Quantum Computing Exercises # 14

Ronald de Wolf

May 10, 2011
(to be handed in before or at the start of the lecture on May 17)

1. Let E be an arbitrary 1-qubit unitary. We know that it can be written as

$$E = \alpha_0 I + \alpha_1 X + \alpha_2 Y + \alpha_3 Z,$$

for some complex coefficients α_i. Show that $\sum_{i=0}^{3} |\alpha_i|^2 = 1$. *Hint: Compute the trace* $\text{Tr}(E^* E)$ *in two ways, and use the fact that* $\text{Tr}(AB) = 0$ *if A and B are distinct Paulis, and* $\text{Tr}(AB) = \text{Tr}(I) = 2$ *if A and B are the same Pauli.*

2. (a) Write the 1-qubit Hadamard transform H as a linear combination of the four Pauli matrices.

   (b) Suppose an H-error happens on the first qubit of $\alpha|\overline{0}\rangle + \beta|\overline{1}\rangle$ using the 9-qubit code. Give the various steps in the error-correction procedure that corrects this error.

3. Give a quantum circuit for the encoding of Shor's 9-qubit code, i.e., a circuit that maps $|00^8\rangle \mapsto |\overline{0}\rangle$ and $|10^8\rangle \mapsto |\overline{1}\rangle$. Explain why the circuit works.

4. Show that there cannot be a quantum code that encodes one logical qubit into $2k$ physical qubits while being able to correct errors on up to k of the qubits. *Hint: Proof by contradiction. Given an unknown qubit* $\alpha|0\rangle + \beta|1\rangle$, *encode it using this code. Split the $2k$ qubits into two sets of k qubits, and use each to get a copy of the unknown qubit. Then invoke the no-cloning theorem.*