Isochronous dynamical systems, the arrow of time and the definition of deterministic chaos

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Summary

Any (autonomous) dynamical system can be *extended* or *modified*, obtaining thereby a *new* (autonomous) dynamical system involving a constant *T*---the value of which can be freely assigned---and featuring the following two properties: (i) all solutions of the new model are *isochronous* (completely periodic in all their degrees of freedom with the assigned period *T*); (ii) starting from *generic* initial data, the time evolution of the new dynamical system over time intervals of order $\tilde{T} << T$ is *essentially identical* to that of the original dynamical system, up to a *constant* rescaling of time and of corrections of order $\tilde{T}/T$. These findings entail that, in some sense, “*isochronous systems are not rare*” and moreover that such systems may feature an “*extremely complicated* “ time-evolution. They are also valid in the context of *Hamiltonian* dynamics; they are in particular applicable to the most general many-body problem (provided it is, overall, translation-invariant), entailing remarkable observations about statistical mechanics, thermodynamics and the issue of the “arrow of time” for macroscopic physics. Since *completely periodic* systems are *maximally superintegrable* (possessing the maximal number of functionally independent constants of motion compatible with the time evolution not being frozen), these findings also entail that *any* (Hamiltonian) dynamics can be *embedded* into a *superintegrable* (Hamiltonian) dynamics; and again, that “*integrable (indeed, superintegrable) Hamiltonian systems are not rare*” and that such systems may feature an “*extremely complicated* “ time-evolution.

All these findings have been obtained together with **François Leyvraz**. Some of them are reported in a recent monograph (F. Calogero, *Isochronous systems*, Oxford University Press, 2008); others are more recent, see references listed below. An even more recent finding demonstrates how to modify an *arbitrary* (autonomous) dynamical system so that the (also autonomous) modified system is *isochronous* (with an *arbitrarily assigned* period *T*) yet its dynamics for an *arbitrary* fraction (of course, less than unity) of its (periodic) time evolution is *exactly identical* to that of the original system (F. Calogero and F. Leyvraz, “Isochronous systems, the arrow of time, and the definition of deterministic chaos”, in preparation).

Main references

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