Quantum Computing Exercises # 10

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(to be handed in before or at the start of the lecture on Apr 19)

1. Prove the *quantum no-cloning theorem*: there does not exist a 2-qubit unitary U that maps

$$|\phi\rangle|0\rangle \mapsto |\phi\rangle|\phi\rangle$$

for every qubit $|\phi\rangle$. *Hint: consider what U has to do when $|\phi\rangle = |0\rangle$, when $|\phi\rangle = |1\rangle$, and when $|\phi\rangle$ is a superposition of these two.*

2. (a) Give the density matrix that corresponds to a 50-50 mixture of $|0\rangle$ and $|1\rangle$.
   (b) Give the density matrix that corresponds to a 50-50 mixture of $|+\rangle = \frac{1}{\sqrt{2}}(|0\rangle + |1\rangle)$ and $|-\rangle = \frac{1}{\sqrt{2}}(|0\rangle - |1\rangle)$.

3. (a) Give a quantum random access code that encodes 2 classical bits into 1 qubit, such that each of the two classical bits can be recovered from the quantum encoding with success probability $p \geq 0.85$. *Hint: It suffices to use pure states with real amplitudes as encoding. Try to "spread out" the 4 encodings $|\phi_{00}\rangle$, $|\phi_{01}\rangle$, $|\phi_{10}\rangle$, $|\phi_{11}\rangle$ in the 2-dimensional real plane as best as possible.*
   (b) Give a good upper bound (as a function of n) on the maximal success probability p for a random access code that encodes n classical bits into 1 qubit.

4. Consider the Hadamard code C that encodes $n = 2$ bits x_1x_2 into a codeword of $N = 4$ bits.
   (a) Give the 4-bit codeword $C(11)$.
   (b) What are the states $|\phi_x\rangle$ that arise as quantum random access code when we apply the LDC lower bound proof of the lecture to C?
   (c) What is the measurement used for recovering x_2 in that proof?