Quantum Computing Exercises # 12

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(to be handed in before or at the start of the lecture on May 3)

1. Suppose Alice and Bob share an EPR-pair $\frac{1}{\sqrt{2}}(|00\rangle + |11\rangle)$.
   (a) Let U be a unitary with real entries. Show that the following two states are the same:
   (1) the state obtained if Alice applies U to her qubit of the EPR-pair;
   (2) the state obtained if Bob applies the transpose U^T to his qubit of the EPR-pair.
   (b) What state do you get if both Alice and Bob apply a Hadamard transform to their qubit of the EPR-pair? *Hint: you could write this out, but you can also get the answer almost immediately from part (a) and the fact that* $H^T = H^{-1}$.

2. Give a classical strategy using shared randomness for the CHSH game, such that Alice and Bob win the game with probability at least 3/4 for every possible input x, y (note the order of quantification: the same strategy has to work for every x, y). *Hint: For every fixed input* x, y*, there is a classical strategy that gives a wrong output only on that input, and that gives a correct output on all other possible inputs. Use the shared randomness to randomly choose one of those deterministic strategies.*

3. Consider three space-like separated players: Alice, Bob, and Charlie. Alice receives input bit x, Bob receives input bit y, and Charlie receives input bit z. The input satisfies the promise that $x \oplus y \oplus z = 0$. The goal of the players is to output bits a, b, c, respectively, such that $a \oplus b \oplus c = \text{OR}(x, y, z)$. In other words, the outputs should sum to 0 (mod 2) if $x = y = z = 0$, and should sum to 1 (mod 2) if $x + y + z = 2$.
   (a) Show that every classical deterministic strategy will fail on at least one of the 4 allowed inputs.
   (b) Show that every classical randomized strategy has success probability at most 3/4 under the uniform distribution on the four allowed inputs xyz.
   (c) Suppose the players share the following entangled 3-qubit state:
   $$\frac{1}{2}(|000\rangle - |011\rangle - |101\rangle - |110\rangle).$$
   Suppose each player does the following: if his/her input bit is 1, apply H to his/her qubit, otherwise do nothing. Describe the resulting 3-qubit superposition.
   (d) Using (c), give a quantum strategy that wins the above game with probability 1 on every possible input.