| 0 | 00 | 000 | 0000 | $\begin{aligned} & \text { o} \\ & \text { or } \\ & \stackrel{\rightharpoonup}{3} \\ & 0 \end{aligned}$ |
| :---: | :---: | :---: | :---: | :---: |
|  |  |  | 0001 |  |
|  |  | 001 | 0010 |  |
|  |  |  | 0011 |  |
|  | 01 | 010 | 0100 | $\begin{aligned} & 0 \\ & 0 \\ & 0 \end{aligned}$ |
|  |  |  | 0101 |  |
|  |  | 011 | 0110 |  |
|  |  |  | 0111 | $0$ |
| 1 | 10 | 100 | 1000 | $\frac{\xi}{i}$ |
|  |  |  | 1001 |  |
|  |  | 101 | 1010 | $\stackrel{\Im}{0}$ |
|  |  |  | 1011 |  |
|  | 11 | 110 | 1100 | $\underset{\sim}{e}$ |
|  |  |  | 1101 |  |
|  |  | 111 | 1110 |  |
|  |  |  | 1111 |  |

Figure 5.1. The symbol coding budget. The 'cost' $2^{-l}$ of each codeword (with length $l$ ) is indicated by the size of the box it is written in. The total budget available when making a uniquely decodeable code is 1 .
You can think of this diagram as showing a codeword supermarket, with the codewords arranged in aisles by their length, and the cost of each codeword indicated by the size of its box on the shelf. If the cost of the codewords that you take exceeds the budget then your code will not be uniquely decodeable.

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| $a_{i}$ | $c\left(a_{i}\right)$ | $l_{i}$ |
| :---: | :---: | :---: |
| a | 1000 | 4 |
| $C_{0}:$ |  |  |
| b | 0100 | 4 |
| c | 0010 | 4 |
| d | 0001 | 4 |$\quad$| $C_{3}:$ |  |  |  |  |  |
| :--- | :--- | :--- | :--- | :--- | :--- |$\quad$|  |
| :--- | :--- | :--- | :--- | :--- | :--- |



| $C_{6}:$ |  |  |  |  |
| :---: | :--- | :---: | :---: | :---: |
| $a_{i}$ | $c\left(a_{i}\right)$ | $p_{i}$ | $h\left(p_{i}\right)$ | $l_{i}$ |
| a | 0 | $1 / 2$ | 1.0 | 1 |
| b | 01 | $1 / 4$ | 2.0 | 2 |
| c | 011 | $1 / 8$ | 3.0 | 3 |
| d | 111 | $1 / 8$ | 3.0 | 3 |


| $C_{0}$ |  |  |  |
| :---: | :---: | :---: | :---: |
| 0 | 00 | 000 | 0000 |
|  |  |  | -10001 |
|  |  | 001 |  |
|  |  |  | 0011 |
|  | 01 | 010 | 0100 |
|  |  |  | 0101 |
|  |  | 011 | 0110 |
|  |  |  | 0111 |
| 1 | 10 | 100 |  |
|  |  |  | 1001 |
|  |  | 101 | 1010 |
|  |  |  | 1011 |
|  | 11 | 110 | 1100 |
|  |  |  | 1101 |
|  |  | 111 | 1110 |
|  |  |  | 1111 |



| $a_{i}$ | $p_{i}$ | $\log _{2} \frac{1}{p_{i}}$ | $l_{i}$ | $c\left(a_{i}\right)$ |
| :--- | :--- | ---: | ---: | :--- |
| a | 0.0575 | 4.1 | 4 | 0000 |
| b | 0.0128 | 6.3 | 6 | 001000 |
| c | 0.0263 | 5.2 | 5 | 00101 |
| d | 0.0285 | 5.1 | 5 | 10000 |
| e | 0.0913 | 3.5 | 4 | 1100 |
| f | 0.0173 | 5.9 | 6 | 111000 |
| g | 0.0133 | 6.2 | 6 | 001001 |
| h | 0.0313 | 5.0 | 5 | 10001 |
| i | 0.0599 | 4.1 | 4 | 1001 |
| j | 0.0006 | 10.7 | 10 | 1101000000 |
| k | 0.0084 | 6.9 | 7 | 1010000 |
| l | 0.0335 | 4.9 | 5 | 11101 |
| m | 0.0235 | 5.4 | 6 | 110101 |
| n | 0.0596 | 4.1 | 4 | 0001 |
| o | 0.0689 | 3.9 | 4 | 1011 |
| p | 0.0192 | 5.7 | 6 | 111001 |
| q | 0.0008 | 10.3 | 9 | 110100001 |
| r | 0.0508 | 4.3 | 5 | 11011 |
| s | 0.0567 | 4.1 | 4 | 0011 |
| t | 0.0706 | 3.8 | 4 | 1111 |
| u | 0.0334 | 4.9 | 5 | 10101 |
| v | 0.0069 | 7.2 | 8 | 11010001 |
| w | 0.0119 | 6.4 | 7 | 1101001 |
| x | 0.0073 | 7.1 | 7 | 1010001 |
| y | 0.0164 | 5.9 | 6 | 101001 |
| z | 0.0007 | 10.4 | 10 | 1101000001 |
| - | 0.1928 | 2.4 | 2 | 01 |



Figure 5.6. Huffman code for the English language ensemble (monogram statistics).
ure 5.6. This code has an expected length of 4.15 bits; the entropy of the ensemble is 4.11 bits. Observe the disparities between the assigned codelengths and the ideal codelengths $\log _{2} 1 / p_{i}$.


| Context <br> (sequence thus far) |  |  |  |
| :--- | ---: | ---: | ---: |
|  | $P(\mathrm{a})=0.425$ | $P(\mathrm{~b})=0.425$ | $P(\square)=0.15$ |
| b | $P(\mathrm{a} \mid \mathrm{b})=0.28$ | $P(\mathrm{~b} \mid \mathrm{b})=0.57$ | $P(\square \mid \mathrm{b})=0.15$ |
| bb | $P(\mathrm{a} \mid \mathrm{bb})=0.21$ | $P(\mathrm{~b} \mid \mathrm{bb})=0.64$ | $P(\square \mid \mathrm{bb})=0.15$ |
| bbb | $P(\mathrm{a} \mid \mathrm{bbb})=0.17$ | $P(\mathrm{~b} \mid \mathrm{bbb})=0.68$ | $P(\square \mid \mathrm{bbb})=0.15$ |
| bbba | $P(\mathrm{a} \mid \mathrm{bbba})=0.28$ | $P(\mathrm{~b} \mid \mathrm{bbba})=0.57$ | $P(\square \mid \mathrm{bbba})=0.15$ |

Figure 6.4 shows the corresponding intervals. The interval b is the middle 0.425 of $[0,1)$. The interval bb is the middle 0.567 of b , and so forth.

Figure 6.4. Illustration of the arithmetic coding process as the sequence bbba $\square$ is transmitted.

