English Text over Erasure Channel

10% erasures:

A_ she said_this she l_oked down at her han_s, _nd was surpris_d to _ee _hat she had put on one of th_ Rabbit's _ittle white _id_gloves wh_le she was talking._'How CAN I have done that?'_she_th__ght. '_ m_st be growing small again.' She got up and _ent to the table to measure hersel_ by it,_a_d_fo_nd that,_as n_arly as she could gu___, __e was now

20% erasures:

a_out _wo fe__ high,_an_ _as_go__g _n__h__nking rapid__: she _oon found out that t_e _ause of this was the fa_ she_wa_ holding, _nd__he dropp_d it has_ily,_just i_ t__e to_avoid shrinking away altog_ther.

'That__AS a narrow es_ape!' said ___ce,_a good deal _righ__ned at _h_ s_dde_ change,_but very glad_to _in_ __rsel_ s_ill_in existence; '_nd

English Text over Erasure Channel

30% erasures:

n__ __r th__gard_n!'__nd _he_r__ wit___ll _p_e_ __c__to _he_li_tl_ door:

_u_, _l__! the litt_e doo_ w_s__hut __ain, a__ t_e_li__l_ g_lde_ key was

lying on t_e _l_ss_t_ble__s be_ore, 'and _hing_ are wor_e th_n _v_r,'

t_ought_t__ po_r child, 'f_r_I neve_ __s _o ___ll_as th__ be_o_e, _e_e_!

And I_dec_are i_'__t__ bad,_tha___t is!'

40% erasures:

_s __e_s___ t_es__w_rds h__ _oot_sli_ped, and_in_ano_her ___ent,____as_!

she wa_ up___ _e_ c_in in _alt water. _e_ first _d_a _as t_at s__

had____eh__ fal_e_ int__t_____a,_'_nd i_ th__ _ase_I_can go_ba_k_by

r_il_ay,'__he s_i_ to_her_e__. (Alice had __e_ to_the se___d___nc__in

__r_li__, an_ h__ c_me_t_ th_ gener__ ___cl_____, __a___her__e_ _ou __

English Text over Erasure Channel

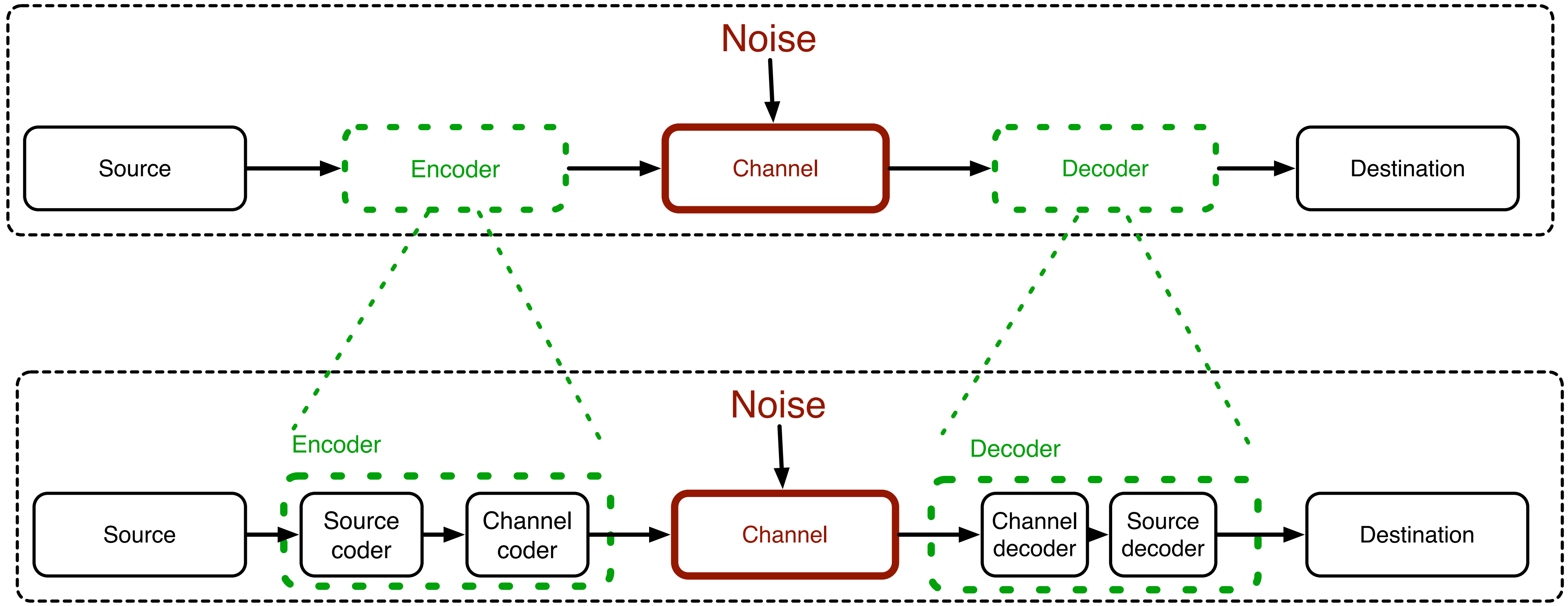
50% erasures:

___on t___Eng_i__ __a______ ____ a ____e_ __ ba___ng ma_h_n_s in_th_ s__, s___ _h_____n__ig_i___in_the san_ _____woode_ s_____,__h__ a_r_w of____gin__h____s,_an_ behin_ the__a _ailway st___o_.) H_we__r, _h_ s___ ma_e_ou_ tha_ __e_w_s__n__h_ _o____f _e___ w_i__ she_h_d w_p_ w__n____ __s n_____e___h__h.

60% erasures:

'__w_sh_l_h___'___r____s__m___!' _a_d ___c_, as_s__ s_am_a_out,______g ___fi___h__ ______t. '___h_l____ ____s_ed_fo_ _t___w, I__u_po__, __ b___g _r_w_ed _n __________r_! _h__ _l_L __ _ q______hi__, __ ____u_e! Ho_e_e_, _v_ry____g is _u_e___o-__y.' _____th______ h_ar_____eth__g s____h_____bo____n____ p___ __l___l_ ___

Source-channel separation



- For (time-varying) DMC we can design the source encoder and channel coder separately and still get optimum performance

- Not true for:
  - Correlated Channel and Source
  - Multiple access with correlated sources
  - Broadcast channel

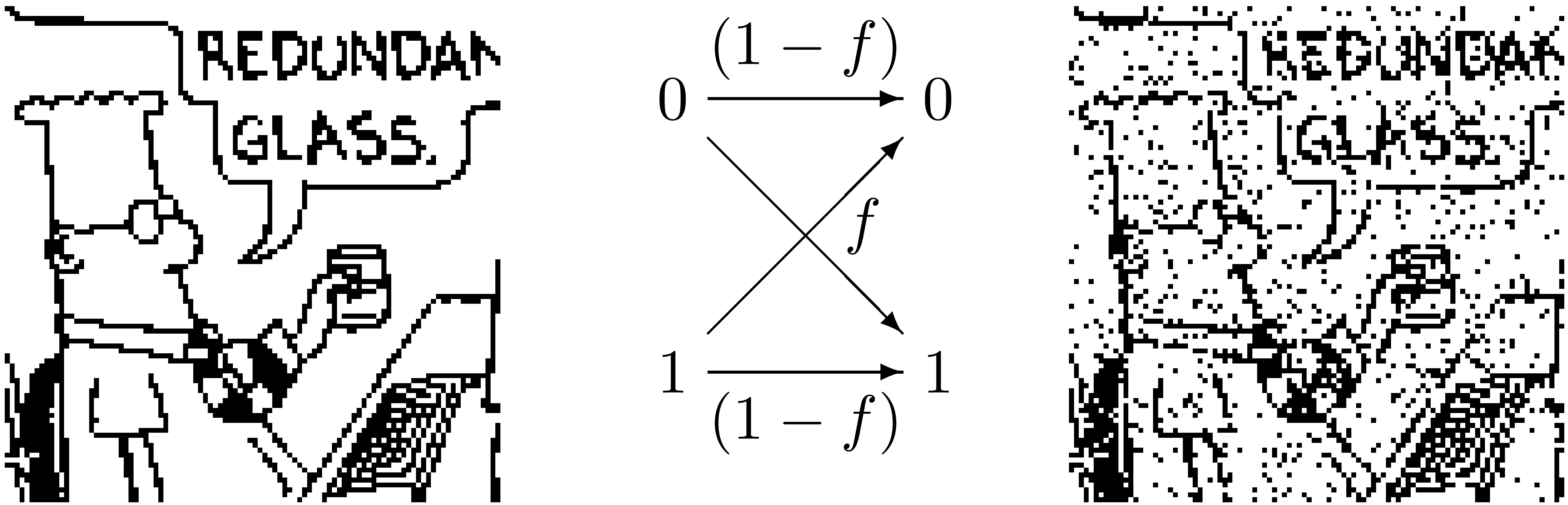


Figure 1.5. A binary data sequence of length 10 000 transmitted over a binary symmetric channel with noise level $f = 0.1$. [Dilbert image Copyright©1997 United Feature Syndicate, Inc., used with permission.]

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Received sequence $\mathbf{r}$	Likelihood ratio $\frac{P(\mathbf{r} \mid s=1)}{P(\mathbf{r} \mid s=0)}$	Decoded sequence $\hat{\mathbf{s}}$
000	γ^{-3}	0
001	γ^{-1}	0
010	γ^{-1}	0
100	γ^{-1}	0
101	γ^{1}	1
110	γ^{1}	1
011	γ^{1}	1
111	γ^{3}	1

Algorithm 1.9. Majority-vote decoding algorithm for R_3. Also shown are the likelihood ratios (1.23), assuming the channel is a binary symmetric channel; $\gamma \equiv (1-f)/f$.

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$\mathbf{s}$	0	0	1	0	1	1	0
$\mathbf{t}$	000	000	111	000	111	111	000
$\mathbf{n}$	000	001	000	000	101	000	000
$\mathbf{r}$	000	001	111	000	010	111	000
$\hat{\mathbf{s}}$	0	0	1	0	0	1	0
corrected errors		$\star$					
undetected errors					$\star$		

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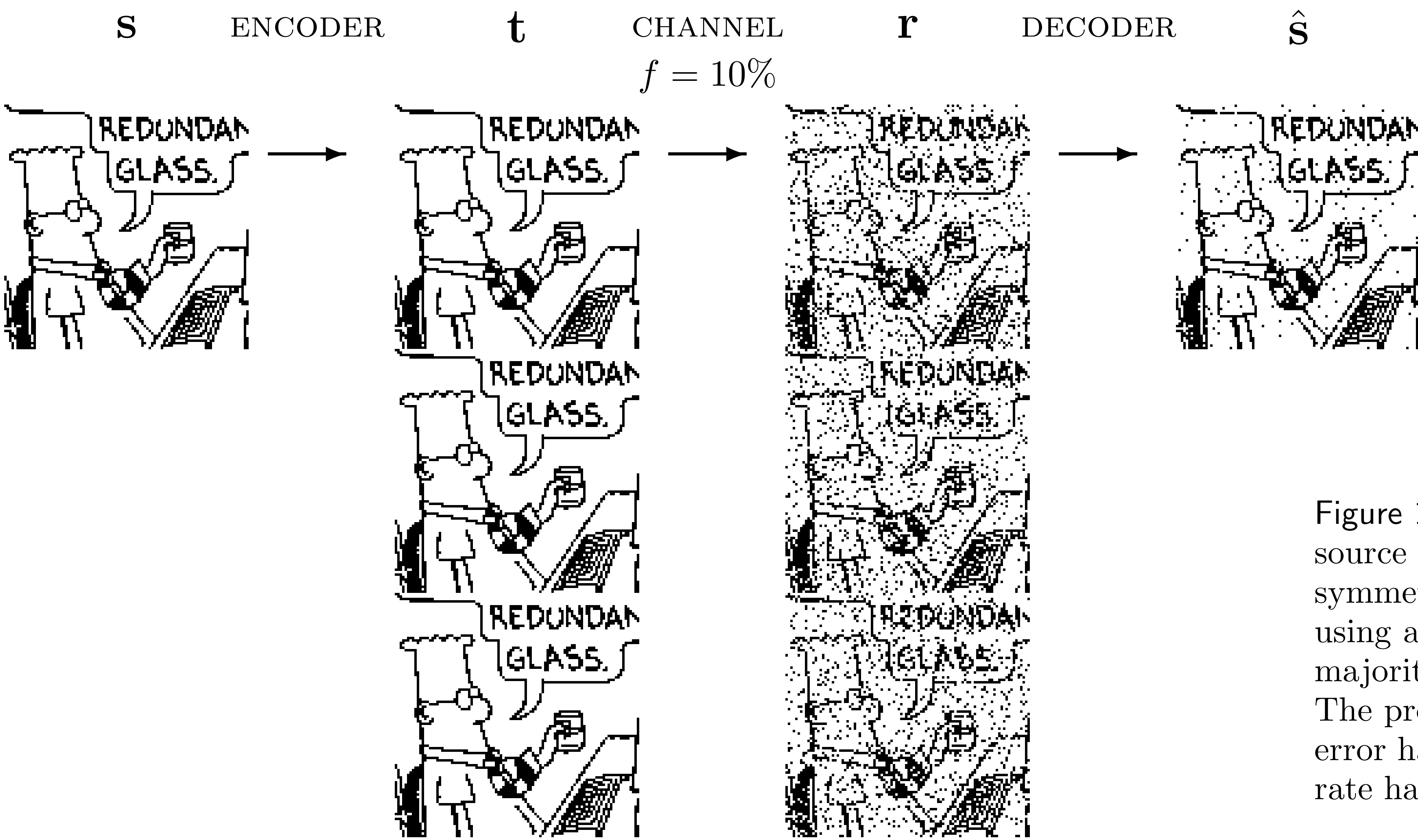

Figure 1.11. Transmitting 10 000 source bits over a binary symmetric channel with $f = 10\%$ using a repetition code and the majority vote decoding algorithm. The probability of decoded bit error has fallen to about 3%; the rate has fallen to 1/3.

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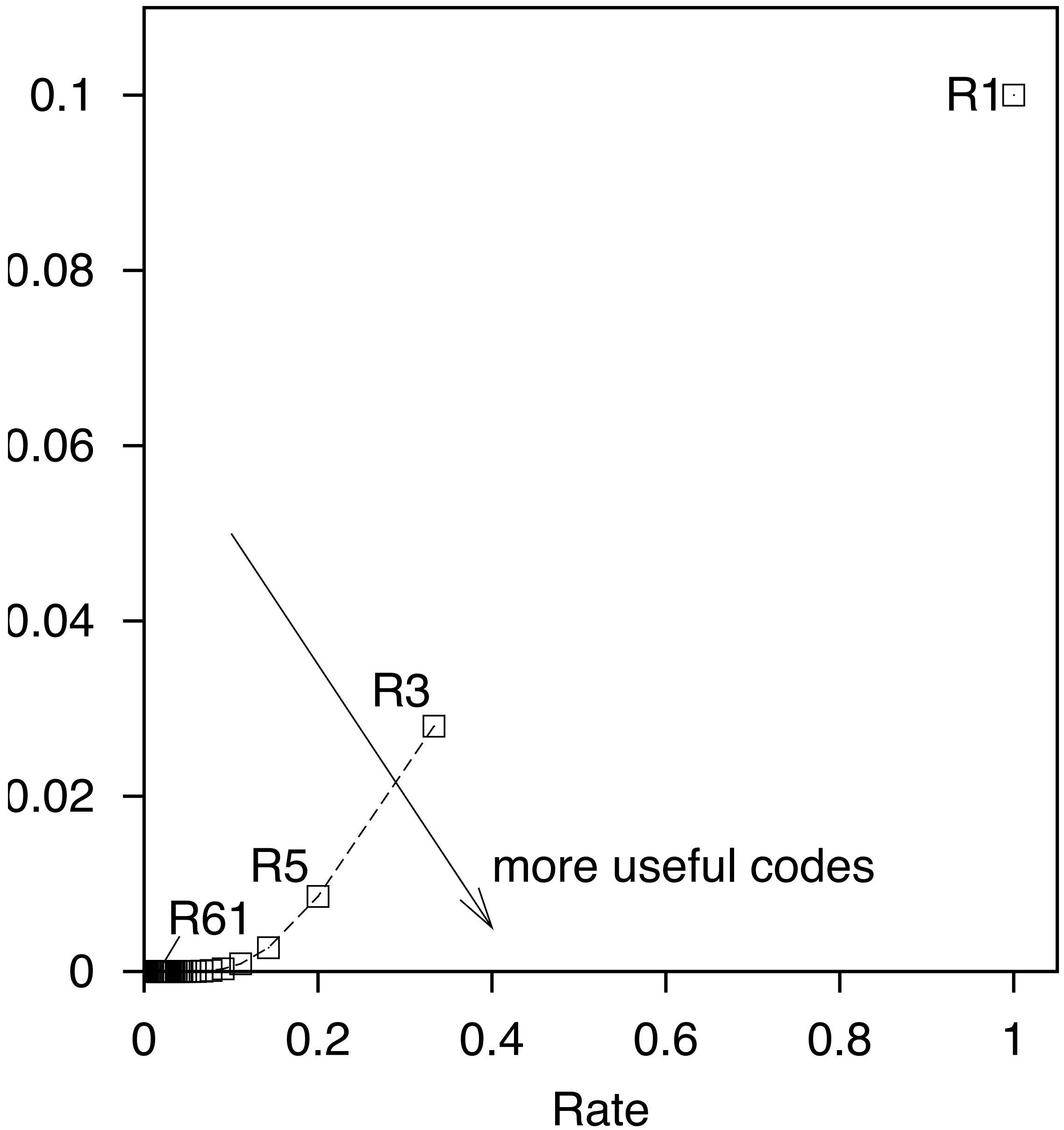

p_b

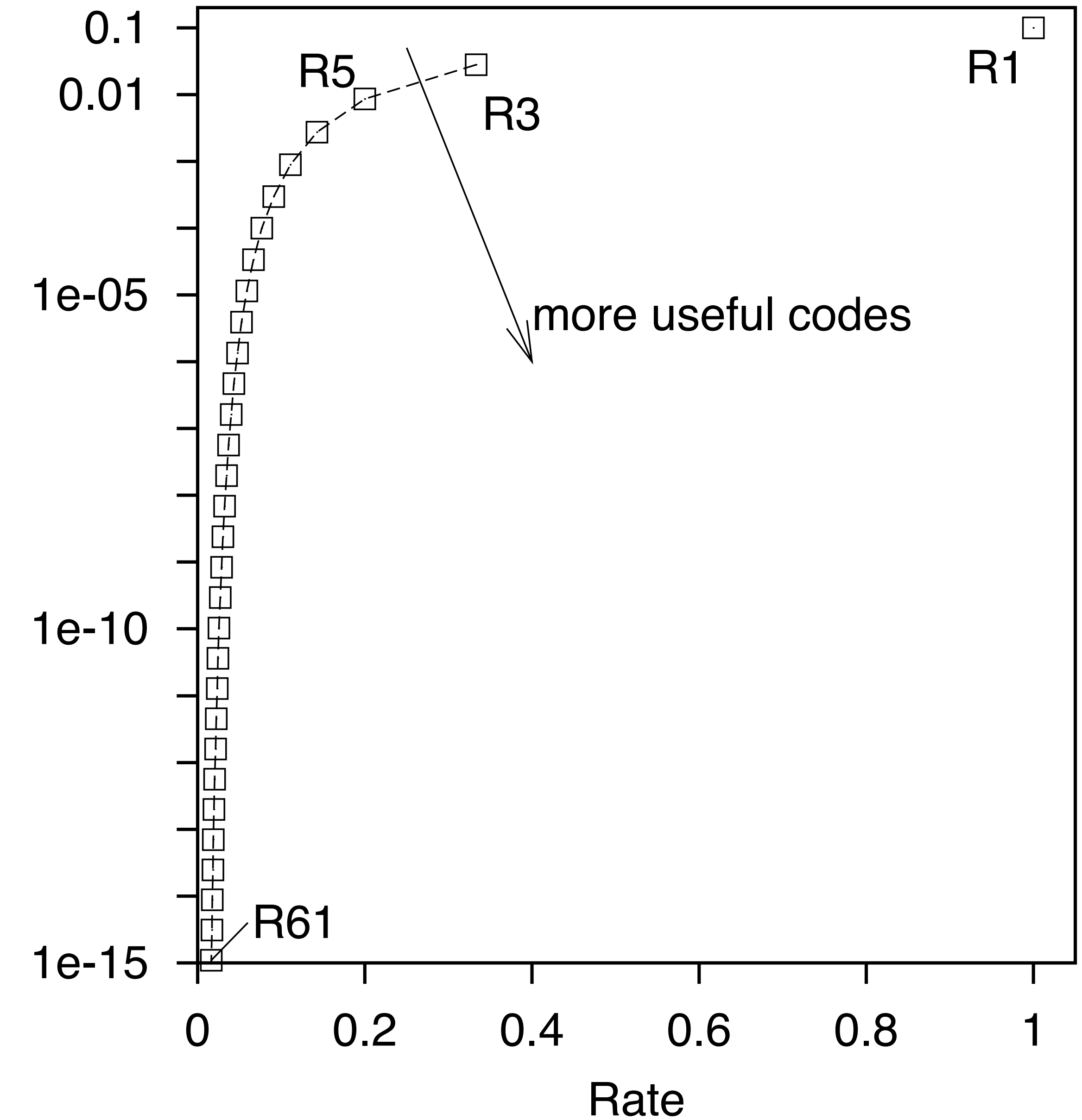

Figure 1.12. Error probability p_b versus rate for repetition codes over a binary symmetric channel with $f = 0.1$. The right-hand figure shows p_b on a logarithmic scale. We would like the rate to be large and p_b to be small.

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s	t	s	t	s	t	s	t
0000	0000000	0100	0100110	1000	1000101	1100	1100011
0001	0001011	0101	0101101	1001	1001110	1101	1101000
0010	0010111	0110	0110001	1010	1010010	1110	1110100
0011	0011100	0111	0111010	1011	1011001	1111	1111111

Table 1.14. The sixteen codewords $\{\mathbf{t}\}$ of the $(7,4)$ Hamming code. Any pair of codewords differ from each other in at least three bits.

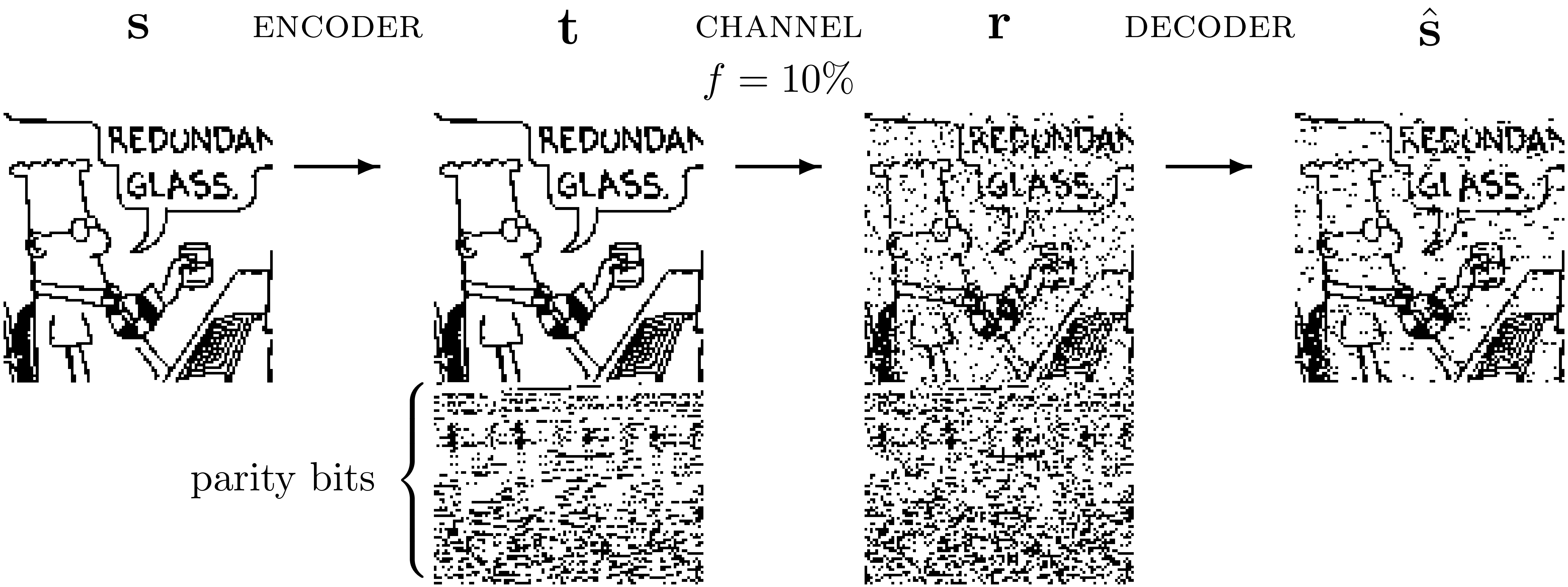

Figure 1.17. Transmitting 10 000 source bits over a binary symmetric channel with $f = 10\%$ using a $(7, 4)$ Hamming code. The probability of decoded bit error is about 7%.

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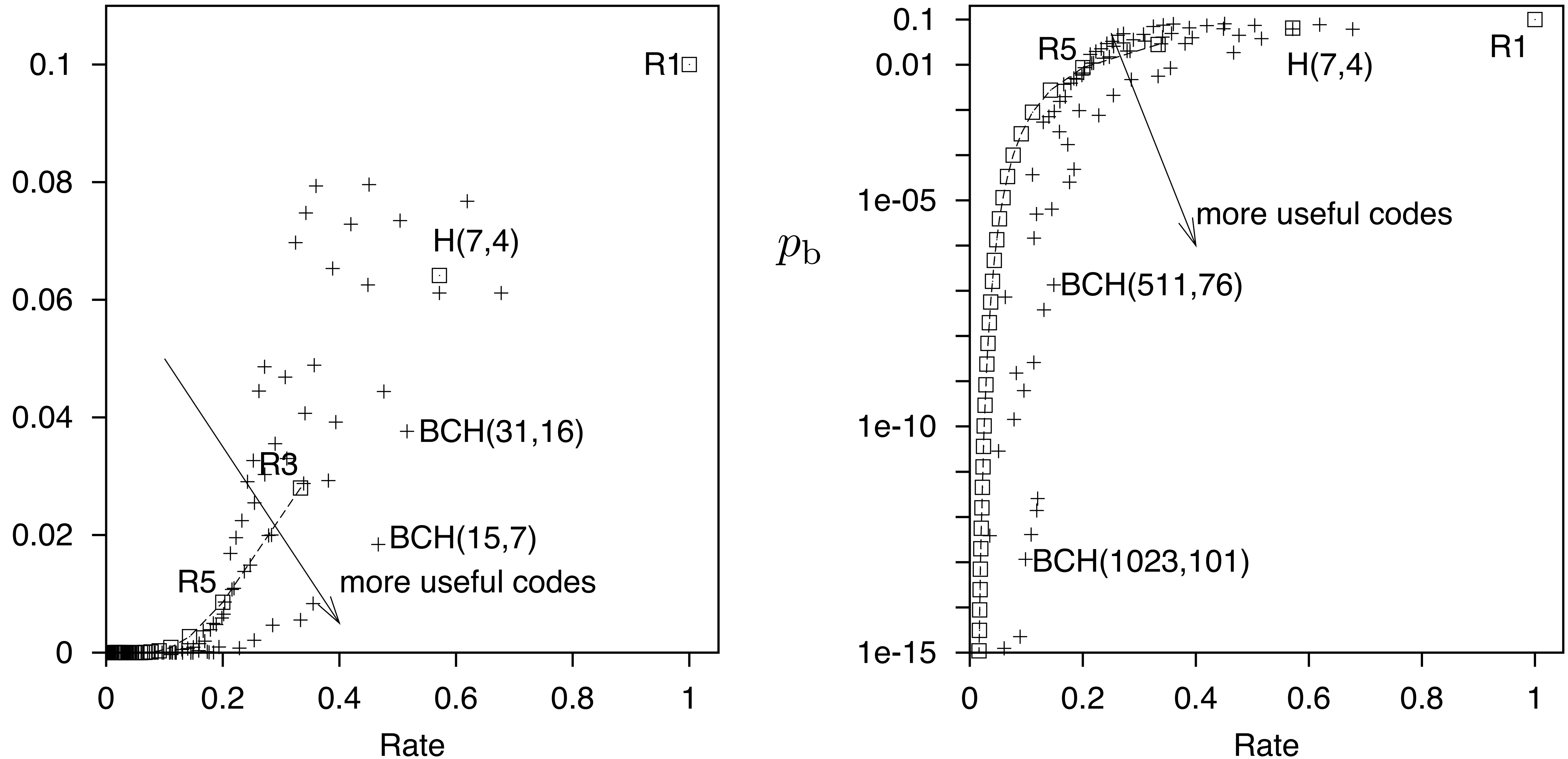

Figure 1.18. Error probability p_b versus rate R for repetition codes, the $(7, 4)$ Hamming code and BCH codes with blocklengths up to 1023 over a binary symmetric channel with $f = 0.1$. The righthand figure shows p_b on a logarithmic scale.

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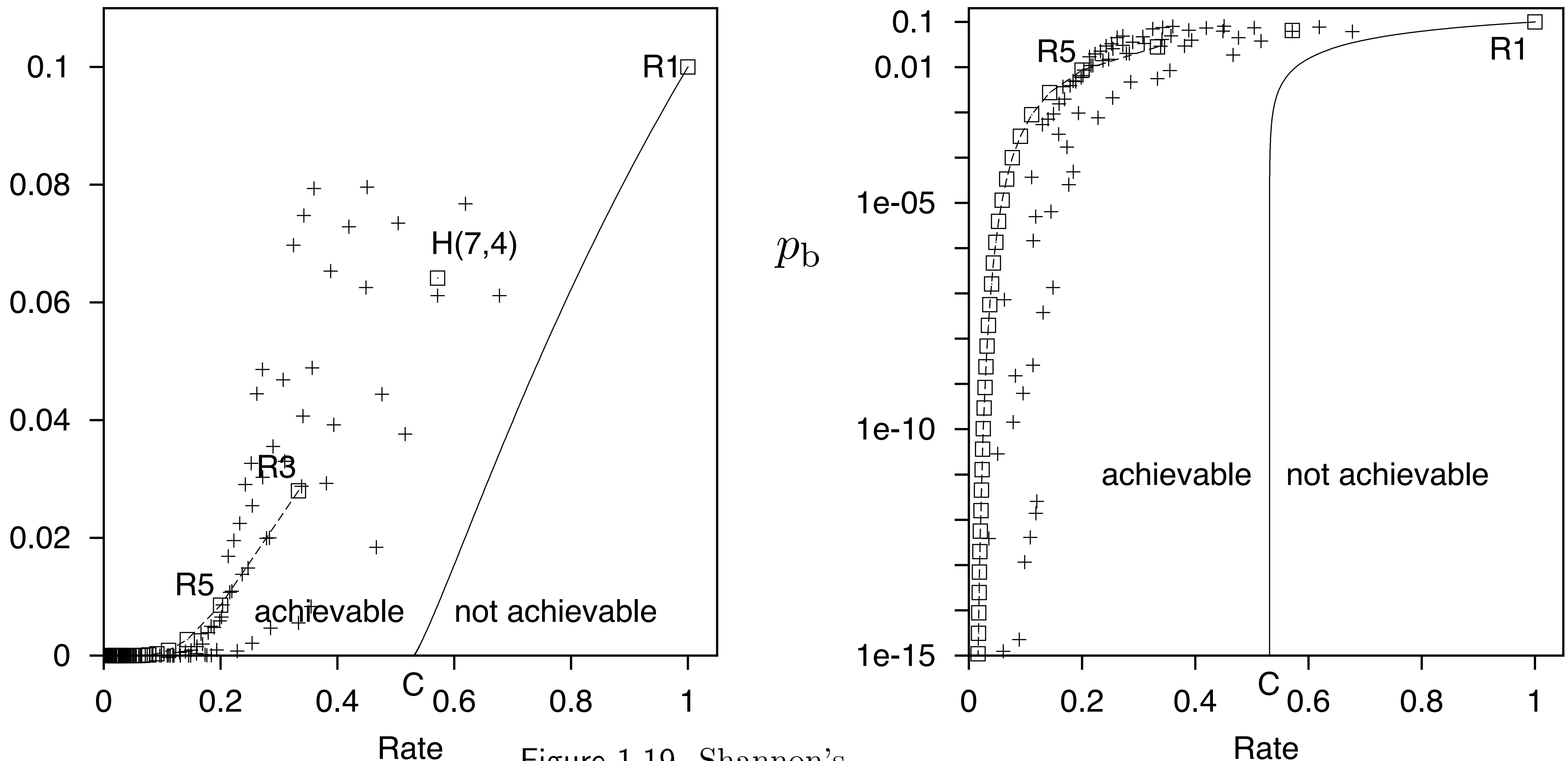

Figure 1.19. Shannon's noisy-channel coding theorem. The solid curve shows the Shannon limit on achievable values of (R, p_b) for the binary symmetric channel with $f = 0.1$. Rates up to $R = C$ are achievable with arbitrarily small p_b. The points show the performance of some textbook codes, as in figure 1.18.

The equation defining the Shannon limit (the solid curve) is $R = C/(1 - H_2(p_b))$, where C and H_2 are defined in equation (1.35).

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$C \simeq 0.53$. Let us consider what this means in terms of noisy disk drives. The repetition code R_3 could communicate over this channel with $p_{\mathrm{b}} = 0.03$ at a rate $R = 1/3$. Thus we know how to build a single gigabyte disk drive with $p_{\mathrm{b}} = 0.03$ from three noisy gigabyte disk drives. We also know how to make a single gigabyte disk drive with $p_{\mathrm{b}} \simeq 10^{-15}$ from sixty noisy one-gigabyte drives (exercise 1.3, p.8). And now Shannon passes by, notices us juggling with disk drives and codes and says:

'What performance are you trying to achieve? 10^{-15}? You don't need *sixty* disk drives – you can get that performance with just *two* disk drives (since 1/2 is less than 0.53). And if you want $p_{\mathrm{b}} = 10^{-18}$ or 10^{-24} or anything, you can get there with two disk drives too!'

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[Strictly, the above statements might not be quite right, since, as we shall see, Shannon proved his noisy-channel coding theorem by studying sequences of block codes with ever-increasing blocklengths, and the required blocklength might be bigger than a gigabyte (the size of our disk drive), in which case, Shannon might say 'well, you can't do it with those *tiny* disk drives, but if you had two noisy *terabyte* drives, you could make a single high-quality terabyte drive from them'.]

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