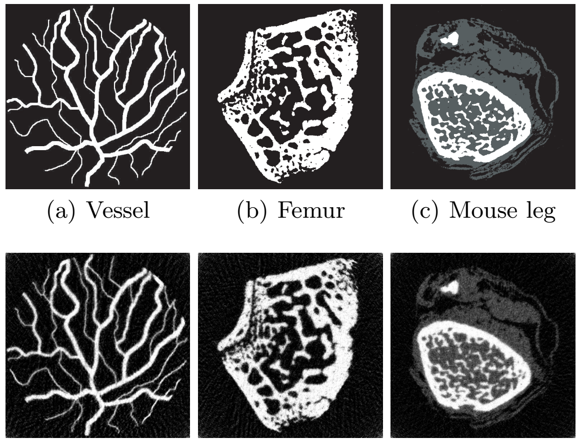
If there are more than two segmentation classes, it is usually not possible to compute the total squared projection error for each possible set of candidate thresholds, due to the vast number of candidates. However, using the update operation from Section 2, it is possible to compute a *local minimum* of the projection error in reasonable time. A simple algorithm for this case is the following: first, determine initial thresholds $\boldsymbol{\tau}$, possibly using another automated procedure, such as fitting Gaussian functions to the histogram. Next, compute A, $\bar{Q}$ and $\bar{\boldsymbol{c}}$. In an iteratively loop, compute for each threshold the effect of a small increase and a small decrease of that threshold on the total projection error. Among all these possible steps, select the one that results in the largest decrease of the total projection error. The algorithm terminates if no step can be found that decreases the total error. The initial estimate $\boldsymbol{\tau}_0$ can be computed using another automated procedure, such as fitting Gaussian functions to the histogram.

4 Results and discussion

In order to validate our proposed threshold selection method, simulation experiments were set up. For this purpose, three phantom images of size 512×512 were constructed: a binary *vessel* image representing a vessel tree, a binary *femur* image, and a grey valued *mouse leg* image (see Fig 2). From these images, CT projections were simulated as follows. First, the Radon transform of the images was computed, resulting in a sinogram for which each data point represents the line integral of attenuation coefficients. Then, (noiseless) CT projection data were generated where a mono-energetic X-ray beam was assumed. The projections were then polluted with Poisson distributed noise where the number of photons per detector element was varied from $5 \times 10^2 - 5 \times 10^4$. Next, the noisy sinogram of the attenuation coefficients was obtained by dividing the CT projection data by the maximum intensity and computing the negative logarithm. Finally, the simulated, noisy CT reconstructions were obtained by applying a SIRT algorithm. For each case, 10 independent noisy sinograms were generated. We refer to [9] for further details on image formation in CT and on the SIRT algorithm.

The proposed tomogram thresholding technique was then compared to commonly applied thresholding methods [10]. First, a parametric optimal thresholding technique was implemented where the image histogram was fitted to a mixture probability density function (two gaussian functions) from which an optimal threshold was derived [5]. Next, the commonly used iterative threshold selection scheme of Otsu was implemented [6]. This is also the default thresholding method used in Matlab. As a final method, k-means clustering was applied to the image histogram.

For each simulated reconstruction, global thresholds were computed. Then, the number of misclassified pixels for each method, referred to as N_m, was compared to the number of misclassified pixels N_{opt} of the 'optimally thresholded



(a) Vessel (b) Femur (c) Mouse leg

(d) Vessel (e) Femur (f) Mouse leg

Fig. 2. (a-c) Phantom images ; (d-f) Simulated CT reconstructions from 90 projections.

image'. The latter image can be found by an exhaustive search over all possible threshold values and comparing the thresholded image to the original, noiseless image. From those numbers, a measure for the number of correctly classified pixels for each method is given by: $R = 100 * N_m/N_{opt}$which will be referred to as the classification performance ratio. For each method, R is evaluated as a function of the number of photons per detector element employed during simulation of the CT sinograms.

Typical results of the simulation experiments are shown in Fig. 3. Since the classification performance ratio is based on N_{opt}, this number is denoted above the figure for each data point. The error bars around each point indicate the range of the results for the 10 independent experiments. From the figure, it is clear that the proposed projection based method outperforms conventional thresholding techniques with respect to the classification performance ratio. Similar results were obtained for the vessel and mouse-leg images. For the vessel image, the classification performance ratio of our proposed method was above 92% in all tests, whereas the best performing alternative method, k-means clustering, dropped to 80% in the test with the highest count per detector element. For all tests, the running time was around 10s on a standard desktop PC.

5 Conclusions

Global grey value thresholding is a trivial, yet often used segmentation technique. The search for the optimal grey level threshold is, however, far from trivial. Many procedures have been proposed to select the grey value threshold based on the image histogram. In our paper, we have presented an innovative approach to find the optimal threshold grey levels by exploiting the available projection data. The

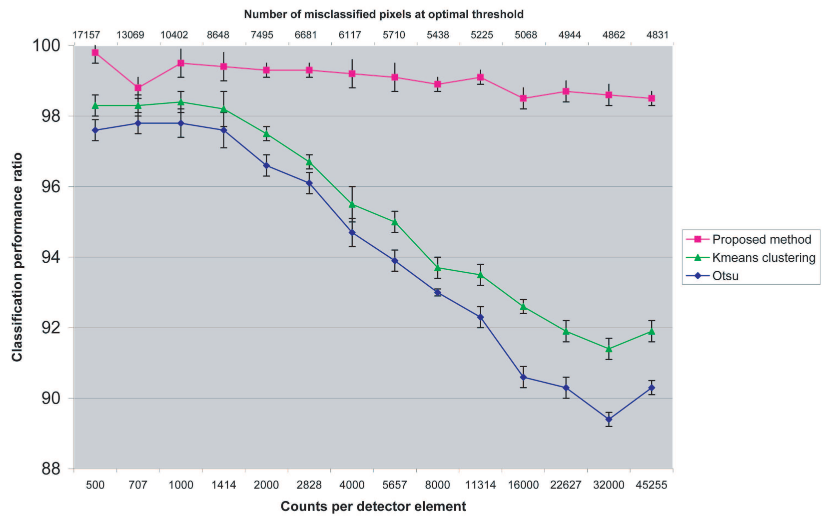


Fig. 3. Thresholding results of two commonly applied thresholding methods and the proposed method for the femur image (cfr Fig. 2(b)). The numbers at the top of the figure indicate the least possible number of misclassified pixels for each case

results on simulated CT-reconstructions show that our proposed method obtains superior results compared to established histogram-based methods.

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