

# Tolstoy's Mathematics in "War and Peace"

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The nineteenth century Russian author Leo Tolstoy based his egalitarian views on sociology and history on mathematical and probabilistic views and also proposed a mathematical theory of waging war.

## 1 Introduction

It is interesting to consider the excursions of mathematicians and scientists into prose and poetry, and conversely and less known, the explorations of poets and novelists into mathematics.

An example of the first is Luitzen E.J. Brouwer's excursion into literature and environmentalism [1], an appeal *avant la lettre* to save the earth's natural environment from human pollution. In particular he wants to abolish the technology that enables man's supremacy over nature and the physics and mathematics that makes this possible. Only pure ('intuitionistic') mathematics that by its nature is unapplied and unapplicable for evil purposes, and which is the ultimate creation of the noble mind, should be saved.

In another direction, the great Russian mathematician Andrei N. Kolmogorov was particularly interested in the form and structure of the poetry by the Russian author Pushkin [3]. He also remarks [4]: "what real meaning is there, for example, in asking how much information is contained in 'War and Peace'? Is it reasonable to include this novel in the set of 'possible novels,' or even to postulate some probability distribution for this set? Or, on the other hand, must we assume that the individual scenes in this book form a random sequence with 'stochastic relations' that damp out quite rapidly over a distance of several pages?"

The answer to the latter question is decidedly 'no'. There is a ubiquitous general theme in 'War and Peace', namely, the idea that single individuals cannot influence in any sense the course of history (contrary to what is assumed in common history writing), but that the course of history is

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determined by the confluence of myriad motions of the infinitesimally small individual human acts of free will, much as a flock of birds wheels about in unison without any apparent governor. Here we have individual humans as interchangeable atoms of ideal gas that in combination determine effects on macroscopic scales such as heat and pressure, as in the nineteenth century statistical physics of H. von Helmholtz. It serves to justify egalitarian doctrine. Helmholtz is also the author of the unrelated whitticism, so true and so unknown to politicians and managers of science: “Whoever in the pursuit of science, seeks after immediate practical utility may rest assured that he seeks in vain,” [2].

It is unknown and perhaps unlikely that the great Russian novelist Count Leo Nikolayevich Tolstoy was aware of Helmholtz’s work (or directly related work); however he may have been exposed to Laplace’s [5] *Essay* exposing the world as a mechanistic ensemble of moving and colliding particles that by their combined microscopic actions produce macroscopic effects. This is all the more likely because of the immense influence of Laplace’s work, combined with Tolstoy’s interest in justice and believable testimony in his role as country noble passing judgement on his people. The *Essay* in fact treats in great detail matters of believable testimony and probability of proper justice—this was a time when relevant matters mattered.

An issue for Tolstoy is unknowability and uncertainty: he is not really seeking a usable model so much as a *reductio ad absurdum* to show the futility of the quest for explanations of wars’ outcomes. All in all, this is not a matter of saying that the future is in the laps of the gods, but rather that it is deterministic and determined precisely, but practically and possibly in principle unknowable by humans. Much like Laplace’s viewpoint in the *Essay* where a demon knowing the positions and velocities of all particles can perfectly predict the future and reconstruct the past, while to the imperfect human mind not all information can be available in a snapshot and so it is reduced to ignorance or at best probabilistic reasoning.

The author of ‘War and Peace’ had an intense interest in mathematical approaches to the sciences, as appears from his proposals to found sociology, history, and the science of war as a mathematical discipline, much like mathematician John von Neumann proposed to found the science of economy as a mathematical discipline in [7].

Tolstoy’s views on the matter are set forth at great length in ‘War and Peace’, by many regarded as the greatest novel in any language. Based on, or perhaps called upon as justification for, Tolstoy’s egalitarian philosophy, it is set forth passionately in long interludes littered through the later parts of this great novel. Recall that the book is ostensibly about the doings

and adventures of a group of aristocratic people, and in the descriptions of great battles, at the time of Napoleon’s invasion in the bleak reaches of great Russia. Closer inspection reveals that one of the main themes of the tale is the insignificance and expendability of the particular heroes—like Napoleon—in the sweep of history: the events would have unfolded in the same way irrespective of the so-called main figures. We base our treatment on Rosemary Edmonds 1957 translation into English published in Penguin Classics[6] (part I, 1972 printing; part II, revised 1978 printing). I will refer to the page numbers as [WP, xx].

## 2 Mathematical Sociology

Tolstoy disagrees with the view of history that ascribes the evolution of events to individuals:

“One might have supposed that the historians, who ascribe the actions of the masses to the will of one man, would have found it impossible to fit the flight of Napoleon’s armies into their theory, considering that during this period of the campaign [in Russia] the French did all they could to bring about their own ruin, and that not a single movement of that rabble of men . . . betrayed a hint of rhyme or reason. But no! Mountains of volumes have been written by historians . . . [with] accounts of Napoleon’s masterly arrangements and deeply considered plans . . .” [WP, 1266]

Not only that individuals cannot be the main governors of the making of History, but:

“It is beyond the power of the human intellect to encompass *all* the causes of a phenomenon.” . . . “the human intellect . . . snatches at the first comprehensible approximation to a cause and says: ‘There is the cause.’” Tolstoy goes on [WP, 1168] to explain “in historical events (where the actions of men form the subject of observation) the primeval conception of a case was the will of the gods, succeeded later on by the will of those who stand on the historical foreground—the heroes of history.”

On page [WP, 1342] Tolstoy continues to unmask common misconceptions of traditional views of History:

“Why did things happen thus, and not otherwise? Because they did so happen. ‘*Chance* created the situation; *genius* made use of it,’ says history. But what is *chance*? What is *genius*? The words *chance* and *genius* do not denote anything that actually exists, and therefore they cannot be defined. These two words merely indicate a certain degree of comprehension of the phenomena. I do not know why a certain event occurs; I suppose that I

cannot know: therefore I do not try to know, and I talk about *chance*. I see a force producing effects beyond the scope of ordinary human agencies; I do not understand why this occurs, and I cry *genius*.”

Now we come to the true view of history, in the spirit of the so successful natural sciences. The “unreasonable effectiveness of mathematics in science” as phrased by E. Wigner, must be extended *avant la lettre* to sociology and political history [WP, 977]:

“To elicit the laws of history we must leave aside kings, ministers, and generals, and select for study the homogeneous, infinitesimal elements which influence the masses. No one can say how far it is possible for a man to advance in this way to an understanding of the laws of history; but it is obvious that this is the only path to that end, and that the human intellect has not, so far, applied in this direction one-millionth of the energy which historians have devoted to describing the deeds of various kings, generals and ministers, and propounding reflections of their own concerning those deeds.”

How then is this proper view of history obtained? Tolstoy discusses the continuity of motion that was captured in laws by dividing continuity into units. He observes that this can be done in a wrong way, [WP, 974]:

“Take, for instance, the well-known sophism of the ancients which set out to prove that Achilles would never catch up with the tortoise that had the start on him, even though Achilles traveled ten times as fast as the tortoise: by the time Achilles has covered the distance that separated him from the tortoise, the tortoise has advanced one-tenth of that distance ahead of him. While Achilles does this tenth the tortoise gains a hundredth, and so on *ad infinitum*. This problem appeared to the ancients insoluble. The absurdity of the finding (that Achilles can never overtake the tortoise) follows from arbitrarily separating the motion into separate units, whereas the motion of Achilles and the tortoise was continuous.

By adopting smaller and smaller units of motion we only approximate the solution of the problem but never reach it. It is only by admitting infinitesimal quantities and their progression up to a tenth, and taking the sum of that geometrical progression, that we arrive at the solution of the problem.”

Now we come to the heart of the matter: Tolstoy’s proposal of a differential and integral analysis of history [WP, 974–975]:

“A new branch of mathematics, having attained the art of reckoning with infinitesimal, can now yield solutions to other more complex problems of motion which before seemed insoluble. This new branch of mathematics,

which was unknown to the ancients,<sup>1</sup> by admitting the conception, when dealing with problems of motion, of the infinitely small and thus conforming to the chief condition of motion (absolute continuity), corrects the inevitable error which human intellect cannot but make if it considers separate units of motion instead of continuous motion. In the investigation of the laws of historical movement precisely the same principle operates.

The march of humanity, springing as it does from an infinite multitude of individual wills, is continuous. The discovery of the laws of this continuous movement is the aim of history. But to arrive at these laws of continuous motion resulting from the sum of all those human volitions, human reason postulates arbitrarily, separated units. The first proceeding of a historian is to select at random a series of successive events and examine them apart from others, though there is and can be no *beginning* to any event, for an event flows without break in continuity from another. The second method is to study the actions of some one man—a king or a commander—as though their actions represented the sum of many individual wills; whereas the sum of the individual wills never finds expression in the activity of a single historical personage.

... Only by assuming an infinitesimal small unit for observation—a differential of history (that is, the common tendencies of men)—and arriving at the art of integration (finding the sum of the infinitesimals) can we hope to discover the laws of history.”

### 3 Mathematics of War

The causality involved in war defies simple analysis, Tolstoy says, but is reached by the integration of the infinitesimal individual causes, [WP, 1184]:

“An infinite amount of freely acting forces (and nowhere is a man freer than during a life and death struggle) influence the course taken by a battle, and that course can never be known beforehand and never coincides with the direction it would have taken under the impulsion of any single force.

If simultaneously and variously directed forces act on a given body, the direction which that body will take cannot be the course of anyone of the forces individually—it will always follow an intermediate, as it were, shortest path, or what is presented in mechanics by the diagonal of a parallelogram of forces.”

In [WP, 1223—1224] Tolstoy outlines the mathematics of war and goes into an explicit calculation that is patently false:

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<sup>1</sup>Apart from Archimedes and Eudoxos [PV].

“Military science says, the greater the numbers [of an army] the greater the strength. . . . For military science to make this assertion is like defining energy in mechanics by reference to the mass only. It is like saying that the momenta of moving bodies will be equal or unequal according to the equality or inequality of their masses. But momentum (or ‘quantity of motion’) is the product of mass and velocity. So in warfare the strength of an army is the product of its mass and of something else, some unknown factor  $x$ .”

He goes on to debate what this unknown  $x$  may stand for and rejects the common explanations, especially the interpretation of  $x$  as the amount of genius of the commanding general. He goes on to say that [WP, 1224]:

“We must accept the unknown and see it for what it is: the more or less active desire to fight and face danger. Only then, expressing the known historical facts by means of equations, shall we be able to compare the relative values of the unknown factor; only then may we hope to arrive at the unknown itself.

If ten men, battalions or divisions, fighting fifteen men, battalions or divisions, beat the fifteen—that is, kill or capture them all while losing four themselves, the loss will have been four on one side and fifteen on the other. Therefore, the four are equal to the fifteen, and we may write  $4x = 15y$ . In other words,  $x$  is to  $y$  as 15 is to 4. Though this equation does not yet give us the absolute value of the unknown factor, it does give us a ratio between two unknowns. And by putting a whole variety of historical data (battles, campaigns, periods of warfare, and so on) into the form of such equations, a series of figures will be obtained which must involve the laws inherent in equations and will in time reveal them.”

This argument of Tolstoy is remarkable. He compares the loss of the conquering army with the total of the vanquished army—perhaps on the grounds that the vanquished army is totally lost. Testing the idea by inserting more extreme figures, such as that an army of 1,000,000 men beats a small army of 10 men, while the conquering army loses one man, we obtain the equation  $x = 10y$ . This means that the fighting spirit of the million-men army exceeded necessarily the fighting spirit of the minuscule ten-men army tenfold. The problem with Tolstoy’s reasoning here is that he equates the ratio of the loss of the conquering army (irrespective of the size of the total army) and the total of the beaten army (however small) with the ratio of the fighting spirit of the beaten army and that of the conquering army. In our opinion this reasoning is hard to defend in general as is shown by substituting extreme numbers as above. The general drift of the argument is of course reasonable. Note that (contrary to the intention of the author) the variables  $x$  and  $y$  may contain the quality of the commanders (much as

the quality of performance of a good symphony orchestra greatly depends on the quality of the conductor).

## 4 Conclusion

It is seldom the case that a great author deems fit to incorporate extensive discussions about mathematical foundations of social sciences in a major literary novel. It is much more common that scientists strive for literary redemption. Tolstoy is one of the rare examples of the former. In fact, he gives definite proposals to mathematize history, sociology, and the sciences of war in line with the rational inclination of the nineteenth century.

## A Infinitesimal methods in Tolstoy

Above we have extracted a few salient parts of “War and Peace” advocating the use of the methods of the calculus to study history. We used only fragments to keep the pace; nonetheless it may be useful to provide some more complete quotations in an appendix. The following fragments of “War and Peace” are taken from the Virginia Tech gopher site.

### A.1 Third book, Third part, Chapter 1

Absolute continuity of motion is not comprehensible to the human mind. Laws of motion of any kind become comprehensible to man only when he examines arbitrarily selected elements of that motion; but at the same time, a large proportion of human error comes from the arbitrary division of continuous motion into discontinuous elements. There is a well known, so-called sophism of the ancients consisting in this, that Achilles could never catch up with a tortoise he was following, in spite of the fact that he traveled ten times as fast as the tortoise. By the time Achilles has covered the distance that separated him from the tortoise, the tortoise has covered one tenth of that distance ahead of him: when Achilles has covered that tenth, the tortoise has covered another one hundredth, and so on forever. This problem seemed to the ancients insoluble. The absurd answer (that Achilles could never overtake the tortoise) resulted from this: that motion was arbitrarily divided into discontinuous elements, whereas the motion both of Achilles and of the tortoise was continuous.

By adopting smaller and smaller elements of motion we only approach a solution of the problem, but never reach it. Only when we have admitted the

conception of the infinitely small, and the resulting geometrical progression with a common ratio of one tenth, and have found the sum of this progression to infinity, do we reach a solution of the problem.

A modern branch of mathematics having achieved the art of dealing with the infinitely small can now yield solutions in other more complex problems of motion which used to appear insoluble.

This modern branch of mathematics, unknown to the ancients, when dealing with problems of motion admits the conception of the infinitely small, and so conforms to the chief condition of motion (absolute continuity) and thereby corrects the inevitable error which the human mind cannot avoid when it deals with separate elements of motion instead of examining continuous motion.

In seeking the laws of historical movement just the same thing happens. The movement of humanity, arising as it does from innumerable arbitrary human wills, is continuous.

To understand the laws of this continuous movement is the aim of history. But to arrive at these laws, resulting from the sum of all those human wills, man's mind postulates arbitrary and disconnected units. The first method of history is to take an arbitrarily selected series of continuous events and examine it apart from others, though there is and can be no beginning to any event, for one event always flows uninterruptedly from another.

The second method is to consider the actions of some one man- a king or a commander- as equivalent to the sum of many individual wills; whereas the sum of individual wills is never expressed by the activity of a single historic personage.

Historical science in its endeavor to draw nearer to truth continually takes smaller and smaller units for examination. But however small the units it takes, we feel that to take any unit disconnected from others, or to assume a beginning of any phenomenon, or to say that the will of many men is expressed by the actions of any one historic personage, is in itself false.

It needs no critical exertion to reduce utterly to dust any deductions drawn from history. It is merely necessary to select some larger or smaller unit as the subject of observation- as criticism has every right to do, seeing that whatever unit history observes must always be arbitrarily selected.

Only by taking infinitesimally small units for observation (the differential of history, that is, the individual tendencies of men) and attaining to the art of integrating them (that is, finding the sum of these infinitesimals) can we hope to arrive at the laws of history.

The first fifteen years of the nineteenth century in Europe present an extraordinary movement of millions of people. Men leave their customary

pursuits, hasten from one side of Europe to the other, plunder and slaughter one another, triumph and are plunged in despair, and for some years the whole course of life is altered and presents an intensive movement which first increases and then slackens. What was the cause of this movement, by what laws was it governed? asks the mind of man.

The historians, replying to this question, lay before us the sayings and doings of a few dozen men in a building in the city of Paris, calling these sayings and doings "the Revolution"; then they give a detailed biography of Napoleon and of certain people favorable or hostile to him; tell of the influence some of these people had on others, and say: that is why this movement took place and those are its laws.

But the mind of man not only refuses to believe this explanation, but plainly says that this method of explanation is fallacious, because in it a weaker phenomenon is taken as the cause of a stronger. The sum of human wills produced the Revolution and Napoleon, and only the sum of those wills first tolerated and then destroyed them.

"But every time there have been conquests there have been conquerors; every time there has been a revolution in any state there have been great men," says history. And, indeed, human reason replies: every time conquerors appear there have been wars, but this does not prove that the conquerors caused the wars and that it is possible to find the laws of a war in the personal activity of a single man. Whenever I look at my watch and its hands point to ten, I hear the bells of the neighboring church; but because the bells begin to ring when the hands of the clock reach ten, I have no right to assume that the movement of the bells is caused by the position of the hands of the watch.

Whenever I see the movement of a locomotive I hear the whistle and see the valves opening and wheels turning; but I have no right to conclude that the whistling and the turning of wheels are the cause of the movement of the engine.

The peasants say that a cold wind blows in late spring because the oaks are budding, and really every spring cold winds do blow when the oak is budding. But though I do not know what causes the cold winds to blow when the oak buds unfold, I cannot agree with the peasants that the unfolding of the oak buds is the cause of the cold wind, for the force of the wind is beyond the influence of the buds. I see only a coincidence of occurrences such as happens with all the phenomena of life, and I see that however much and however carefully I observe the hands of the watch, and the valves and wheels of the engine, and the oak, I shall not discover the cause of the bells ringing, the engine moving, or of the winds of spring. To that I must entirely

change my point of view and study the laws of the movement of steam, of the bells, and of the wind. History must do the same. And attempts in this direction have already been made.

To study the laws of history we must completely change the subject of our observation, must leave aside kings, ministers, and generals, and the common, infinitesimally small elements by which the masses are moved. No one can say in how far it is possible for man to advance in this way toward an understanding of the laws of history; but it is evident that only along that path does the possibility of discovering the laws of history lie, and that as yet not a millionth part as much mental effort has been applied in this direction by historians as has been devoted to describing the actions of various kings, commanders, and ministers and propounding the historians' own reflections concerning these actions.

## **A.2 Fourth book, Second part, Chapter 8**

A countless number of free forces (for nowhere is man freer than during a battle, where it is a question of life and death) influence the course taken by the fight, and that course never can be known in advance and never coincides with the direction of any one force.

If many simultaneously and variously directed forces act on a given body, the direction of its motion cannot coincide with any one of those forces, but will always be a mean- what in mechanics is represented by the diagonal of a parallelogram of forces.

If in the descriptions given by historians, especially French ones, we find their wars and battles carried out in accordance with previously formed plans, the only conclusion to be drawn is that those descriptions are false.

## **A.3 Second epilogue, Chapter 11**

History examines the manifestations of man's free will in connection with the external world in time and in dependence on cause, that is, it defines this freedom by the laws of reason, and so history is a science only in so far as this free will is defined by those laws.

The recognition of man's free will as something capable of influencing historical events, that is, as not subject to laws, is the same for history as the recognition of a free force moving the heavenly bodies would be for astronomy.

That assumption would destroy the possibility of the existence of laws, that is, of any science whatever. If there is even a single body moving freely,

then the laws of Kepler and Newton are negated and no conception of the movement of the heavenly bodies any longer exists. If any single action is due to free will, then not a single historical law can exist, nor any conception of historical events.

For history, lines exist of the movement of human wills, one end of which is hidden in the unknown but at the other end of which a consciousness of man's will in the present moves in space, time, and dependence on cause.

The more this field of motion spreads out before our eyes, the more evident are the laws of that movement. To discover and define those laws is the problem of history.

From the standpoint from which the science of history now regards its subject on the path it now follows, seeking the causes of events in man's freewill, a scientific enunciation of those laws is impossible, for however man's free will may be restricted, as soon as we recognize it as a force not subject to law, the existence of law becomes impossible.

Only by reducing this element of free will to the infinitesimal, that is, by regarding it as an infinitely small quantity, can we convince ourselves of the absolute inaccessibility of the causes, and then instead of seeking causes, history will take the discovery of laws as its problem.

The search for these laws has long been begun and the new methods of thought which history must adopt are being worked out simultaneously with the self-destruction toward which- ever dissecting and dissecting the causes of phenomena- the old method of history is moving. All human sciences have traveled along that path. Arriving at infinitesimals, mathematics, the most exact of sciences, abandons the process of analysis and enters on the new process of the integration of unknown, infinitely small, quantities. Abandoning the conception of cause, mathematics seeks law, that is, the property common to all unknown, infinitely small, elements.

In another form but along the same path of reflection the other sciences have proceeded. When Newton enunciated the law of gravity he did not say that the sun or the earth had a property of attraction; he said that all bodies from the largest to the smallest have the property of attracting one another, that is, leaving aside the question of the cause of the movement of the bodies, he expressed the property common to all bodies from the infinitely large to the infinitely small. The same is done by the natural sciences: leaving aside the question of cause, they seek for laws. History stands on the same path. And if history has for its object the study of the movement of the nations and of humanity and not the narration of episodes in the lives of individuals, it too, setting aside the conception of cause, should seek the laws common to all the inseparably interconnected infinitesimal elements of free will.

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