Quantum Computing Exercises # 2

Ronald de Wolf

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(to be handed in before or at the start of the lecture on Feb 15)

1. Construct a CNOT from two Hadamard gates and one controlled-Z (the controlled-Z gate maps $|11\rangle \mapsto -|11\rangle$ and acts like the identity on the other basis states).

2. A SWAP-gate interchanges two qubits: it maps basis state $|a, b\rangle$ to $|b, a\rangle$. Implement a SWAP-gate using a few CNOTs.

3. Using an ancilla qubit, it is possible to avoid doing any intermediate measurements in a quantum circuit. Show how this can be done.

   Hint: instead of measuring the qubit, apply a CNOT that "copies" it to a new $|0\rangle$-qubit, which is then left alone until the end of the computation. Analyze what happens.

4. During the lecture we showed that a query of the type $|i, b\rangle \mapsto |i, b \oplus x_i\rangle$ (where $i \in \{1, ..., n\}$ and $b \in \{0, 1\}$) can be used to implement a phase-query, i.e., one of the type $|i\rangle \mapsto (-1)^{x_i}|i\rangle$. Is the converse possible: can a query of the first type be implemented using phase-queries, and possibly some ancilla qubits and other gates? If yes, show how. If no, explain why not.

5. Give a randomized classical algorithm (i.e., one that can flip coins during its operation) that makes only two queries to x, and decides the Deutsch-Jozsa problem with success probability at least 2/3 on every possible input. A high-level description is enough, no need to write out the classical circuit.

6. Suppose our N-bit input x satisfies the following promise:
   either (1) the first $N/2$ bits of x are all 0 and the second $N/2$ bits are all 1; or (2) the number of 1s in the first half of x plus the number of 0s in the second half, equals $N/2$. Modify the Deutsch-Jozsa algorithm to efficiently distinguish these two cases (1) and (2).