Tentamen 2IW50

14 november 2005, 9:00 - 12:00

This examination consists of five questions, which are weighted equally heavy. The book, the course notes or other written material may be used during this examination.

1. Consider the following formulas, where p, q are atomic propositions:

   (A) $\mathbf{A}(\mathbf{G}\,\mathbf{F}\,(p \vee q))$

   (B) $p \wedge (\mathbf{AF}\,q) \wedge \neg(\mathbf{E}[p\,\mathbf{U}\,q])$

   (a) For each of the formulas **(A), (B)**:

      i. Is the formula in LTL? Is it in CTL? Is it in ACTL* ?

      ii. Is the formula equivalent to an ACTL* formula? (explain)

   (b) For each of the formulas **(A), (B)**:

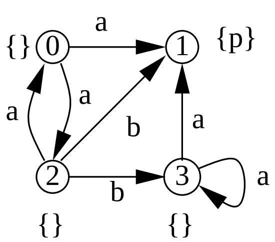
      i. draw a Kripke structure with a single initial state in which it holds

      ii. draw a Kripke structure with one initial state in which it doesn't hold

      (so in total 4 Kripke structures are required)

   (c) **Only** for formula **(B)**: transform it to an equivalent formula in the μ-calculus, assuming a is the only transition label.

2. Consider the following mixed Kripke structure.



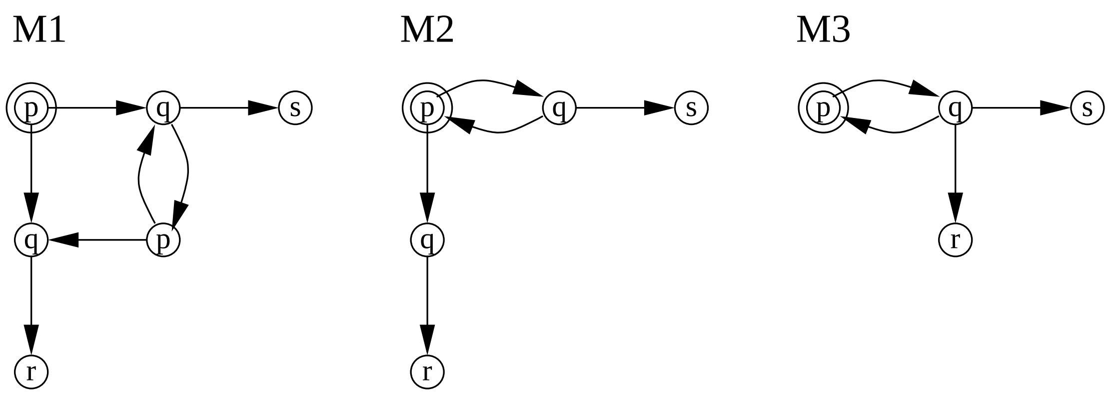
   (a) Use the Emerson-Lei algorithm (Figure 7.2 in the book) to determine in which states the formula

   $$\mu X.\, \mu Y.\, \Big(p \vee [a]X \vee \langle b\rangle Y\Big)$$

   holds (where the modal operators bind stronger than $\vee$). Show the intermediate approximations.

   (b) What is the worst-case time complexity of this algorithm? Explain in words the relationship with the (dependent) alternation depth.

3. Consider the following three Kripke structures, where $\{p, q, r, s\}$ are the atomic propositions. Determine whether the following properties hold. If so, give the concrete (bi)simulation. If not, give a formula in CTL* that witnesses this fact.



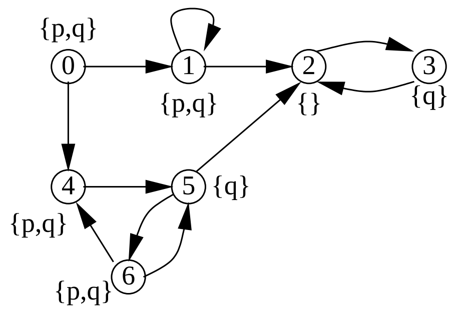
(a) $M_1 \equiv M_2$

(b) $M_2 \equiv M_3$

(c) $M_2 \preceq M_3$

(d) $M_3 \preceq M_2$

4. Consider the following Kripke Structure and temporal formulas:



(C) $\mathbf{EG}\, q$

(D) $\mathbf{E}[p\, \mathbf{U}\, (\mathbf{AF}\, \neg q)]$

In all three computations, show the intermediate stages!

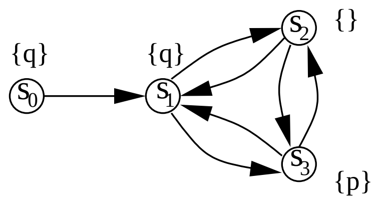
(a) Determine the set of states where **(C)** holds, using the standard CTL model checking algorithm, based on graph algorithms (Chapter 4.1 of the book).

(b) Determine the set of states where **(C)** holds, using symbolic CTL model checking algorithm, based on fixed point characterisations (Chapter 6.2.2 of the book). However, you don't have to represent sets of states by BDDs, but may use explicit set notation.

(c) Similarly, determine the set of states where **(D)** holds, using the symbolic CTL model checking algorithm.

5. We define two Kripke structures with fairness constraints as follows. Both M_1 and M_2 have states, transitions, and labels as in the figure below. But, M_1 has fairness constraints $\mathcal{F}_1$ and M_2 has fairness constraints $\mathcal{F}_2$, where

$$\mathcal{F}_1 := \{\{s_2\}, \{s_3\}\}$$
$$\mathcal{F}_2 := \{\{s_1\}\}$$



Now, which of the following propositions hold according to the semantics of fair CTL*? Explain, and give a witness or counterexample when possible.

(a) $M_1, s_0 \models_f \mathbf{AF}\, p$

(b) $M_2, s_0 \models_f \mathbf{AF}\, p$

(c) $M_1, s_0 \models_f \mathbf{EG}\, [q\, \mathbf{U}\, p]$

(d) $M_2, s_0 \models_f \mathbf{EG}\, [q\, \mathbf{U}\, p]$