Examination 2IW50: *Algorithms for Model Checking*

January 17, 2006, 9:00–12:00

This examination consists of four questions, the weights of which are given in boldface. The book, the course notes or other written material may be used during this examination.

(20) 1. Determine whether the following propositions hold or not. Give an argument.

(a) Every ACTL formula is a formula in LTL.

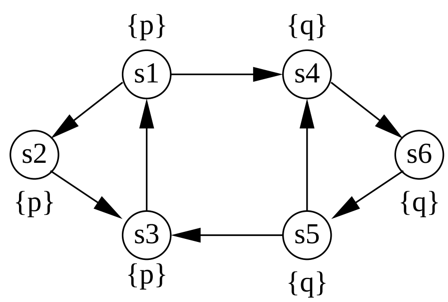
(b) Every LTL formula in positive normal form is a formula in ACTL*.

(c) The state transformer τ defined by $\tau(Z) = \mathbf{AX}\,\neg(p \vee \neg\mathbf{EG}\,Z)$ is monotonic.

(d) If two Kripke Structures simulate each other then they satisfy the same CTL formulas.

(e) The strongly connected components of a directed graph can be determined in polynomial time (in the size of the graph).

(30) 2. Consider the following Kripke Structure $\mathcal{M}$:



(a) Use the fixed point characterization of **AU** to determine the set of states of $\mathcal{M}$ in which the formula $\mathbf{A}[p\,\mathbf{U}\,q]$ holds. Show the intermediate steps.

Consider the following set of fairness constraints:

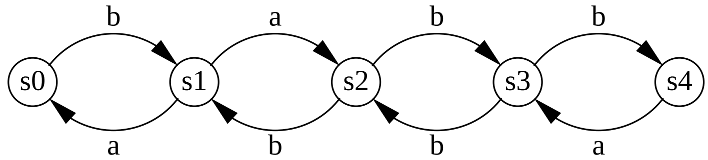
$$\mathcal{F} = \{\{s1, s2, s3\}, \{s4, s5, s6\}\}$$

(b) Give examples of a fair path and an unfair path in $\mathcal{M}$ with respect to $\mathcal{F}$.

(c) Use the semantics of CTL with fairness constraints to determine the set of states of $\mathcal{M}$ where $\mathbf{A}[p\,\mathbf{U}\,q]$ *fairly* holds with respect to $\mathcal{F}$.

(d) Use the standard graph based model checking algorithm for CTL with fairness (Section 4.1.1 in the book) to determine in which states of $\mathcal{M}$ the formula $\mathbf{EG}\,(\mathbf{EX}\,\mathbf{EX}\,q)$ *fairly* holds with respect to $\mathcal{F}$. Show intermediate steps.

(25) 3. Consider the following labeled transition system:



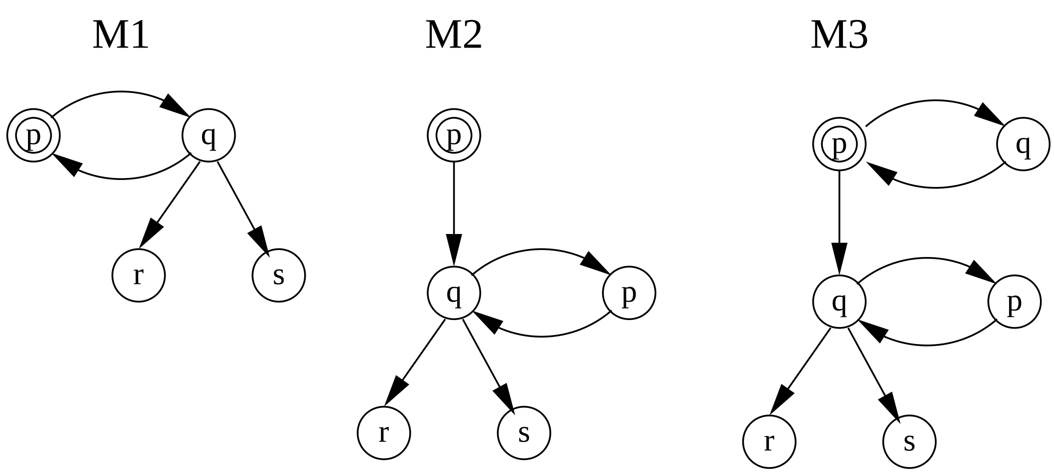
(a) Use the Emerson-Lei algorithm (Figure 7.2 in the book) to determine in which states the following formula holds, and show the intermediate approximations:

$$\nu Y.\left(\langle a\rangle \mu X.\,(Y \vee [b]X)\right)$$

(b) What is the *dependent* alternation depth of this formula? Explain your answer.

(c) Consider the (standard) translation of CTL formulas into the μ-calculus. What is the dependent alternation depth of the resulting formulas?

(25) 4. Consider the following three Kripke structures, where $\{p, q, r, s\}$ is the set of atomic propositions. Determine whether the following properties hold. If so, give the concrete (bi)simulation. If not, give a formula in CTL* that witnesses this fact.



(a) $M_1 \equiv M_2$

(b) $M_2 \equiv M_3$

(c) $M_2 \preceq M_3$

(d) $M_3 \preceq M_2$