Synthesis for Idle Speed Control of an Automotive Engine*

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Abstract. The problem of maintaining the crankshaft speed of an automotive engine within a given set interval (*idle speed control*), is formalized as a constrained control problem using a hybrid model of the engine. The control problem is difficult because the system has delays and a large number of constraints. The approach for the synthesis of a controller for this system is based on the theory developed for affine systems on polytopes. A structured control synthesis procedure is applied in which constraints for state and input variables are backward propagated from the controlled output (the crankshaft speed) across successive subsystems.

1 Introduction

In the automotive industry, increased performance, safety and time-to-market pressure require the use of complex control algorithms with guaranteed properties. Best practices in this industry are based on extensive experimentation and tuning of parameters for the control algorithm and for the engine model. This procedure needs a substantial overhaul to eliminate long re-design cycles and potential safety problems after the car is introduced in the market. Using more accurate models and control algorithms with guaranteed properties reduces greatly the need for extensive experimentation and points to potential problems early in the design cycle. In this paper, we investigate this strategy for the idle control problem of a four-cylinder engine.

In particular, we address the problem by proposing

– a hybrid piecewise affine model of a four cylinder in–line engine that represents more faithfully the dynamical behavior of the engine than the traditional *mean-value models*;
– the theory of control of affine systems on polytopes.

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The hybrid model for the four–cylinder automotive engine was developed by PARADES in close cooperation with Magneti Marelli Powertrain (see ([1, 2]). The model consists of the series connection of three sub–models: the intake manifold, the cylinders, and the crankshaft.

The difficulty of the problem lies in the load variations coming from the intermittent use of devices powered by the engine, such as the air conditioning system and the steering wheel servo-mechanism, which may cause engine stalls. The reformulation of the hybrid model as a piecewise affine hybrid system allows to derive an efficient synthesis procedure for a controller that satisfies the specifications.

The paper is organized as follows: the model is formulated as a piecewise-affine hybrid system in Section 2. Section 3 contains concepts and results for control of affine systems on polytopes. A control law is formulated for maintaining the crankshaft speed at a set interval even in the presence of disturbances in Section 4.

2 Hybrid model of the engine

In this section, to set the stage for our control synthesis strategy, the hybrid model of the engine described in [1] is cast in the framework of piecewise-affine hybrid systems. Piecewise-affine hybrid systems[4] have been proposed in [5, 6] and are based on piecewise-linear systems as introduced by E.D. Sontag, see [8].

Definition 1. *A* (time-invariant continuous-time) piecewise-affine hybrid system *(PAHS) consists of an automaton $(Q, E_{in} \cup E_{cd}, f)$ in combination with a $|Q|$-tuple of affine systems on polytopes parametrized by $q \in Q$ that interact in the following way. At a discrete state $q \in Q$, the continuous state x_q evolves according to the affine dynamical system,*

$$\dot{x}_q(t) = A(q)x_q(t) + B(q)u(t) + a(q), \;\; x_q(t_0) = x_q^+, \tag{1}$$

$$y(t) = C(q)x_q(t) + D(q)u(t) + c(q), \tag{2}$$

with $x_q \in X_q$ and $u \in U$. The state set X_q for all $q \in Q$, the input set U, and the output set Y are assumed to be polyhedral sets. As soon as a discrete input event $e \in E_{in}$ is applied, or an event generated by the continuous dynamics occurs, $e \in E_{cd}$, because the continuous state has reached the guard $G_q(e) \subseteq \partial X_q$, a discrete transition takes place according to the transition map f and the reset map is applied to the continuous state at the past discrete state to yield the initial condition at the new discrete state:

$$\text{if } x_{q^-}^- = \lim_{s \uparrow t} x_{q^-}(s) \in G_{q^-}(e) \text{ or if } e \in E_{in} \text{ occurs, then}$$

$$\begin{aligned} q^+ &= f(q^-, x_{q^-}, e), \\ x_{q^+}^+ &= A_r(q^-, e, q^+)x_{q^-}^- + b_r(q^-, e, q^+). \end{aligned}$$

[4] The class of piecewise-affine hybrid systems has been proven to be equivalent with that of mixed logic-dynamical systems in discrete-time and with linear complementary systems, see [3].

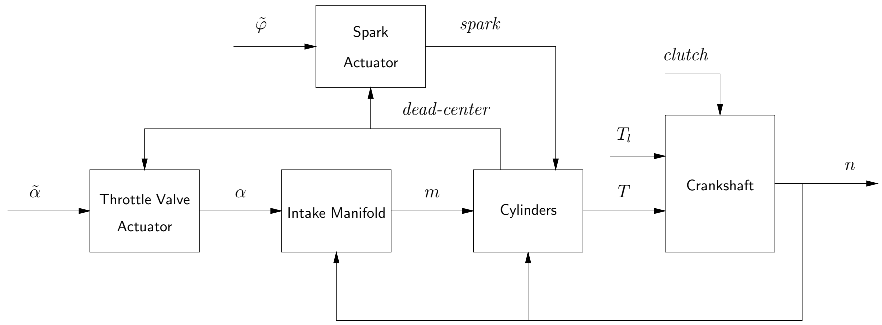

Fig. 1. Engine hybrid model for idle speed control.

At the new discrete state q^+, the evolution of the new continuous state x_{q^+} is described by differential equation (1), with q replaced by q^+, and with initial value $x^+_{q^+}$.

We recall also the notions of invariant, robust invariant, and controlled robust invariant sets that will be used in the rest of the paper.

Definition 2. *A subset of the state set of an autonomous dynamic system is said to be* forward invariant *(or* positively invariant*) if for any initial state in the subset the state trajectory for all future times remains inside the state set. A subset of a dynamic system with input is said to be* controlled forward invariant *if there exists an input function such that the closed-loop system has a forward invariant set.*

A subset of the state set of an autonomous dynamic system is said to be a robust (forward) invariant subset *with respect to a set of disturbance signals if for any initial condition in the subset and for any disturbance signal the resulting state trajectory remains inside the subset. A subset of a dynamic system with input is said to be a* controlled robust (forward) invariant subset *with respect to disturbances if there exists an input function such that the closed-loop system has a robust (forward) invariant subset.*

As shown in Figure 1, the engine hybrid model is composed of three interacting subsystems, namely the *intake manifold*, the *cylinders* and the *crankshaft*, plus the spark and throttle valve actuators. In idle speed control, the output of interest is the crankshaft speed n, whose evolution depends on the engine torque T, the load torque T_l acting on the crankshaft, and the state of the clutch (either open or closed). The engine torque T is a function of the spark ignition timing and the mass m of air-fuel mixture loaded in the cylinder during the intake stroke (the mixture is assumed to be stoichiometric):

$$T = \eta(\varphi)(Gm + T_0) \tag{3}$$

where φ denotes the spark advance[5] and $\eta(\varphi)$ denotes the spark ignition efficiency. The latter is a strictly increasing function defined on the interval of feasible spark advances $[-15, 20]$, with $\eta(-15) = 0.6$ and $\eta(20) = 1$. The spark advance is positive if ignition occurs during the compression stroke and negative if it occurs during the expansion stroke. The *spark actuator* models the delay of the spark actuation system. Due to this delay, the engine control has to issue the desired value of spark advance $\tilde{\varphi}$ at the beginning of the compression stroke, for each engine cycle.

The mass m of air-fuel mixture is controlled by the throttle plate position α and is subject to the dynamics of the cylinder filling due to the intake manifold.

The *throttle valve actuator* describes the synchronization of throttle control with the engine cycle. The throttle valve is driven by sequences of commands $\tilde{\alpha}$, with a time separation of 5 msec, and synchronized with dead–center events[6]: each sequence starts with the first command triggered by a dead–center event, the number of commands in the sequences depend on the time between two consecutive dead–center events, i.e. on the crankshaft speed. Throttle valve commands $\tilde{\alpha}$ are nonnegative and bounded from above by 20 degrees.

Due to the necessary synchronization between the engine cycle and throttle valve and spark ignition actuators, the *cylinders* model returns the dead–center event signals to both actuators.

The hybrid model of the engine has 6 discrete states and a 10–dimensional continuous state, i.e.

$$Q = \{S_-, S, S_+, S^L_-, S^L, S^L_+\} \quad \text{and} \quad x = (\alpha, \tau, p, m_C, m_E, \varphi, \varphi_N, T, n, \theta)$$

with: α the throttle valve angular position, τ a timer introduced to generate throttle valve command events, p the intake manifold pressure, m_C the mass of air-fuel mixture in the cylinder currently in compression stroke, m_E the mass of air-fuel mixture in the cylinder currently in expansion stroke, φ the spark advance, φ_N the spark advance in the next expansion, T the engine torque, n the crankshaft speed, and θ the crankshaft angle.

The discrete behavior of the hybrid system, namely the transitions between the discrete states and the reset maps, is represented in Figure 2. Transitions are triggered by

- input events in $E_{in} = \{on, off\}$, modeling opening and closing of the clutch, respectively;
- events generated by the continuous dynamics in $E_{cd} = \{\ \theta = 180, \theta = 180 - \varphi, \theta = -\varphi, \tau = 5\ \}$ and modeling, respectively, the reaching of a dead–center, the actuation of a positive spark advance, the actuation of a negative spark advance, and the reading of a throttle valve command.

[5] The spark advance is defined as the difference between the angular position of the crankshaft at the end of the compression stroke and its value at ignition time.

[6] In 4–stroke 4–cylinder in–line engines, at any time each cylinder is in a different stroke (intake, compression, expansion, exhaust). Stroke transitions, occurring when pistons reach either a top or a bottom dead–center, are synchronous.

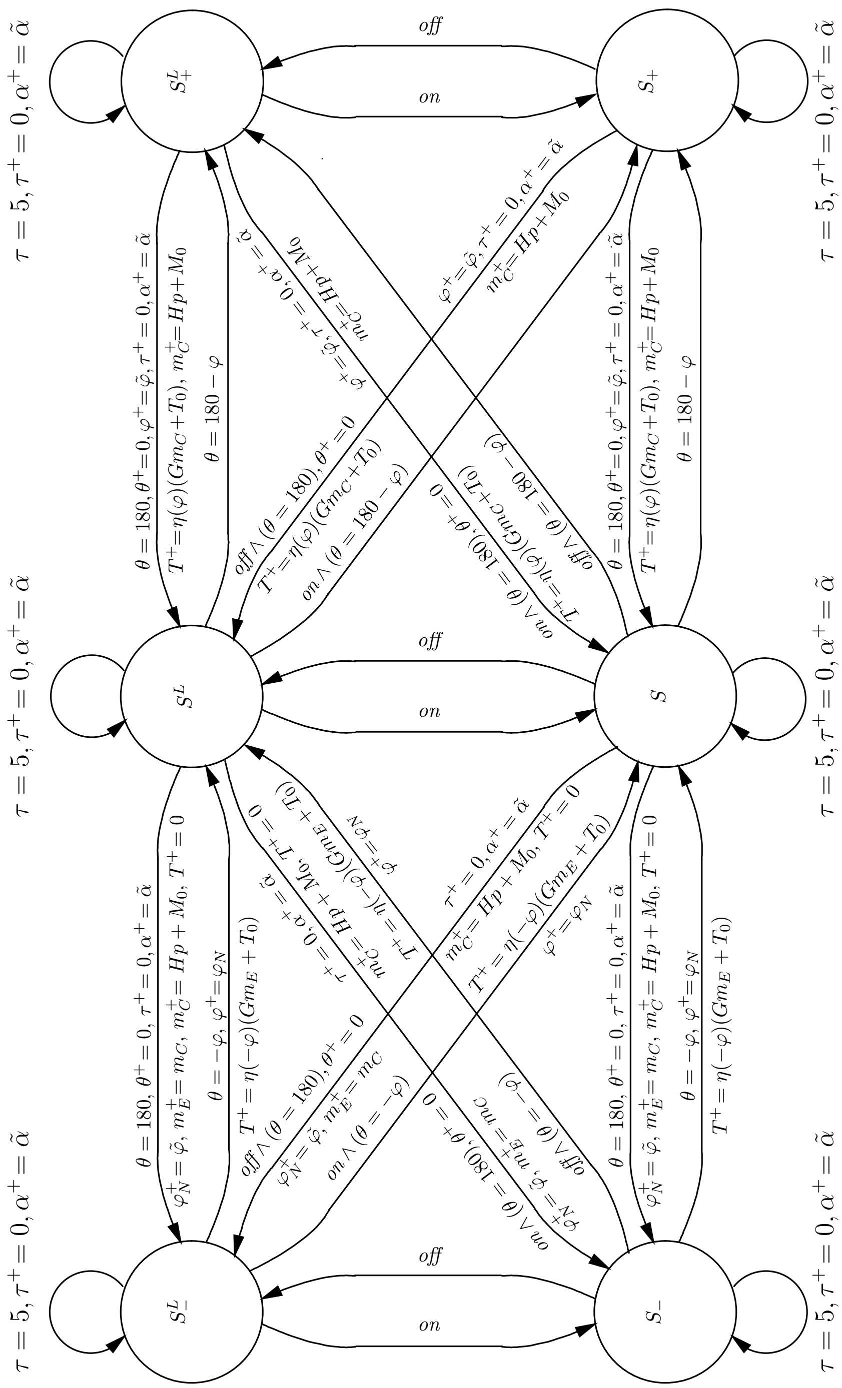

Fig. 2. Model of the discrete behavior of the engine.

Variables $\alpha, m_C, m_E, \varphi, \varphi_N, T$ evolve as piecewise constant signals (i.e. according to the dynamic $\dot{x} = 0$) and are updated by the reset maps defined in Figure 2. Note that variables m_E and φ_N are used only to model negative spark advance.

Consider first the discrete states S, S_+ and S_-, corresponding to the clutch in open state. In state S, the cylinder in expansion is generating torque, and the cylinder in compression has not received the spark command yet.

If the spark advance φ is positive then, when $\theta = 180 - \varphi$ the spark is ignited and a transition to S_+ takes place. In state S_+ the spark command has been given for the cylinder in compression, while the cylinder in expansion is generating torque. At the next dead-center, i.e. when $\theta = 180$, a transition from S_+ to S occurs:

- the crankshaft angle θ is reset;
- the desired value of spark advance φ for the next cylinder is received;
- the value of the engine torque T produced by the cylinder entering the expansion stroke is computed ($T^+ = \eta(\varphi)(Gm_C + T_0)$);
- the mass of mixture m_C loaded by the cylinder at the end of the intake stroke is determined ($m_C^+ = Hp + M_0$).

From state S, if a negative spark advance φ is applied then, when the next dead–center event is generated (i.e. $\theta = 180$), a transition to S_- takes place:

- the crankshaft angle θ is reset;
- the desired value of spark advance φ_N for the next cycle (related to the cylinder currently in compression stroke) is received;
- the mass of mixture m_C loaded by the cylinder at the end of the intake stroke is determined ($m_C^+ = Hp + M_0$);
- the engine torque T is reset, since spark has not been ignited yet for the cylinder currently in expansion stroke.

In phase S_-, the cylinder in expansion is waiting for the spark command and the cylinder in compression has not received the spark command yet. No torque is generated in this case. When the specified value φ of negative spark advance is reached, spark ignition occurs with the transition from S_- to S and the corresponding value of engine torque is produced ($T^+ = \eta(-\varphi)(Gm_E + T_0)$).

A timer τ, assuming values in the interval $[0, 5]$, is introduced to model the higher frequency of throttle valve commands. In all transitions produced by the dead–center event $\theta = 180$, the timer is reset and the first value of a new sequence of commands is received from the input $\tilde{\alpha}$. Then, the timer τ evolves with dynamic $\dot{\tau} = 1000$. The self–loop transitions in Figure 2 detect when τ reaches the value 5. When they are activated, 5 msec have been elapsed from the last timer reset and a new throttle valve command is read from the input $\tilde{\alpha}$.

The behavior of the system for the discrete states S_-^L, S^L, S_+^L, related to clutch closed, is analogous. Transitions between states S_-, S, S_+ and states S_-^L, S^L, S_+^L are due to clutch switching, which are modeled by the input events in $E_{in} = \{on, off\}$.

The model is completed by the continuous dynamics of the p, n and θ. The dynamic of the manifold pressure p is identical in all discrete states:

$$\dot{p}(t) = a_p(p(t) - p_0) + b_p(\alpha(t) - \alpha_0) \ , \tag{4}$$

where p_0, α_0 is the equilibrium point around which the linear model has been obtained.

The dynamic of the crankshaft depends on the clutch state:

$$\dot{n}(t) = \begin{cases} a_n n(t) + b_n[T(t) - T_p - T_l(t)] & \text{if } q \in \{S_-, S, S_+\} \\ a_n^L n(t) + b_n^L[T(t) - T_p - T_l(t)] & \text{if } q \in \{S_-^L, S^L, S_+^L\} \end{cases} \quad (5)$$

$$\dot{\theta}(t) = 6n(t) \quad (6)$$

where T_p is a constant term modeling pumping work and friction, $T_l(t) \in [0, T^M]$ is a bounded disturbance modeling variable friction and the action of subsystems powered by the crankshaft, n is in rpm and θ is in degrees.

Finally, since all the continuous state variables are nonnegative and bounded from above, then in each discrete state q the continuous input $(\tilde{\varphi}, \tilde{\alpha})$, state x and output $y = n$ evolve in polyhedral sets U, X_q, Y, respectively. However, the boundaries of X_q that can be reached, causing an internal event which trigger a discrete state transition, are only those defined by $\theta = 180$, for $q \in \{S, S_+, S^L, S_+^L\}$, $\theta = 180 - \varphi$, for $q \in \{S, S^L\}$, $\theta = -\varphi$, for $q \in \{S_-, S_-^L\}$, and $\tau = 5$ in any $q \in Q$.

3 Control synthesis for affine systems on polytopes

Our approach to synthesis for idle speed control is based on concepts and theorems derived for affine systems on polytopes. An important role is played by the so called *control-to-facet* problem. The problem is to guide the closed–loop trajectory of the system from a given point to a particular facet without first crossing or touching any other facet of the polytope. Necessary and sufficient conditions for the existence of a control law that solve the control-to-facet problem exist [5, 7]. We report below some extensions of these results to the case of discrete-time affine systems on simplices.

Problem 1. Consider a discrete-time affine system on a simplex,

$$x(t+1) = Ax(t) + Bu(t) + a, \qquad x(t_0) = x_0$$
$$X_1 = \text{convh}(\{v_1, \ldots, v_{n+1}\})$$

with $A \in \mathbb{R}^{n \times n}$, $B \in \mathbb{R}^{n \times m}$, $a \in \mathbb{R}^n$, and a polytope

$$X_2 = \text{convh}(\{w_1, \ldots, w_m\}) = \{x \in \mathbb{R}^n | n_j^T x \leq k_j, \ \forall j \in \{1, 2, \ldots, m\}\}$$

with $m \geq n+1$, normal $n_j \in \mathbb{R}^n$ pointing out of the polytope, and w_j opposite vertex of normal $n_j \in \mathbb{R}^n$. Determine a control law $g : X_1 \to U$ such that the closed-loop system,

$$x(t+1) = Ax(t) + Bg(x(t)) + a, \ x(t_0) = x_0,$$

is such that for all $t \in T$, $x(t) \in X_1$ implies that $x(t+1) \in X_2$.

A special case of the above problem is that with $X_2 = X_1$, and then the closed-loop system leaves the set X_1 invariant or X_1 is a controlled invariant set.

Proposition 1. *Assume that the polytope X_2 is a simplex, i.e. $m = n+1$. If there exist vectors $u_1, \ldots, u_{n+1} \in U = \mathbb{R}^m$ such that*

$$n_j^T(Aw_i + Bu_i + a) \leq k_j, \quad \forall\, i, j \in \{1, 2, \ldots, n+1\}\ , \tag{7}$$

then the matrix $\begin{pmatrix} w_1^T & 1 \\ \vdots & \vdots \\ w_{n+1}^T & 1 \end{pmatrix}$ *is invertible and the affine control law $g(x) = Fx + h$,*

$$\textit{with} \qquad \begin{pmatrix} F^T \\ h^T \end{pmatrix} = \begin{pmatrix} w_1^T & 1 \\ \vdots & \vdots \\ w_{n+1}^T & 1 \end{pmatrix}^{-1} \begin{pmatrix} u_1^T \\ \vdots \\ u_{n+1}^T \end{pmatrix} \in \mathbb{R}^{(n+1)\times m}\ , \tag{8}$$

solves Problem 1.

Proof. Since X_2 is a simplex, then the matrix with parameters w_j in (8) is invertible, see [7] for a proof. Note that, by (8), for all $j \in \{1, 2, \ldots, n+1\}$, $Fw_j + h = u_j$. The closed-loop system is,

$$x(t+1) = Ax(t) + Bu(t) + a = (A + BF)x(t) + (a + Bh).$$

Because X_2 is a simplex,

$$\exists c \in \mathbb{R}_+^{n+1},\ \sum_{i=1}^{n+1} c_i = 1, \text{ such that, } x \in X_2 \ \Rightarrow\ x = \sum_{i=1}^{n+1} c_i w_i.$$

$$\begin{aligned} u(t) &= Fx(t) + h = \sum_{i=1}^{n+1} c_i(Fw_i + h) = \sum_{i=1}^{n+1} c_i u_i. \\ n_j^T x(k+1) &= n_j^T[Ax(t) + Bu(t) + a] \\ &= \sum_{i=1}^{n+1} c_i n_j^T(Aw_i + Bu_i + a) \leq \sum c_i k_j = k_j,\ \forall j \in \{1, 2, \ldots, n+1\} \\ &\Rightarrow x(t+1) \in X_2. \end{aligned}$$

Note that if $X_2 = X_1 = X$, then X is a forward invariant = subset. If $X_2 \subset X_1$ then $x(t+1) \in X_2 \subset X_1$, hence $x(t+2) \in X_2 \subset X_1$ and X_2 is forward invariant.

Proposition 2. *Problem 1 admits a solution for polytope X_2 not a simplex, if there exists a feasible solution for F and h to the problem*

$$n_j^T(Av_i + B(Fv_i + h) + a) \leq k_j,\ \forall j \in \{1, 2, \ldots, m\},\ \forall i \in \{1, 2, \ldots, n+1\}.$$

A technique for computing feedback parameters F and h, based on a partitioning of the set X_1 is presented in [7].

4 Control synthesis for idle speed control

In this section, we use a structured synthesis method analogous to back–stepping to synthesize a hybrid idle speed control algorithm. In general, the idle speed control problem can be formalized as follows:

Problem 2. Given a set point for the crankshaft speed $n^0 \in (0,\infty)$ and an interval $I_n \subset (0,\infty)$, such that n^0 belongs to the interior of I_n, determine a control law such that the closed-loop system satisfies the following control objectives:

(1) *Bounding the crankshaft speed.* There exists a non trivial robust invariant set $X_{ri} \subseteq \{x \in X | n \in I_n\}$. Thus, if $x_0 \in X_{ri}$ then, for any disturbance signals, $x(t) \in X_{ri}$ and $n(t) \in I_n$ for all $t \geq 0$.
(2) *Asymptotic stability.* In the absence of disturbances, $\lim_{t\to\infty} n(t) = n^0$.
(3) *Attraction.* For any disturbance signals, $\lim_{t\to\infty} x(t) \in X_{ri}$.
(4) *Rejection of disturbances.* For stepwise disturbances, $\lim_{t\to\infty} n(t) = n^0$.

We focus on the control objective (1) formulated above, with $I_n = [750, 850]$ rpm. The problem is difficult because the dynamic system for the crankshaft speed is a system with delays, where the duration of the delays depends on the full dynamics. Our approach to controller synthesis makes use of the structure of the dynamic system. The system consists of a series connection of

1. the intake manifold system,
2. the cylinder system, and
3. the crankshaft system.

First, we synthesize a control law for the torque, which is the input to the crankshaft system. Then, proceeding backward in the system, we produce a control law for the cylinder system with as input the gas mixture and the spark advance angle such that the control objectives are met for the torque. Finally, we devise a control law for the throttle valve, which is the input to the intake manifold system.

Step 1 - Controller synthesis for the crankshaft system. By specification the crankshaft speed n is bounded between 750 rpm and 850 rpm. Furthermore, for idle speed control it is reasonable to bound the engine torque to the range $[0, 30]$ Nm. Then, consider an extended model of the crankshaft with state variables (n, θ, T), defined on the multi-variable box

$$\begin{aligned} B^c &= \{(n,\theta,T) | n \in [750,850], \theta \in [0,180], T \in [0,30]\} \\ &= [750,850] \times [0,180] \times [0,30] \ . \end{aligned}$$

Let F^o and F^e be the facets of B^c lying on the subspaces $\theta = 0$ and $\theta = 180$, respectively, i.e.

$$F^o = [750,850] \times \{0\} \times [0,30] \text{ and } F^e = [750,850] \times \{180\} \times [0,30] \ .$$

Consider the *control-to-facet* problem of determining a subset of B^c such that F^e is the unique exit facet. Since θ monotonically increases in B^c, this problem can be stated as follows:

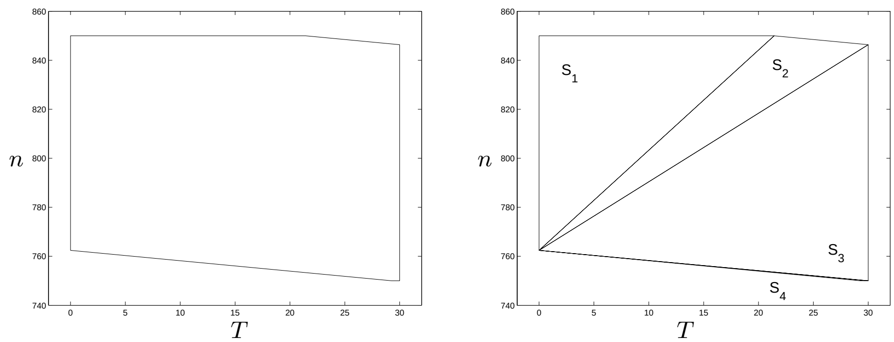

Fig. 3. Set S^c (on the left) and its partition $S_1 \cup S_2 \cup S_3 \cup S_4$ (on the right).

Problem 3. Find a subset $S^c \subset [750, 850] \times [0, 30]$ such that, under any action of the disturbances *clutch* $\in \{on, off\}$ and $T_l(t) \in [0, T^M]$, all trajectories starting from $(n(0), \theta(0), T(0)) = (n_0, 0, T_0) \in F^o$, with $(n_0, T_0) \in S^c$, reach in finite time the exit facet F^e and $(n(t), \theta(t), T(t)) \in B^c$, until F^e has been reached.

Note that the time required to reach the target set F^e depends on the evolution of the crankshaft angle θ, which is affected by both the engine torque $T(t)$ and the disturbances *clutch* and $T_l(t)$. A polyhedral set S^c for which the following proposition holds has been computed:

Proposition 3. *If the initial state* (n_0, θ_0, T_0) *of crankshaft system satisfies* $\theta_0 = 0$ *and* $(n_0, T_0) \in S^c$*, with* S^c *as in Figure 3, then* F^e *is the unique exit facet from* B^c*. Furthermore, the worst torque disturbance* $T_l(t)$ *is constant and equal either to* 0 *or* T^M *and the worst clutch signal is clutch constant and open.*

Let t_k denote the sequence of times at which dead–center events are produced. From the above proposition it follows that

Corollary 1. *If the engine torque is controlled in such a way that*

$$(n^+(t_k), T^+(t_k)) \in S^c, \quad \forall k \in \mathbb{Z}_+^0 \tag{9}$$

then $(n(t), \theta(t), T(t)) \in B^c$ *for all* $t \geq t_0$ *and the control objective (1) of Problem 2 is met.*

To ensure that condition (9) is verified at each dead–center, the crankshaft system is represented by a discretized model that expresses the evolution of the system at dead–center times. The discretized model is obtained by assuming that the engine torque T and the disturbance torque T_l are constant between dead–center times. While assuming T_l constant between dead–center times is justified by Proposition 3, this assumption is actually not verified for T in the case of negative spark advance. In fact, according to the engine hybrid model, the expansion stroke starts in the state S_- (or S_-^L) associated to a zero engine torque and proceeds in the state S (or S^L) with the generation of a constant engine torque. However, since assuming T constant greatly simplify the developments, then this assumption will be kept for the moment and the particular behavior in case of negative spark advance will be handled in Step 2 below. To model the compression delay, we introduce a virtual input $u(k)$ corresponding to the next value of the engine torque. The discretized crankshaft model is

$$n(k+1) = a_d(k)n(k) + b_d(k)(T(k) - T_p - T_l(k)) \tag{10}$$
$$T(k+1) = u(k) \tag{11}$$

where $n(k) = n^+(t_k)$, $T(k) = T^+(t_k)$, and $T_l(k) \in \{0, T^M\}$ models the worst torque disturbance. Model (10-11) is time–varying since its parameters

$$a_d(k) = e^{a_n(t_{k+1}-t_k)} \quad \text{and} \quad b_d(k) = -[1 - e^{a_n(t_{k+1}-t_k)}]b_n/a_n \tag{12}$$

depend on the dead–center time interval length $t_{k+1} - t_k$, which is assumed unknown. However, since the crankshaft speed is bounded by specification, then

$$\frac{30}{850} = \frac{30}{\max n(t)} = t_{min} \ \le t_{k+1} - t_k \le \ t_{max} = \frac{30}{\min n(t)} = \frac{30}{750} \ , \tag{13}$$

and parameters $a_d(k), b_d(k)$ are bounded as follows

$$e^{a_n t_{max}} = \underline{a}_d \ \le a_d(k) \le \ \overline{a}_d = e^{a_n t_{min}} \tag{14}$$
$$-[1 - e^{a_n t_{min}}]b_n/a_n = \underline{b}_d \ \le b_d(k) \le \ \overline{b}_d = -[1 - e^{a_n t_{max}}]b_n/a_n \ . \tag{15}$$

Then, condition (9) of Corollary 1 is verified once the following problem is solved:

Problem 4. Find a feedback $u(k) = g(n(k), T(k))$ and a set $X \subseteq S^c$ such that

(1) X is a robust invariant set for the crankshaft discretized model (10–11), with time–varying parameters (12) bounded as in (14–15), under feedback $u(k) = g(n(k), T(k))$ and for any action of the disturbances *clutch* $\in \{$*on*, *off*$\}$ and $T_l(t) \in [0, T^M]$;
(2) the feedback $g(n(k), T(k))$ can be implemented by the cylinder model using appropriate values of spark advance φ and mass of gas mixture m.

The design of a feasible feedback $u(k) = g(n(k), T(k))$ and its implementation will be addressed in the next step.

Step 2 - Controller synthesis for the cylinder system. In general, it is not easy to determine a non trivial controlled robust invariant subset X contained in some polyhedral set S^c as specified in point (1) of Problem 4. Therefore an indirect method will be used:

1. Partition the polytope S^c into a finite number of simplices, denoted by $\Pi = \{S_1, \ldots, S_N\}$. This procedure is called *triangulation* (see [7]).
2. Select a subset of these simplices $\Pi^I = \{S_{\ell_1}, \ldots, S_{\ell_M}\} \subseteq \Pi$, which defines a candidate set $X = \cup_{S_p \in \Pi^I} S_p$ on which robust controlled invariance will be tested.

3. Split the set of simplices Π^I into Π_1^I and Π_2^I, with $\Pi_1^I \cup \Pi_2^I = \Pi^I$ and $\Pi_1^I \cap \Pi_2^I = \emptyset$, and determine for each simplex $S_p \in \Pi^I$ whether there exists an affine control law $g_p(x) = F_p x + h_p$ such that the closed-loop system maps robustly in one step S_p to a subset of $\cup_{S_p \in \Pi_2^I} S_p$.
4. If it fails, either choose a different partition Π_1^I, Π_2^I, or a different subset of simplices Π^I, or change[7] the triangulation of S^c.

The set S^c has been partitioned in the subsets $\Pi = \{S_1, S_2, S_3, S_4\}$, represented in Fig. 3. Furthermore, robust controlled invariance has been enforced on $\Pi^I = \{S_1, S_2, S_3\}$, by choosing $\Pi_1^I = \{S_1\}$ and $\Pi_2^I = \{S_2, S_3\}$, so that the robust controlled invariant set $X = S_1 \cup S_2 \cup S_3$ has been determined.

By a result of Benvenuti and Farina (see [4]), due to the boundness (14–15) of the time–varying parameters $a_d(k), b_d(k)$ and the linearity of (12) with respect to the term $e^{a_p(t_{k+1}-t_k)}$, the closed–loop system, obtained by (10-11) and an piecewise affine feedback $g_p(\cdot)$, maps robustly each $S_p \in \Pi^I$ inside $\cup_{S_p \in \Pi_2^I} S_p$ if this is verified for the two systems obtained by replacing $a_d(k), b_d(k)$ with $\underline{a}_d, \overline{b}_d$ and $\overline{a}_d, \underline{b}_d$. For each simplex S_p in $\Pi^I = \{S_1, S_2, S_3\}$, the parameters F_p and h_p of the affine control law

$$g_p(n(k), T(k)) = F_p \begin{bmatrix} n(k) \\ T(k) \end{bmatrix} + h_p \tag{16}$$

are obtained from Proposition 2, where:
- v_i define the simplex $S_p = X_1 = \mathrm{convh}(\{v_1, v_2, v_3\})$
- n_j, k_j define the target polyhedron

$$X_2 = S_2 \cup S_3 = \{x \in \mathbb{R}^2 | n_j^T x \leq k_j,\ \forall j \in \{1, \dots, 4\}\}$$

- A, B and a model the dynamics corresponding to the extreme values of the uncertain parameters and the worst case actions of disturbances

$$A = \begin{bmatrix} A' \\ A' \\ A'^L \\ A'^L \end{bmatrix}, \quad B = \begin{bmatrix} B' \\ B' \\ B' \\ B' \end{bmatrix}, \quad a = \begin{bmatrix} -a' T_p \\ -a'(T_p + T^M) \\ -a'^L T_p \\ -a'^L (T_p + T^M) \end{bmatrix},$$

$$A' = \begin{bmatrix} 0 & 0 \\ \underline{a}_d & \overline{b}_d \\ 0 & 0 \\ \overline{a}_d & \underline{b}_d \end{bmatrix}, \quad A'^L = \begin{bmatrix} 0 & 0 \\ \underline{a}_d^L & \overline{b}_d^L \\ 0 & 0 \\ \overline{a}_d^L & \underline{b}_d^L \end{bmatrix}, \quad B' = \begin{bmatrix} 0 \\ 1 \\ 0 \\ 1 \end{bmatrix}, \quad a' = \begin{bmatrix} 0 \\ \overline{b}_d \\ 0 \\ \underline{b}_d \end{bmatrix} \quad a'^L = \begin{bmatrix} 0 \\ \overline{b}_d^L \\ 0 \\ \underline{b}_d^L \end{bmatrix}$$

where $\underline{a}_d^L$, $\overline{a}_d^L$, $\underline{b}_d^L$, $\overline{b}_d^L$ are defined as in (14–15), with the closed clutch parameters a_n^L, b_n^L in place of the open clutch parameters a_n, b_n.

The envelopes P_1, P_2, P_3 of the one–step ahead projections of simplices S_1, S_2 and S_3, under the proposed piecewise affine feedbacks (16) and considering parameter uncertainties and disturbances, are represented in Figure 4.

[7] A methodology for defining triangulations of S^c suitable for robust controlled invariant set computation is currently under investigation.

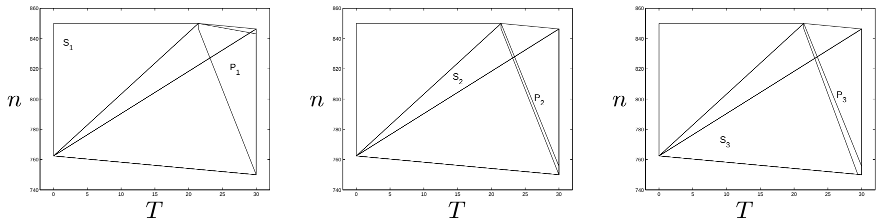

Fig. 4. Envelopes P_1, P_2, P_3 of one–step ahead projections of simplices S_1, S_2, S_3.

The cylinder system has to produce engine torques that implement feedbacks (16), using appropriate values of spark advance φ and mass of gas mixture m. Since the spark ignition efficiency $\eta(\varphi)$ is bounded between 0.6 and 1, then to guarantee feasibility (see point (2) of Problem 4), the amount of mass m loaded during intake has to satisfy

$$0.6(Gm + T_0) \leq g_p(n,T) \leq (Gm + T_0) \qquad \forall (n,T) \in S_1 \cup S_2 \cup S_3 \ . \tag{17}$$

Then, the mass of mixture is controlled to the interval $[m_{min}, m_{max}]$, with

$$m_{min} = \frac{1}{G}\left[\left(\max_{v_i \in S_1, S_2, S_3} F_p v_i + h_p\right) - T_0\right], \qquad m_{max} = \frac{1}{0.6} m_{min} \ . \tag{18}$$

Feedbacks (16) are implemented by a modulation of the spark advance efficiency computed at the dead–center time t_k as follows

$$\tilde{\varphi} = \begin{cases} \phi_1 & \text{if } \phi_1 \geq 0 \\ \phi_2 & \text{if } \phi_1 < 0 \end{cases}$$

with ϕ_1 and ϕ_2 obtained for $[n(t_k), T(t_k)]^T \in S_p$ from the following expressions[8]

$$\phi_1 = \eta^{-1}\left(\frac{F_p[n(t_k) \quad T(t_k)]^T + h_p}{Gm_C(t_k) + T_0}\right) \tag{19}$$

$$\eta(\phi_2)(Gm_C(t_k) + T_0) = \frac{1 - e^{a_n \frac{30}{n(t_k)}}}{1 - e^{a_n\left[\frac{30}{n(t_k)}\left(1 - \frac{\phi_2}{180}\right)\right]}}\left[F_p\begin{pmatrix} n(t_k) \\ T(t_k)\end{pmatrix} + h_p\right] \ . \tag{20}$$

Step 3 - Controller synthesis for the intake manifold system. From the target interval for the mass of mixture (18), the corresponding interval $[p_{min}, p_{max}]$, with

$$p_{min} = \frac{m_{min} - M_0}{H} \quad \text{and} \quad p_{max} = \frac{m_{max} - M_0}{H},$$

[8] The fraction on the right–hand side of (20) takes into account that, for negative spark advance, the engine torque is produced only in the second part of the expansion stroke, its value is then appropriately increased to achieve the same result on $n(t_{k+1})$.

for the intake manifold pressure is derived. An intake manifold controller that keeps the value of the intake manifold pressure inside the interval $[p_{min}, p_{max}]$ at each dead center is designed using the control-to-facet approach. The intake manifold controller is an affine feedback

$$\alpha(k) = f_1 p(k) + h_1 \tag{21}$$

obtained by applying the results of Proposition 2 with

- $X_1 = \text{convh}(\{v_1, v_2\})$, with $v_1 = p_{min}$ and $v_2 = p_{max}$
- $X_1 = X_2$, with $n_1 = -n_2 = 1$, $k_1 = p_{max}$ and $k_2 = -p_{min}$
- dynamic parameters

$$A = \begin{bmatrix} \underline{a}_p \\ \overline{a}_p \end{bmatrix}, \quad B = \begin{bmatrix} \overline{b}_p \\ \underline{b}_p \end{bmatrix}, \quad a = \begin{bmatrix} (1-\underline{a}_p)p_0 - \overline{b}_p\alpha_0 \\ (1-\overline{a}_p)p_0 - \underline{b}_p\alpha_0 \end{bmatrix}, \qquad \text{with}$$
$$\underline{a}_p = e^{a_p t_{max}}, \overline{a}_p = e^{a_p t_{min}}, \underline{b}_p = [e^{a_p t_{min}} - 1]\frac{b_p}{a_p}, \overline{b}_p = [e^{a_p t_{max}} - 1]\frac{b_p}{a_p} \ .$$

The implementation of the intake manifold controller (21) exploits the fact that the throttle valve is actuated with a frequency higher than the dead–center event frequency. In fact, the throttle valve is driven by sequences of commands, with a time separation of 5 msec, and synchronized with dead–center events. Since $t_{min} = \frac{30}{850} > 30$ msec and $t_{max} = \frac{30}{750} = 40$ msec, then for each intake stroke a sequence of 8 commands is applied.

The sequence of throttle valve commands for the k–th intake stroke, lasting from time t_k to time t_{k+1}, is obtained as follows:

- the first command at time t_k, synchronized with the dead–center event at the beginning of the intake stroke, is given by (21) with $p(k) = p(t_k)$;
- the following 7 commands, given at times $t_h = t_k + \ell 5$ msec, with $\ell = 1 : 7$, are computed with a feedback of the most recent value of intake manifold pressure measurement $p(t_h)$, on the basis of the updated information on the time to the $(k+1)$-th dead–center:

$$\alpha(t_h) = \frac{\bar{p}(k+1) - e^{a_p\tau}p(t_h) + (1-e^{a_p\tau})p_0 + [1-e^{a_p\tau}]b_p/a_p\,\alpha_0}{-[1-e^{a_p\tau}]b_p/a_p}$$

  with $\tau = \frac{30}{n(t_h)}\left(1 - \frac{\theta}{180}\right)$ the estimated time to the end of the intake stroke and

$$\bar{p}(k+1) = e^{a_p \frac{30}{n(t_k)}} p(t_k) - \left[1 - e^{a_p \frac{30}{n(t_k)}}\right] \frac{b_p}{a_p} \left(f_1 p(t_k) + h_1\right) \tag{22}$$

  the target intake manifold pressure value given by feedback (21).

Some simulation results of the proposed controller are reported in Figure 5.

5 Concluding remarks

We presented a control synthesis procedure for idle speed control. The procedure is based on the formulation of the problem as an affine hybrid system control

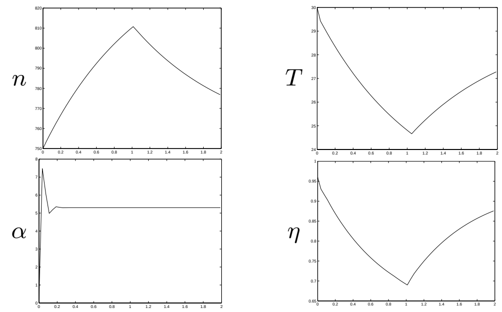

Fig. 5. Simulation results with a torque load T_l acting at time $t = 1\,\text{sec}$.

problem on polytopes. The control synthesis uses a novel combination of the back–stepping procedure, control for affine systems on polytopes, the theory of controlled invariant sets, and interactive control design. In the proposed approach, input and state constraints are handled by partitioning the continuous–state space in polytopes and designing feedback laws for each polytope such that, in the evolution of the controlled system, the continuous state is driven only on those polytopes for which the constraints are verified.

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