Comment on "Spontaneous Branching of Anode-Directed Streamers between Planar Electrodes"

Arrayás, Ebert, and Hundsdorfer [1] (see also [2,3]) describe a 2D numerical simulation of branching of a negative streamer between parallel-plate electrodes. The mechanism of branching is identified as Laplacian instability of the streamer tip. The branching, however, may be caused by numerical instability. Besides, the model [1] does not take into account photoionization, which dramatically changes the physics of streamer propagation. This Comment aims to indicate a possible source of numerical instability of the algorithm [1] and to emphasize the role of photoionization in streamer dynamics.

Unlike classical hydrodynamics, the simulation of a plasma with self-consistent electric field requires that the numerical scheme satisfies a "consistency" condition. The model [1] contains continuity equations for electrons and ions coupled by the Poisson equation for the electric field. From this system it immediately follows that the total current is conserved:

$$\nabla \cdot \left(\frac{\partial \mathbf{E}}{\partial t} + \frac{e}{\varepsilon_0} \mathbf{j} \right) = 0, \tag{1}$$

where $\mathbf{E}$ is electric field and $\mathbf{j}$ is electron flux.

Consistency means that finite-difference equations must conserve total current on a computational grid. Equation (1) shows that such schemes are constructed using *essentially the same finite-difference expressions for divergence* $\nabla \cdot \mathbf{E}$ [4] *and* $\nabla \cdot \mathbf{j}$ in the Poisson and continuity equations, respectively (as in, e.g., [5–7]). This is guaranteed if the densities and potential are given at the nodes, whereas components of the field and the electron flux are calculated at the half-integer points (on a staggered grid). Conversely, the total current is not conserved if different approximations of $\nabla \cdot \mathbf{E}$ and $\nabla \cdot \mathbf{j}$ are used (as in [8,9]).

The third-order upwind-biased scheme used in [1] requires the values of electron flux $\mathbf{j} = -D\nabla\sigma - \sigma\mathbf{E}$ at the nodes (here notations and dimensionless variables are those of [1]). For that reason it seems natural to calculate the field also at the nodes. In that case, however, Eq. (1) on a finite-difference level is *not* fulfilled.

Nonconservation of total current may lead to an unphysical instability of streamer tip. This numerical instability is particularly dangerous in large fields due to an exponential collisional source term in the continuity equations. Therefore, it would be very desirable for the authors of [1] to publish these details of their numerical model.

The standard theory of the Laplacian instability (see, e.g., [10]) tells that a child filament has no characteristic radius. However, a characteristic scale may arise due to photoionization, which is not accounted for in [1]. Photoionization creates precursor electrons ahead of the tip and dramatically changes the physics of streamer propagation [8]: the streamer moves faster than the local drift velocity of electrons and Firsov's theory is not valid. 2D simulation of a positive streamer in air with photoionization shows that in a strong uniform field the streamer head rapidly expands in the radial direction and retains its form [11], whereas in a nonuniform field it exhibits 2D "branching" [12]. The characteristic radius of a child filament appears to be proportional to the absorption length of the photoionizing radiation.

The crucial test for verification of the branching mechanism would, therefore, be a fully 3D simulation of the streamer using a consistent numerical scheme and taking into account the photoionization source of the charged particles.

A. A. Kulikovsky*,†
Institute for Materials and Processes
in Energy Systems (IWV-3)
Research Center "Jülich"
D-52425 Jülich, Germany

Received 22 July 2002; published 12 November 2002
DOI: 10.1103/PhysRevLett.89.229401
PACS numbers: 52.80.Mg, 05.45.–a, 47.54.+r, 51.50.+v

*Electronic address: akul@srcc.msu.su
†On leave from Moscow State University Research Computing Center, 119902 Moscow, Russia.

[1] M. Arrayás, U. Ebert, and W. Hundsdorfer, Phys. Rev. Lett. **88**, 174502 (2002); Phys. Rev. Focus **9**, 19 (16 April 2002), http://focus.aps.org/
[2] A. Rocco, U. Ebert, and W. Hundsdorfer, Phys. Rev. E **66**, 035102 (2002).
[3] P. Ball, Nature (London) (http://www.nature.com/nsu/020408/020408-4.html).
[4] A five-point equation for potential V then is obtained using $\mathbf{E} = -\nabla V$.
[5] S. Dhali and P. F. Williams, J. Appl. Phys. **62**, 4696 (1987).
[6] P. A. Vitello, B. M. Penetrante, and J. N. Bardsley, Phys. Rev. E **49**, 5574 (1994).
[7] A. A. Kulikovsky, Phys. Rev. E **57**, 7066 (1998).
[8] C. Wu and E. E. Kunhardt, Phys. Rev. A **37**, 4396 (1988).
[9] M. C. Wang and E. E. Kunhardt, Phys. Rev. A **42**, 2366 (1990).
[10] J. Kertesz, in *Random Fluctuations and Pattern Growth: Experiments and Models*, NATO ASI, Ser. E, edited by H. E. Stanley and N. Ostrowsky (Kluwer, Dordrecht, 1988), Vol. 157, pp. 42–55.
[11] A. A. Kulikovsky, J. Phys. D **30**, 441 (1997).
[12] A. A. Kulikovsky, J. Phys. D **33**, 1514 (2000).

 0031-9007/02/89(22)/229401(1)$20.00