Quantum Computing Exercises # 8

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(to be handed in before or at the start of the lecture on Apr 5)

1. Consider a 2-bit input $x = x_0x_1$ with an oracle $O_x : |i\rangle \mapsto (-1)^{x_i}|i\rangle$. Write out the final state of the following 1-query quantum algorithm: $HO_xH|0\rangle$. Give a degree-2 polynomial $p(x_0, x_1)$ that equals the probability that this algorithm outputs 1 on input x. What function does this algorithm compute?

2. Consider polynomial $p(x_1, x_2) = 0.3 + 0.4x_1 + 0.5x_2$, which approximates the 2-bit OR function. Write down the symmetrized polynomial $q(x_1, x_2) = \frac{1}{2}(p(x_1, x_2) + p(x_2, x_1))$. Give a single-variate polynomial r such that $q(x) = r(|x|)$ for all $x \in \{0, 1\}^2$.

3. Let f be the N-bit Parity function, which is 1 if its input $x \in \{0, 1\}^N$ has odd Hamming weight, and 0 if the input has even Hamming weight (assume N is an even number).

   (a) Give a quantum algorithm that computes Parity with success probability 1 on every input x, using $N/2$ queries. *Hint: think of Exercise 1.*

   (b) Show that this is optimal, even for quantum algorithms that have error probability $\leq 1/3$ on every input *Hint: show that the symmetrized approximate polynomial r induced by the algorithm has degree at least N.*

4. Suppose we have a T-query quantum algorithm that computes the N-bit AND function with success probability 1 on all inputs $x \in \{0, 1\}^N$. In the lecture we showed that such an algorithm has $T \geq N/2$ (we showed it for OR, but the same argument works for AND). Improve this lower bound to $T \geq N$.

5. Let f be the N-bit Majority function, which is 1 if its input $x \in \{0, 1\}^N$ has Hamming weight $> N/2$, and 0 if the input has Hamming weight $\leq N/2$ (assume N is even). Use the adversary method to show that every bounded-error quantum algorithm for computing Majority, needs $\Omega(N)$ queries. *Hint: when defining the relation R, consider that the hardest task for this algorithm is to distinguish inputs of weight $N/2$ from inputs of weight $N/2 + 1$.*