

**Mathematics of Road Congestion**  
Empirics, models and applications

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8/25/09

**TU Delft** Delft University of Technology  
Challenge the future

## Introduction

### Network load and performance degradation

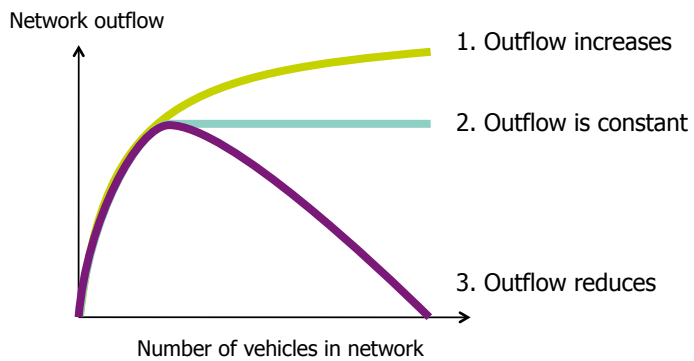
- Consider average relation between number of vehicles in network (accumulation) and performance (number of vehicles flowing out of the network)
- How does average performance (throughput, outflow) relate to accumulation of vehicles?
- What would you expect based on analogy with other networks?
  - Think of a water pipe system where you increase water pressure
  - What happens?

**TU Delft** CWI Queuing Colloquium 2 | 07

# Network traffic flow fundamentals

## Coarse model of network dynamics

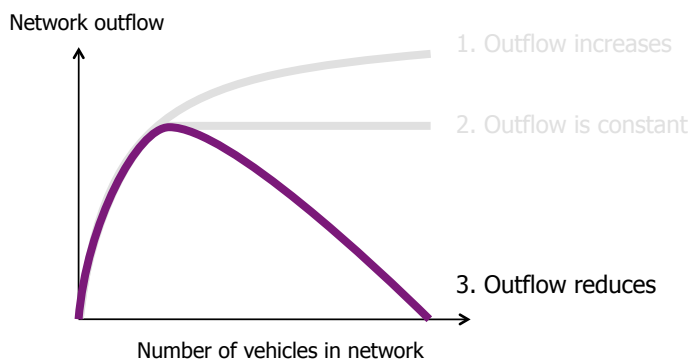
- Fundamental relation between network outflow (rate at which trip end) and accumulation



# Network traffic flow fundamentals

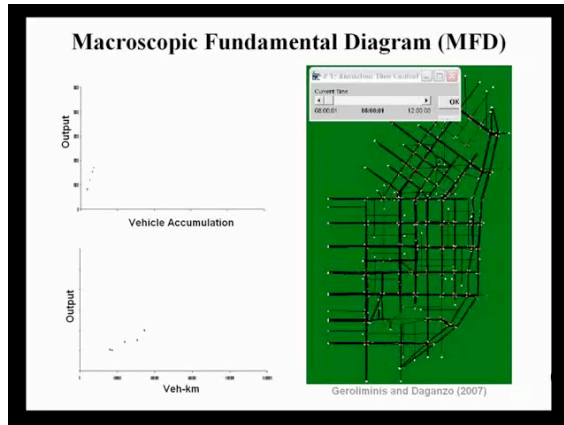
## Coarse model of network dynamics

- Fundamental relation between network outflow (rate at which trip end) and accumulation



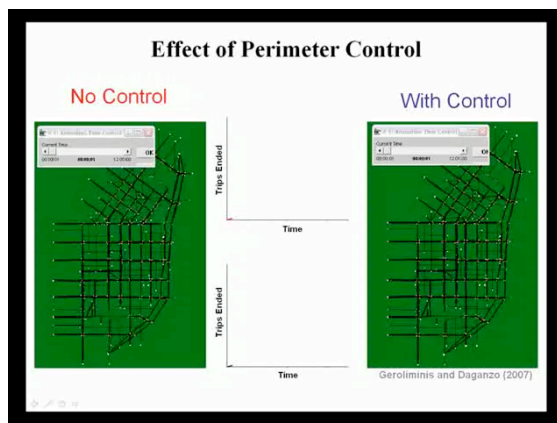
# Network traffic flow fundamentals

## Demand and performance degradation



# Network traffic flow fundamentals

## Implications for traffic network control



# Introduction

## Lecture overview

- Traffic congestion phenomena: examples and empirics
- Modeling traffic congestion in road networks
  - Model components of network models
  - Modeling principles and paradigms
  - Examples and case studies
- Intermezzo Braess paradox
- Model application examples
  - Traffic State Estimation and Prediction
  - Controlling congestion waves
  - Optimization of evacuation plans
- **Microscopic and macroscopic perspectives!**

# 1.

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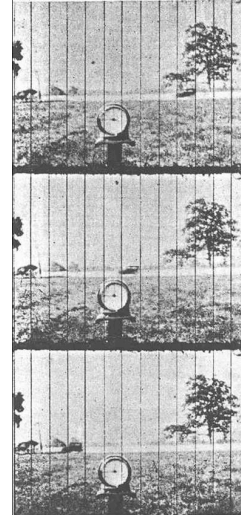
## *Traffic Congestion Phenomena*

### *Empirical Features of Traffic Congestion*

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## Historical perspective

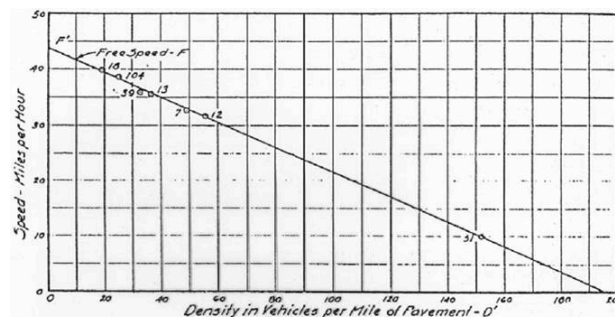
### Bruce Greenshields



## First model of traffic congestion

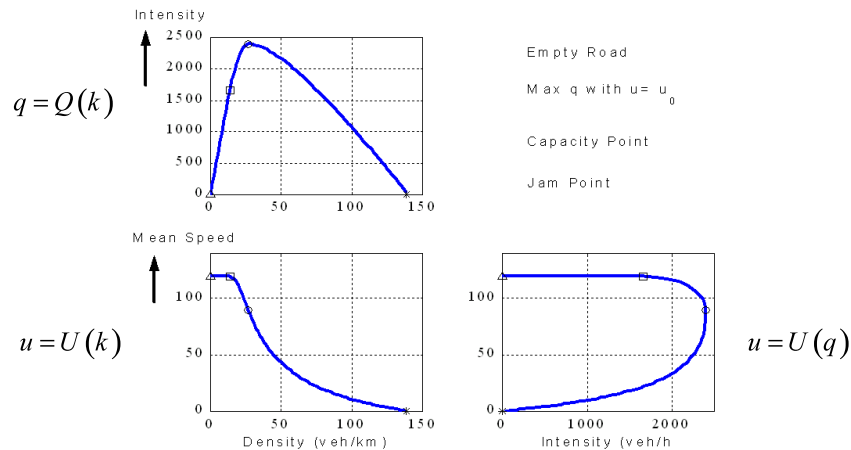
### Fundamental diagram

- Relation between traffic density and traffic speed:  $u = U(k)$
- Underlying behavioral principles? (density =  $1/\text{average distance}$ )



# Fundamental diagrams

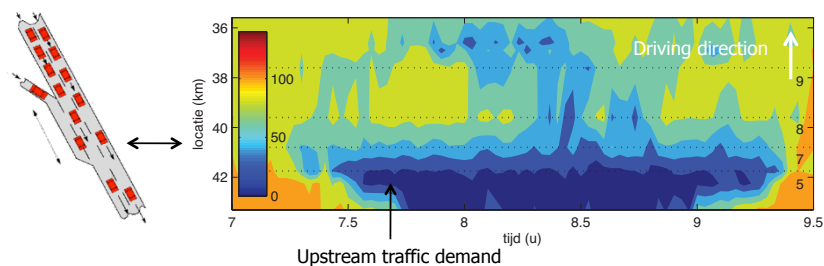
Different representations using  $q = k \times u$



# Dynamic properties

Traffic congestion at bottleneck (on-ramp)

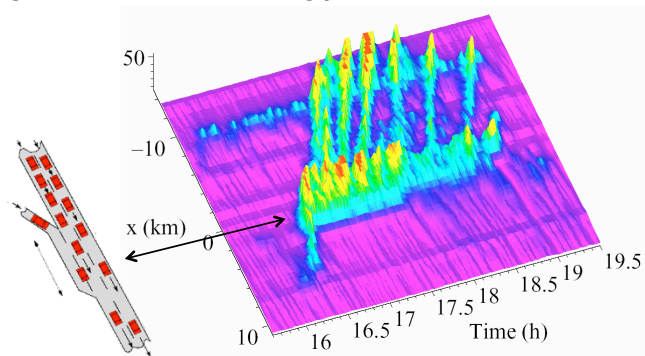
- Traffic from on-ramp is generally able to merge onto mainstream
- Resulting capacity (supply) is lower than demand
- Queue occurs upstream of bottleneck and moves upstream as long as upstream demand > flow in queue (shockwave theory)



## Dynamic features of road congestion

### Capacity funnel, instability, wide moving jams

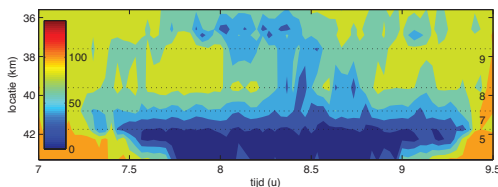
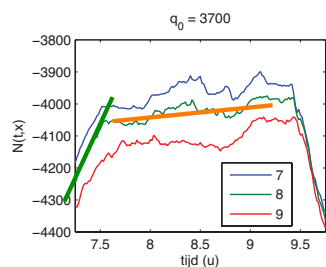
- Capacity funnel and capacity drop
- Self-organisation of wide moving jams



## Capacity funnel and capacity drop

### Free flow capacity and queue discharge rate

- On-set of congestion often occurs downstream of bottleneck
- Capacity before congestion > queue-discharge rate
- Use of (slanted cumulative curves) clearly reveals this
- $N(t,x)$  = # vehicles passing location  $x$  at time  $t$
- Slope = flow

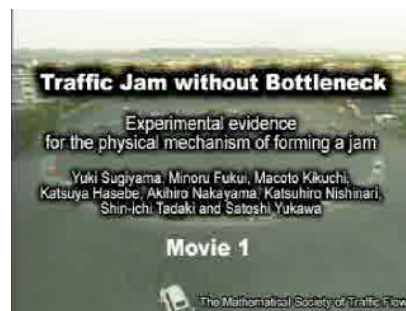


$$N'(t,x) = N(t,x) - q_0 \cdot t$$

## Instability and wide moving jams

### Emergence and dynamics of start-stop waves

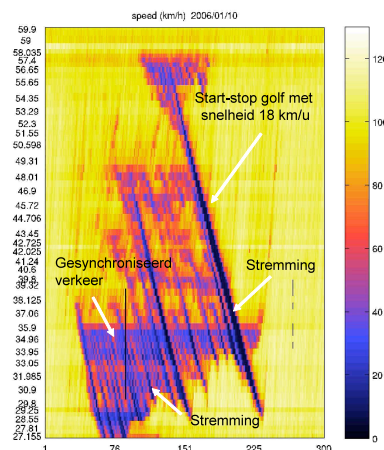
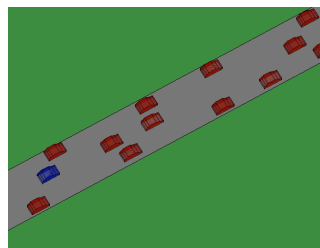
- In certain density regimes, traffic is highly unstable
- So called 'wide moving jams' (start-stop waves) self-organize frequently (1-3 minutes) in these high density regions



## Instability and wide moving jams

### Emergence and dynamics of start-stop waves

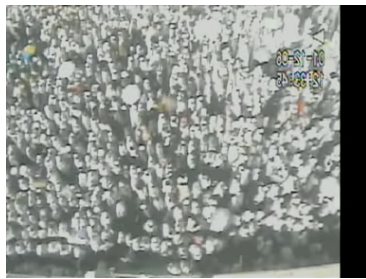
- Wide moving jams can exist for hours and travel past bottlenecks
- Density in wide moving jam is very high (jam-density) and speed is low



## Pedestrian flow congestion

### Start-stop waves in pedestrian flow

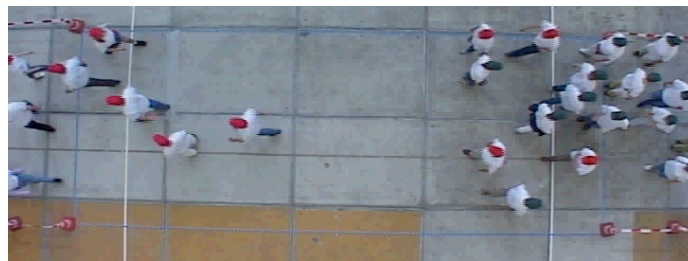
- Example of Jamarat bridge shows self-organized stop-go waves in pedestrian traffic flows



## Pedestrian flow congestion

### Other self-organization phenomena

- Self-organization is common in pedestrian flows
- E.g. bi-directional pedestrian flows show dynamic lane formation



# 2.

## *Traffic Flow Modeling*

*Microscopic and macroscopic approaches to describe flow dynamics*

### Modeling challenge

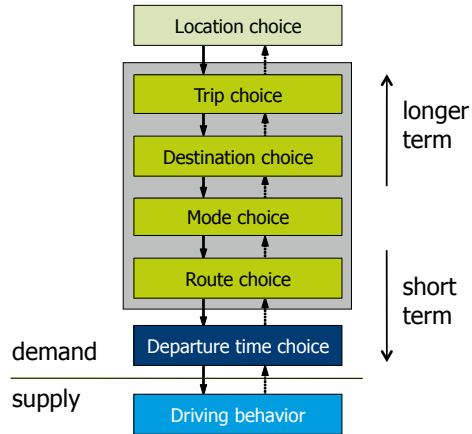
Traffic theory: not an exact science!

- Traffic flow is a result of human decision making and interactions at different behavioral levels (driving, route choice, departure time choice, etc.)
- Characteristics behavior (inter- and intra-driver heterogeneity)
  - Large diversity between driver and vehicle characteristics
  - Intra-driver diversity due to multitude of influencing factors, e.g. prevailing situation, context, external conditions, mood, emotions
- *The* traffic flow theory does not exist (and will probably never exist): this is not Newtonian Physics or thermodynamics
- Challenge is to develop theories and models that represent reality sufficiently accurate for the application at hand

# Network Traffic Modeling

## Model components and processes

- Traffic conditions on the road are end result of many decisions made by the traveler at different decision making levels
- Depending on type of application different levels are in- or excluded in model
- Focus on driving behavior and flow operations



# Modeling approaches

## Microscopic and macroscopic approaches

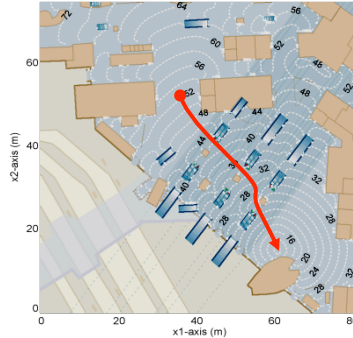
- Two dimensions:
  - Representation of traffic
  - Behavioral rules, flow characteristics

	Individual particles	Continuum
Individual behavior	Microscopic (simulation) models	Gas-kinetic models (Boltzmann equations)
Aggregate behavior	Newell model, particle discretization models	Macroscopic flow models

## Microscopic modeling example

### NOMAD Pedestrian Flow Simulation

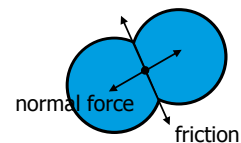
- NOMAD is a microscopic continuum model derived by applying optimal control theory / dynamic game theory
- Model entails 3 behavioral levels:
  - Strategic level: activity scheduling and global route choice
  - Tactical level: local route choice decision-making
  - Operational level: walking, waiting, executing activities



## NOMAD Walker model

### NOMAD Pedestrian Flow Simulation

- Model describes acceleration vector  $\mathbf{a}(t)$
- Distinction between
  - Physical interactions
  - Long-range interactions



$$\mathbf{a}(t) = \mathbf{a}_{\text{physical}}(t) + \mathbf{a}_{\text{control}}(t)$$

- Physical interactions describe normal forces and tangential forces (friction) when pedestrians touch
- Long-range interactions (control model) are derived by applying dynamic game theory

# Control model

## NOMAD Pedestrian Flow Simulation

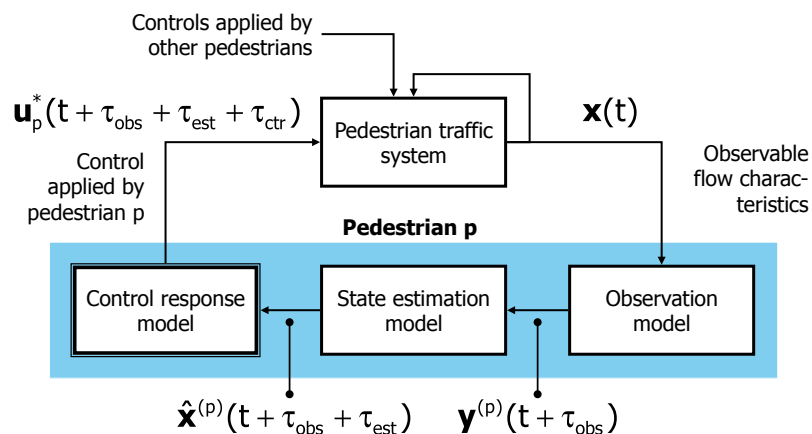
- Control model describes pedestrian interactions
- Main behavioral assumptions (based on psychological research):
  - Pedestrian can be described as optimal, predictive controllers who make short-term predictions of the prevailing conditions, including the anticipated behavior of the other pedestrians
  - Pedestrians minimize 'costs' due to distance between pedestrians, deviations from desired speed and direction, and acceleration
  - Costs are discounted over time, yielding costs:

$$J = \int_t^{\infty} e^{-\eta t} \left[ \frac{1}{2} \mathbf{a}^T \mathbf{a} + c_1 \frac{1}{2} (\mathbf{v}^0 - \mathbf{v})^T (\mathbf{v}^0 - \mathbf{v}) + c_2 \sum_q e^{-\frac{\|\mathbf{r}_q - \mathbf{r}\|}{R_0}} \right]$$

- Pedestrians are largely anisotropic particles

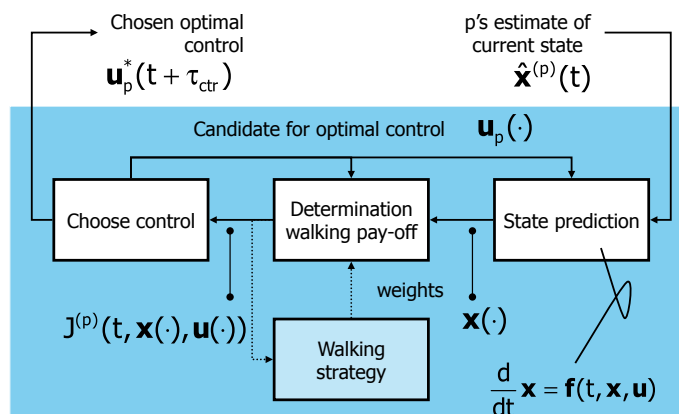
# Pedestrian control cycle

## NOMAD Pedestrian Flow Simulation



## Pedestrian control cycle

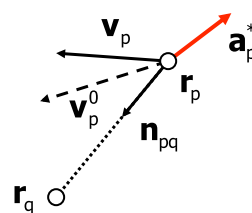
### NOMAD Pedestrian Flow Simulation



## Control model

### NOMAD Pedestrian Flow Simulation

- Simplifying assumption:  $\mathbf{a}_q = 0$
- After specification of state dynamics and cost function  $J$ ; resulting problem is a simple optimal control problem
- Applying Pontryagin's minimum principle yields acceleration term:



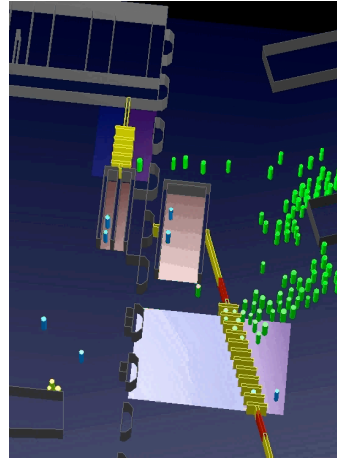
$$\mathbf{a}_{\text{control}}(t + \tau_r) = \frac{\mathbf{v}_p^0 - \mathbf{v}_p}{\tau_p} - A_p^0 \sum_{q \neq p} \mathbf{n}_{pq} e^{-\|r_p - r_q\|/R_p^0}$$

$$\text{where } \tau_p, A_p^0 > 0, \text{ and } \mathbf{n}_{pq} = \frac{\mathbf{r}_q - \mathbf{r}_p}{\|\mathbf{r}_q - \mathbf{r}_p\|}$$

## Pedestrian flow simulation

### NOMAD example

- NOMAD model has been extensively calibrated and validated
- NOMAD reproduces characteristics of pedestrian flow (fundamental diagram, self-organization)
- Applications of model:
  - Assessing Level-of-Service in transfer stations
  - Testing safety in case of emergency conditions (evacuations)
  - Testing alternative designs and Decision Support Tool
  - Hajj strategies and design



## Macroscopic models

### Analogy with gasses, fluids and granular media

- Assume that a traffic flow can be represented as a continuous (compressible) medium
- Let  $k = k(t,x)$ ,  $q = q(t,x)$  and  $u = u(t,x)$  respectively denote the density, flow en average speed (with  $q = ku$ )
- Assume that profiles are sufficiently smooth
- First continuum model (kinematic wave model,1955) consist of conservation of vehicle equation + fundamental diagram:

$$\text{kinematic wave model: } \begin{cases} \frac{\partial k}{\partial t} + \frac{\partial q}{\partial x} = r - s \\ q = Q(k) \end{cases}$$

## Macroscopic models

### Analogy with gasses, fluids and granular media

- Kinematic wave model is simple and well understood
- After 55 years, research is still performed on the model:
  - Model representation in Lagrangian coordinates
  - Hybrid modeling where lane changing vehicles coming from slow lane are considered as moving bottlenecks
  - Multi-class extensions (FastLane)
- Kinematic wave model captures only some of the features of traffic flow; it does not include:
  - Mechanism to predict emergence of start-stop waves (instabilities)
  - Capacity funnel (capacity drop can be included)
  - Smooth solutions instead of shocks
  - Non-equilibrium traffic flow conditions

## Macroscopic models

### Higher-order models

- Model by Payne is based on simple 'car-following' rule
- Describe delayed reaction to downstream flow conditions

$$u(t + T, x) = U(k(t, x + \Delta x)) \quad \text{met} \quad \Delta x = 1/k$$

- Simple Taylor series expansion yields higher-order model:

$$\frac{\partial u}{\partial t} + \underbrace{u \frac{\partial u}{\partial x}}_{\text{convection}} = \underbrace{\frac{U^e(k) - u}{T}}_{\text{relaxation}} - \underbrace{\frac{c_0^2}{k} \frac{\partial k}{\partial x}}_{\text{anticipation}}$$

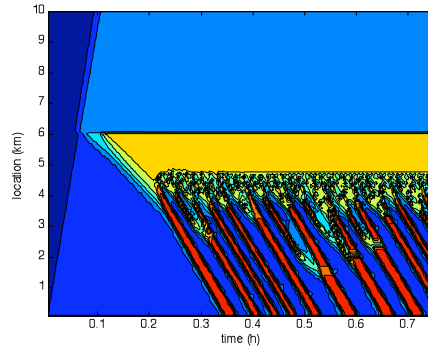
- Model yields some improvements, but also some new drawbacks (misery conservation), such as traffic backing down from queues

## Macroscopic models

### Probabilistic models

- Recent attempt to improve kinematic wave model entails including a dynamic model describing the phase-transition probability
- Phases: free flow, congested flow, wide moving jam
- Simple model can predict some of the observed features

$$\begin{cases} \frac{\partial k}{\partial t} + \frac{dQ}{dk} \frac{\partial k}{\partial x} = r - s \\ \frac{\partial P}{\partial t} + \frac{dQ}{dk} \frac{\partial P}{\partial x} = \pi(k, P) \end{cases}$$



# Intermezzo

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*Need for control?*

*Simple example of the Braess Paradox*

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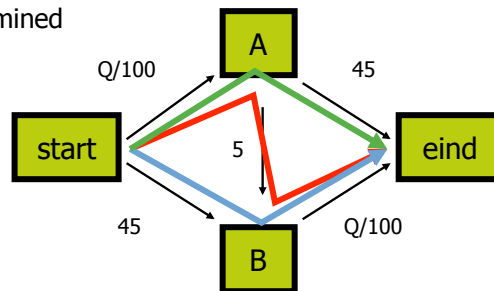
## A well-known game

### Simple network model

$$TT_1(Q) = TT_2(Q) = \frac{Q}{100} + 45$$

$$TT_3(Q) = \frac{Q}{50} + 5$$

- Two actors involved in this game:
  - Network-authority
  - Service provider providing route guidance
- Consider simple network with 3 routes
- Route travel time is determined by route demand  $Q$
- 4000 traveler travel from 'start' to 'end'
- How would the network authority distribute traffic over the routes?



## A well known game

### The Braess paradox

- Service provider aims to improve situation for driver using service, and would guide traveler over the shortest route (being route 2)
- If more drivers have access to information service, system performance will substantially decrease

	Route 1 en 2	Route 3	Totale reistijd
Scenario 1	65 min	45 min	260.000 min
Scenario 2	67 min	49 min	268.000 min
Scenario 3	85 min	85 min	340.000 min

# 3.

## *Solving congestion by optimization?*

*Applications of network flow models to improve traffic conditions*

## Traffic State Estimation & Prediction

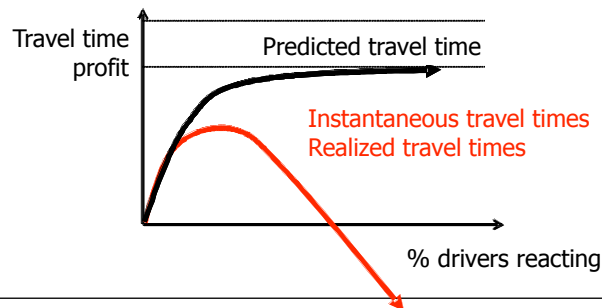
### Applications of Kalman filters

- Traffic information ([www.anwb.nl](http://www.anwb.nl) or from TomTom) provides information about prevailing traffic conditions
- Drivers want to know about future conditions
- Example model application <http://beta.fileradar.nl/>
- State estimation:
  - Data collected from inductive loops on motorway network
  - Kinematic wave model + Kalman filter yields estimate of the current state of the network
- Prediction of flow conditions using kinematic wave model for network shows how queues grow and shrink

## Traffic State Estimation & Prediction

### Applications of Kalman filters

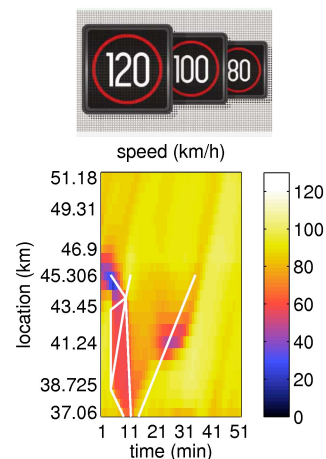
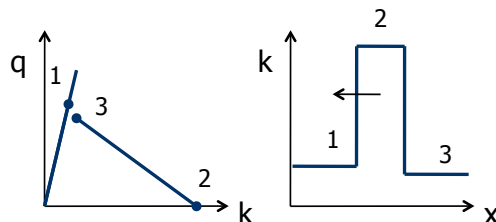
- Traffic information ([www.anwb.nl](http://www.anwb.nl) or from TomTom) provides information about current traffic conditions
- Reaction on 'old' information yields suboptimal network conditions
- Projection: everybody has a TomTom



## Dynamic speed limits

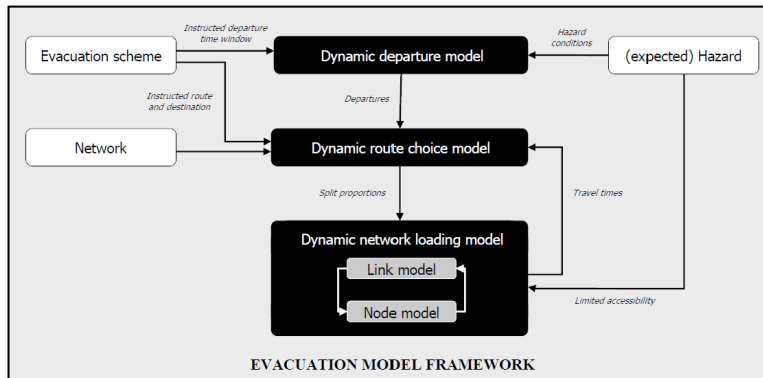
### Using Traffic Flow Theory to improve traffic flow

- Algorithm 'Specialist' to suppress start-stop waves on A12
- Approach is based on reduced flow (capacity drop) downstream of wave
- Reduce inflow sufficiently by speed-limits upstream of wave



# EVAQ model overview

## Model components overview

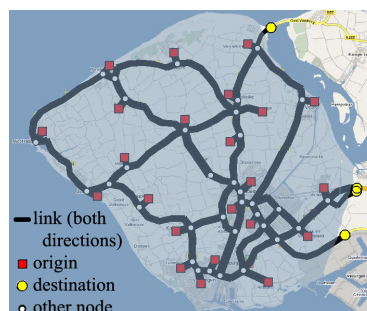


# Case study

## Flooding of Walcheren

- Flood strikes from West to East in six hours
- 120.000 residents need to be evacuated

- Evacuation instructions entail:
  - Departure time
  - Safe haven (or safe destination)
  - Route
 for specific groups of evacuees (e.g. per area code)



## Evacuation Walcheren



## Optimization objectives

### Objective applied in this research

- Maximizing function of the number of arrived evacuees in each time period:

$$J(E) = \sum_t e^{-\beta t} q_E(t)$$

$q_E(t)$  number evacuees arrived in time period  $t$   
 $E$  evacuation scheme

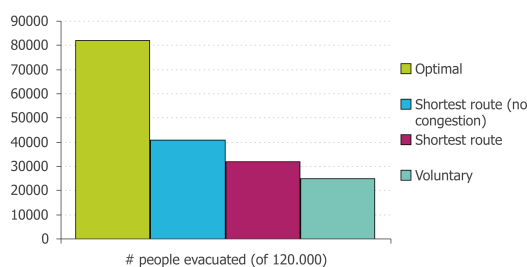
- Evacuate as many people as quickly as possible
- Robust against time at which calamity unfolds
- Use of evacuation simulation model EVAQ to compute  $J(E)$  as function of  $E$



## Example results

### Strategy comparison

- Compared to voluntary evacuation, simple evacuation rules yield significant improvement
- Optimization of evacuation plan yields very significant improvement compared to other scenarios



## Summary

### Main challenges for the future

- Lecture has shown:
  - Some properties of traffic flows and network traffic flow operations
  - Examples of mathematical models of traffic flow
  - Applications of models
- Traffic theory deals with movement of human beings, whose decisions are *by definition* hard to predict
- Furthermore, strong lack of research data has been a problem (most data is macroscopic and on the level of cross-sections)
- Nevertheless, traffic theory has been successful and more and more applications will find their way into practice

## Future challenges in flow theory

### New data, new challenges

- “Meten is Weten” [Kamerling Onnes]:
  - True insight comes from observing phenomena
  - Availability of new data types will strongly enhance understanding



- Our object of knowledge is changing continuously:
  - Environmental, social and political changes will continuously change which aspects of the system needs investigating / improving
  - Demographic / technological innovations change

## Mathematics of Road Congestion

### Empirics, models and applications

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