A mechanism for streamer branching

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We show by a combination of simulations and analytical insight how a freely propagating streamer in a strong homogeneous field can branch. We analyze the minimal continuum model for a negative streamer in a non-ionized and non-attaching gas with impact ionization reaction in local field approximation. Rather than saturating to a mode of stationary propagation, the streamer reaches a state that is intrinsically unstable. Therefore it branches. We explain the nature of the instability.

1. The minimal streamer model

We investigate the minimal streamer model, i.e., a "fluid approximation" with local field-dependent impact ionization reaction in Townsend approximation [1] in a non-attaching and non-ionized gas. In dimensionless units [2–4], the model has the form:

$$\partial_t \sigma - \nabla \cdot (\sigma \mathbf{E} + D \nabla\sigma) = \sigma f(|\mathbf{E}|) , \quad (1)$$

$$\partial_t \rho = \sigma f(|\mathbf{E}|) , \quad (2)$$

$$\rho - \sigma = \nabla \cdot \mathbf{E} \ , \ \mathbf{E} = -\nabla\Phi , \quad (3)$$

$$f(|\mathbf{E}|) = |\mathbf{E}| \, \alpha(|\mathbf{E}|) \left(= |\mathbf{E}| \, e^{-1/|\mathbf{E}|} \text{ in simulation} \right) . \quad (4)$$

Here σ and ρ are the densities of electrons and positive ions, $\mathbf{E}$ is the electric field, Φ the electric potential, D the dimensionless diffusion constant, and $\alpha(|\mathbf{E}|)$ the effective field-dependent cross-section of impact ionization. The mobility of ions is neglected. The same model was investigated by Dhali and Williams [5] and Vitello, Bardsley and Penetrante [6]. The translation to physical units depends on the type and density of the gas. For nitrogen under normal conditions with effective parameters as in [5,6], the unit of time is $\approx$ 3 ps, the unit of length is ≈ 2.3 μm, the unit of field is ≈ 200 kV/cm, and the unit of density is $4.7 \cdot 10^{14}/\text{cm}^3$ [2]. We used $D = 0.1$ which is appropriate for nitrogen.

2. Streamer branching

We recently have found [4] (and our results were reported in Nature [7]) that in contrast to previous expectations [8,9], a streamer propagating in a homogeneous field can branch spontaneously rather than approach a mode of stationary propagation. We observed this in a system identical to that of previous simulations [5,6] with an electrode distance of 5 mm, but with a voltage difference twice as high, namely 50 kV rather than 25 kV. (The question whether branching also would occur in the lower field, but after a longer propagation distance, is open.) To our knowledge, [4] contains the first prediction that a streamer branches in free flight in a homogeneous field within an explicit discharge model like (1) – (4).

For an analytical-physical understanding of the branching process, first the propagation mode of the streamer has to be understood. Simulations of streamer discharges [5,6] have revealed that the space charge is concentrated within a thin layer around its head rather than being smeared out over the full head of the streamer as assumed previously [10,1]. This leads to a different field distribution and a much faster propagation mode, and to a revival of the concept of an ideally conducting streamer formulated by Lozansky and Firsov [11].

Our results on streamer branching can be understood within this framework [4]: After a sequence of transients, the streamer can approach the state of ideal conductivity where the interior is essentially free of field, and the space charge is concentrated in a very thin layer. This state is intrinsically unstable, and therefore leads to spontaneous branching. Mathematically equivalent instabilities, so-called Laplacian instabilities, are observed in viscous fingering, dendritic solidification or certain growth models for bacterial colonies or tumors.

3. New simulation results

Our demonstration of branching in [4] was based on a combination of numerical and analytical results. The simulations revealed that the streamer approached the unstable state, and for the development of the instabilities we relied largely on analytical insight.

The numerical results in [4] as a support for the above statements can be critized for four reasons: (*i*) the initial seed is quite wide, so it is not clear whether one sees an initial transient due to a wide initial condition or the branching of a streamer in free flight. (*ii*) As in earlier simulations, the boundary condition on the electrode is homogeneous Neumann for the densities. This implies that electrons freely travel from the metal of the electrode into the gas. (*iii*) Some wiggly lines in the plots invoked the question whether numerical instabilities dominated the figure [12]. (*iv*) The simulations use cylindrical geometry to allow for effectively 2-dimensional codes.

(*iv*) We keep relying on the analytical argument that the constraint of cylindrical geometry suppresses some instability modes, and that therefore a truely 3-dimensional system would become unstable earlier or at the same time as the system with symmetry constraint, but certainly not later. We have improved all other features. (*iii*) In the new plot in Fig. 1, the spatial resolution is twice as high. The results are much smoother, but nevertheless the branching instability occurs. (*ii*) The boundary

condition on the electrode is changed to homogeneous Dirichlet for the densities, i.e., electrons from the metal cannot enter the gas. (*i*) The initial condition now corresponds to a single electron on the cathode. All other features are unchanged.

The simulation shows a streamer not connected to the electrode. It has the conical shape of a streamer created by a single electron in a homogeneous field as first found in Raether's experiments [10]. The streamer is more narrow than in [4], but becomes unstable and branches after an even shorter travel distance.

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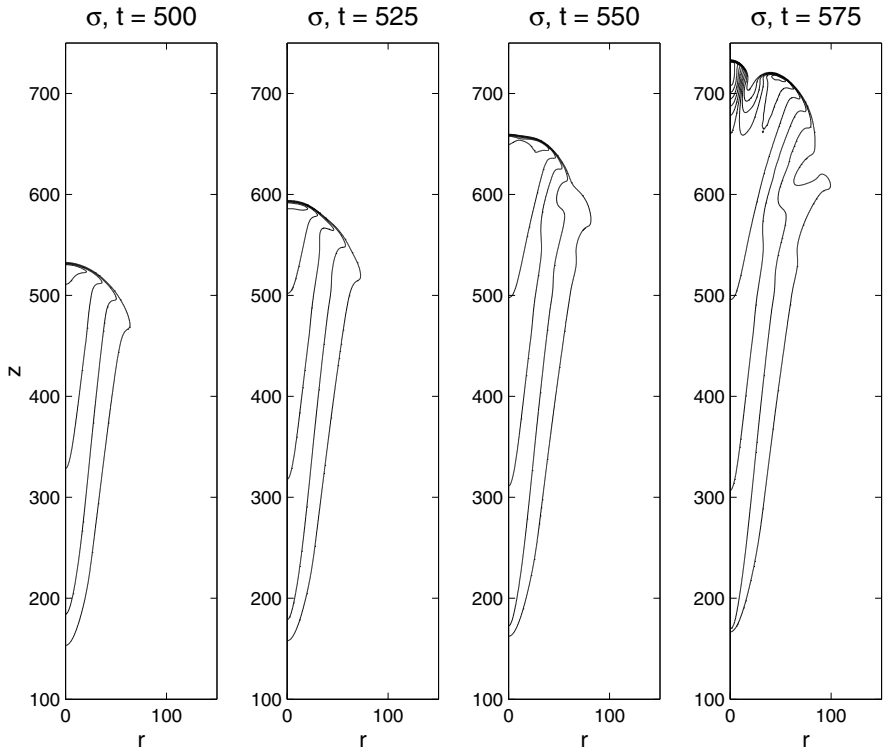

FIG. 1. Evolution of an anode directed streamer in a strong homogeneous background field. The planar cathode is located at $z = 0$ and the planar anode at $z = 2000$. The radial coordinate extends from the origin up to $r = 2000$ to assure homogeneous field conditions. The thin lines denote levels of equal electron density σ with increments of 0.15.

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[12] It should be noted, that the wiggly structures in Fig. 1 of [4] in part are due to the calculation of the contour plots by the graphics program, and not to the underlying numerical data.