

Data Cube Technology

*Erwin M. Bakker & **Stefan Manegold***

<https://homepages.cwi.nl/~manegold/DBDM/>

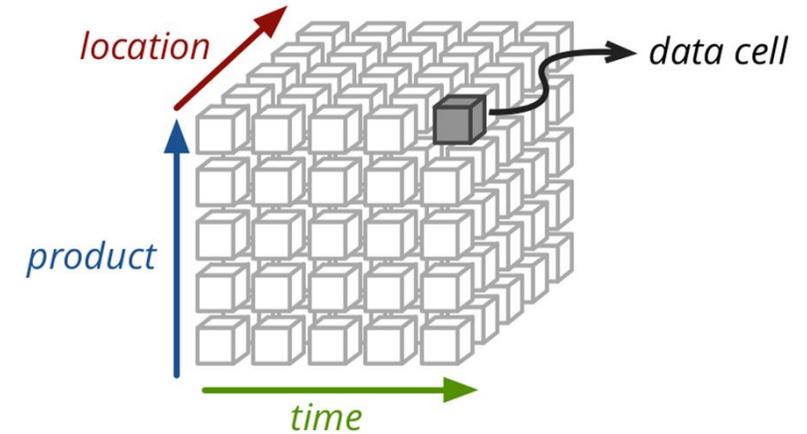
<http://liacs.leidenuniv.nl/~bakkerem2/dbdm/>

s.manegold@liacs.leidenuniv.nl

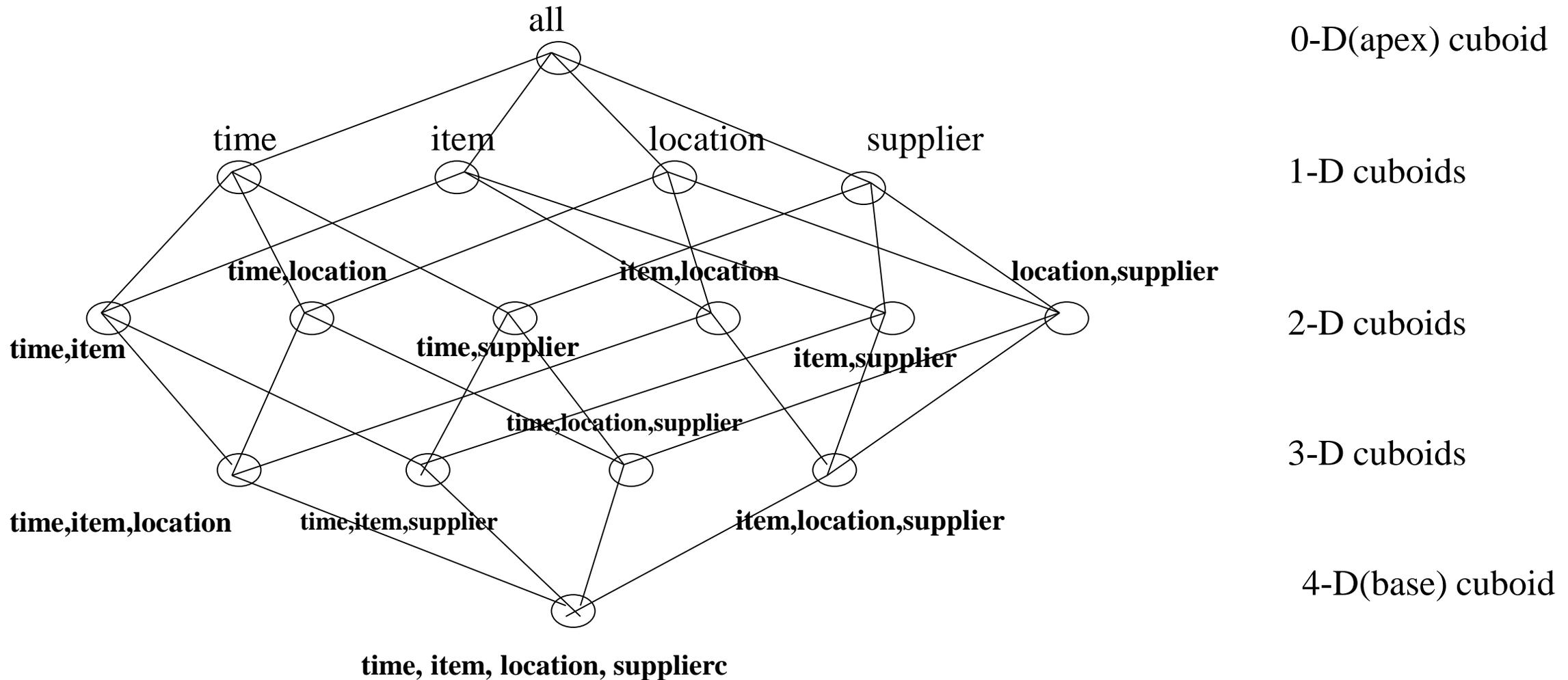
e.m.bakker@liacs.leidenuniv.nl

Chapter 5: Data Cube Technology

- ❑ Data Cube Computation: Basic Concepts
- ❑ Data Cube Computation Methods
- ❑ Processing Advanced Queries with Data Cube Technology
- ❑ Multidimensional Data Analysis in Cube Space
- ❑ Summary



Data Cube: A Lattice of Cuboids



0-D(apex) cuboid

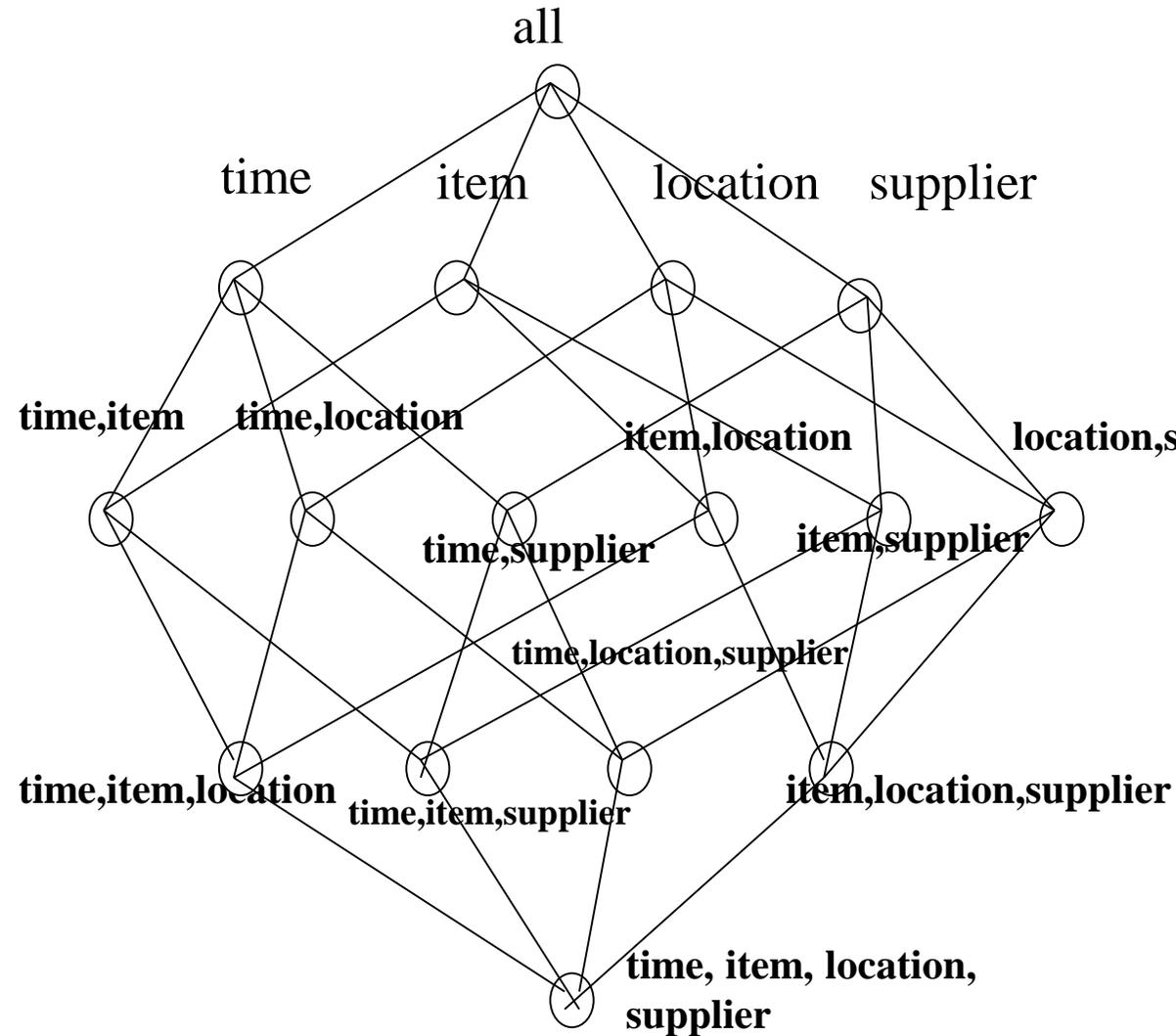
1-D cuboids

2-D cuboids

3-D cuboids

4-D(base) cuboid

Data Cube: A Lattice of Cuboids



0-D(apex) cuboid

- Base vs. aggregate cells
- Ancestor vs. descendant cells

1-D cuboids

- Parent vs. child cells

2-D cuboids

- (*, *, *)
- (*, milk, *, *)
- (*, milk, Urbana, *)

3-D cuboids

- (*, milk, Chicago, *)
- (9/15, milk, Urbana, *)

4-D(base) cuboid

- (9/15, milk, Urbana, Dairy_land)

Cube Materialization: Full Cube vs. Iceberg Cube

- ❑ Full cube vs. iceberg cube

compute cube sales iceberg as

select month, city, customer group, count(*)

from salesInfo

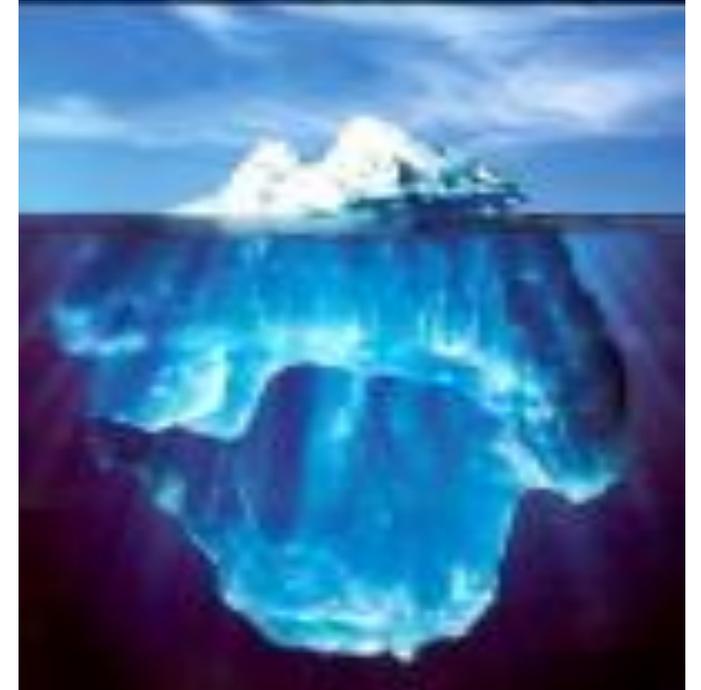
cube by month, city, customer group

having count(*) >= min support



**iceberg
condition**

- ❑ Compute *only* the cells whose measure satisfies the iceberg condition
- ❑ Only a small portion of cells may be “above the water” in a sparse cube
- ❑ Ex.: Show only those cells whose count is no less than 100

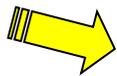


Why Iceberg Cube?

- ❑ Advantages of computing iceberg cubes
 - ❑ No need to save nor show those cells whose value is below the threshold (iceberg condition)
 - ❑ Efficient methods may even avoid computing the un-needed, intermediate cells
 - ❑ Avoid explosive growth
- ❑ Example: A cube with 100 dimensions
 - ❑ Suppose it contains only 2 base cells: $\{(a_1, a_2, a_3, \dots, a_{100}), (a_1, a_2, b_3, \dots, b_{100})\}$
 - ❑ How many aggregate cells if “having count ≥ 1 ”?
 - ❑ Answer: $2^{101} - 6$ (Why?!)
 - ❑ What about the iceberg cells, (i.e., with condition: “having count ≥ 2 ”)?
 - ❑ Answer: 4 (Why?!)

Is Iceberg Cube Good Enough? Closed Cube & Cube Shell

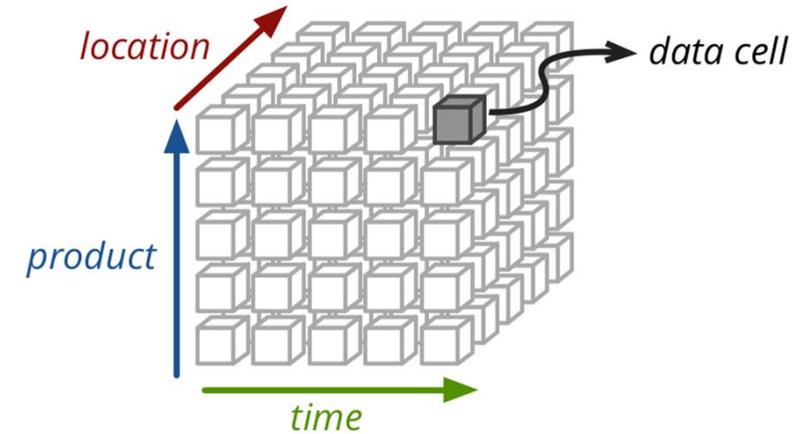
- ❑ Let cube P have only 2 base cells: $\{(a_1, a_2, a_3 \dots, a_{100}):10, (a_1, a_2, b_3, \dots, b_{100}):10\}$
 - ❑ How many cells will the iceberg cube contain if “having count(*) ≥ 10 ”?
 - ❑ Answer: $2^{101} - 4$ (still too big!)
- ❑ **Close cube:**
 - ❑ A cell c is **closed** if there exists no cell d , such that d is a descendant of c , and d has the same measure value as c
Ex. P has only 3 closed cells: $\{(\overline{a_1}, \overline{a_2}): 20, (a_1, a_2, a_3 \dots, a_{100}):10, (a_1, a_2, b_3, \dots, b_{100}):10\}$
(a1,a2,,...,*)*
 - ❑ A **closed cube** is a cube consisting of only closed cells
- ❑ **Cube Shell**
 - ❑ **Cube Shell:** The cuboids involving a small # of dimensions, e.g., 3
 - ❑ Idea: Only compute cube shells, other dimension combinations can be computed on the fly



Q: For $(A_1, A_2, \dots, A_{10})$, how many combinations to compute?

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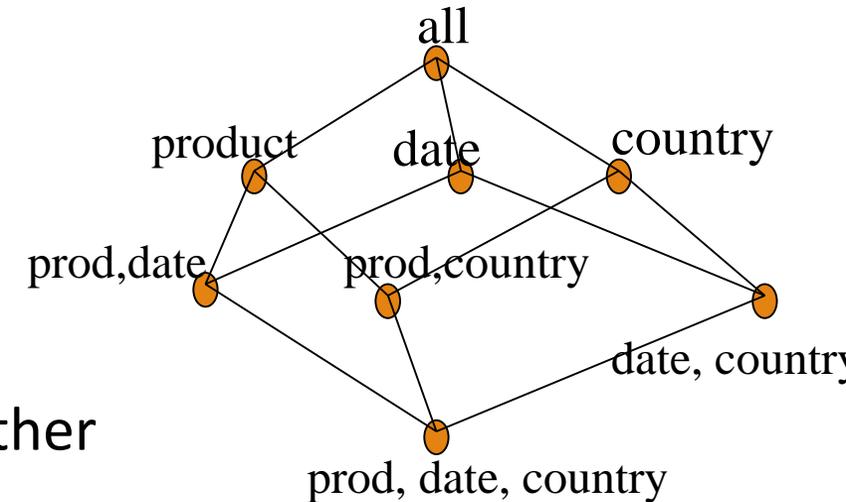
Roadmap for Efficient Computation

- ❑ General computation heuristics (Agarwal et al.'96)
- ❑ Computing full/iceberg cubes: 3 methodologies
 - ❑ Bottom-Up: **Multi-Way** array aggregation (Zhao, Deshpande & Naughton, SIGMOD'97)
 - ❑ Top-down:
 - ❑ BUC (Beyer & Ramarkrishnan, SIGMOD'99)
 - ❑ Integrating Top-Down and Bottom-Up:
 - ❑ Star-cubing algorithm (Xin, Han, Li & Wah: VLDB'03)
- ❑ High-dimensional OLAP:
 - ❑ A Shell-Fragment Approach (Li, et al. VLDB'04)
- ❑ Computing alternative kinds of cubes:
 - ❑ Partial cube, closed cube, approximate cube,



Efficient Data Cube Computation: General Heuristics

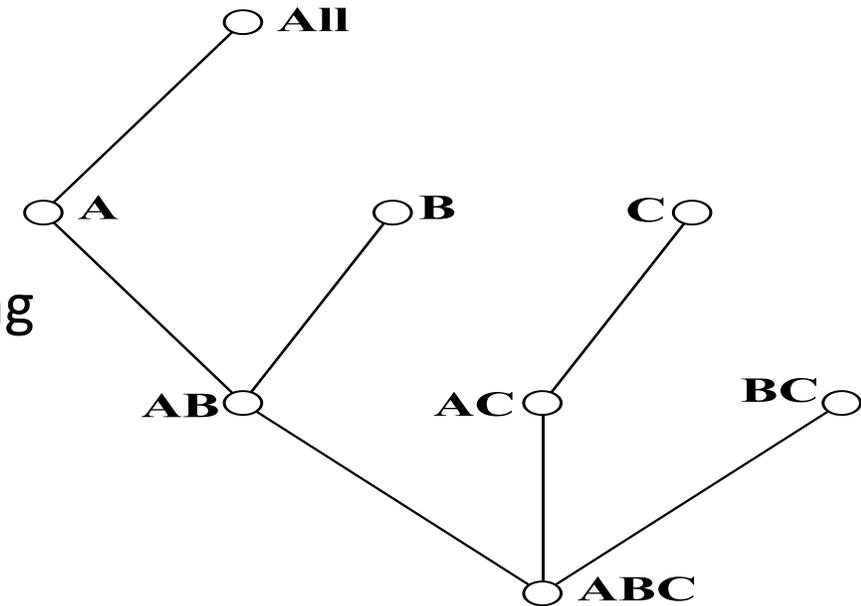
- Sorting, hashing, and grouping operations are applied to the dimension attributes in order to reorder and cluster related tuples
- Aggregates may be computed from previously computed aggregates, rather than from the base fact table
 - **Smallest-child:** computing a cuboid from the smallest, previously computed cuboid
 - **Cache-results:** caching results of a cuboid from which other cuboids are computed to reduce disk I/Os
 - **Amortize-scans:** computing as many as possible cuboids at the same time to amortize disk reads
 - **Share-sorts:** sharing sorting costs cross multiple cuboids when sort-based method is used
 - **Share-partitions:** sharing the partitioning cost across multiple cuboids when hash-based algorithms are used



S. Agarwal, R. Agrawal, P. M. Deshpande, A. Gupta, J. F. Naughton, R. Ramakrishnan, S. Sarawagi. On the computation of multidimensional aggregates. VLDB'96

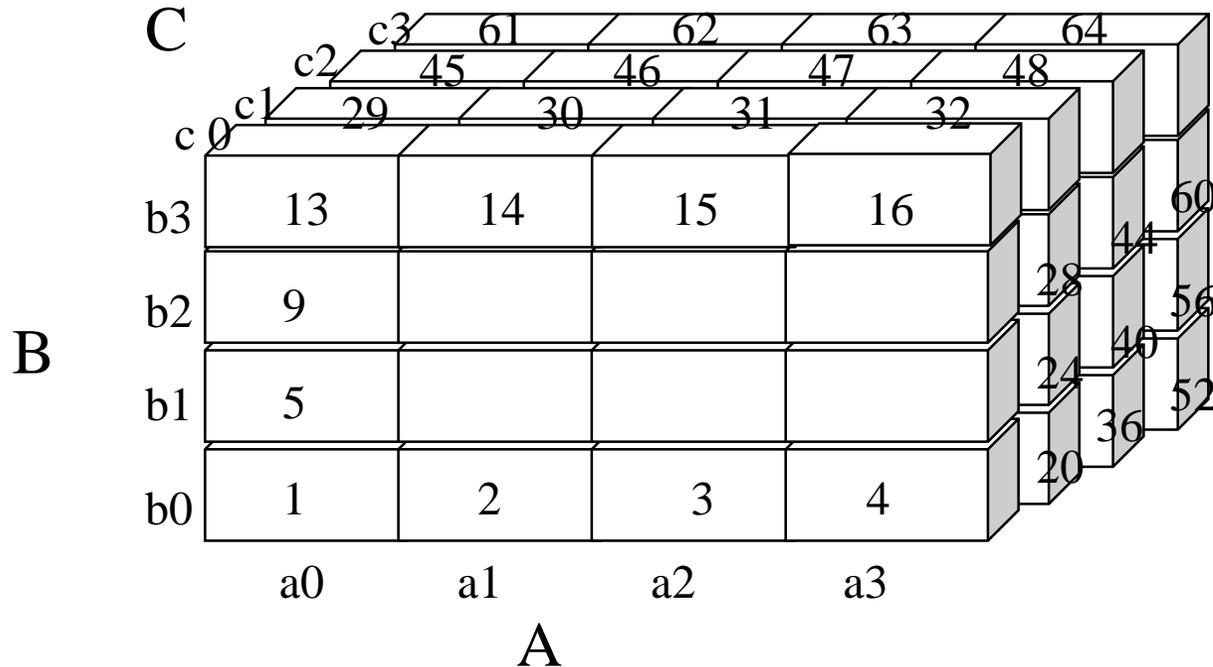
Multi-Way Array Aggregation

- ❑ Array-based “bottom-up” algorithm (from ABC to AB,...)
- ❑ Using multi-dimensional chunks
- ❑ Simultaneous aggregation on multiple dimensions
- ❑ Intermediate aggregate values are re-used for computing ancestor cuboids
- ❑ Cannot do *Apriori* pruning: No iceberg optimization
- ❑ Comments on the method
 - ❑ Efficient for computing the full cube for a small number of dimensions
 - ❑ If there are a large number of dimensions, “top-down” computation and iceberg cube computation methods (e.g., BUC) should be used



Cube Computation: Multi-Way Array Aggregation (MOLAP)

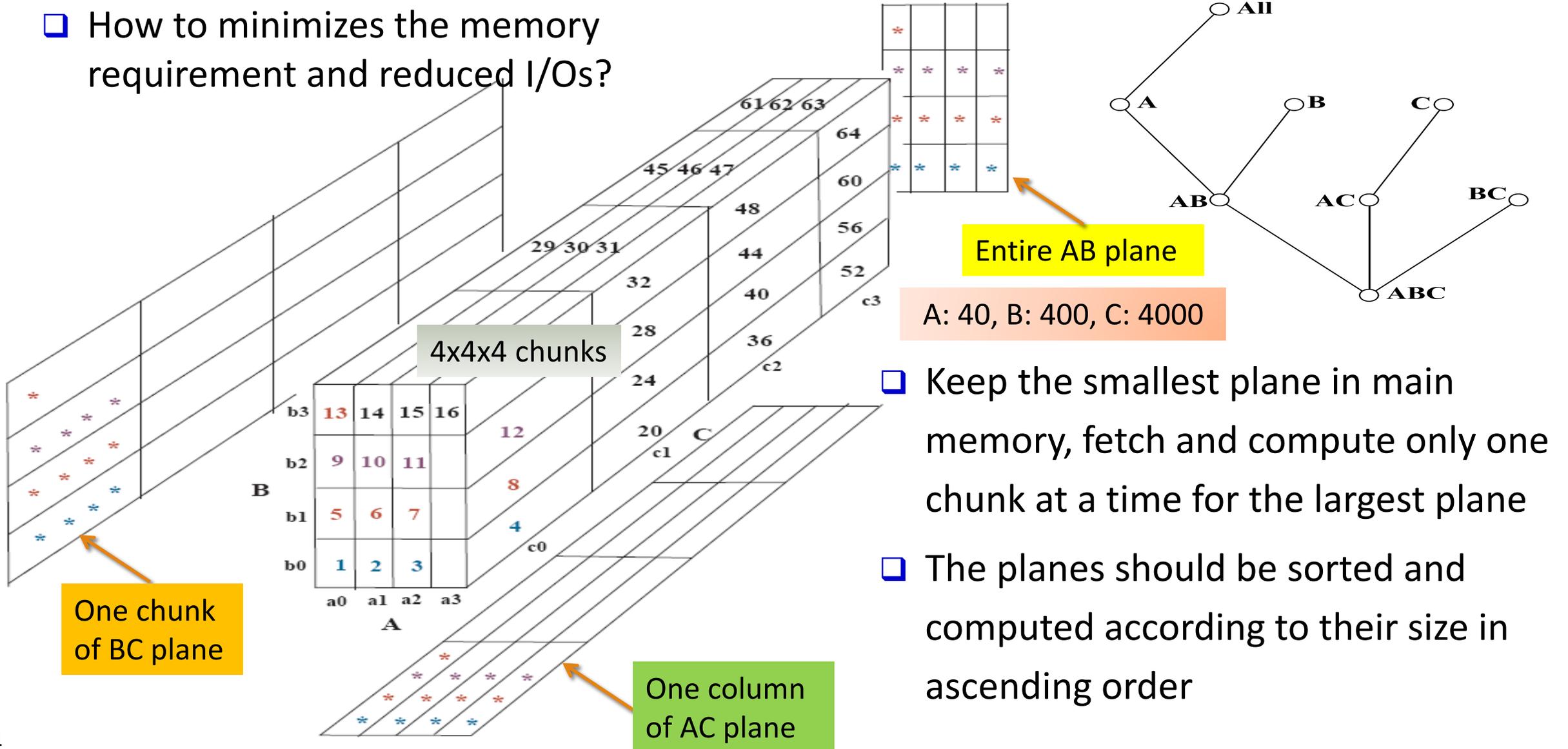
- ❑ Partition arrays into *chunks* (a small subcube which fits in memory).
- ❑ Compressed *sparse array addressing*: (chunk_id, offset)
- ❑ Compute aggregates in “multiway” by visiting cube cells in the order which minimizes the # of times to visit each cell, and reduces memory access and storage cost



What is the best traversing order to do multi-way aggregation?

Multi-way Array Aggregation (3-D to 2-D)

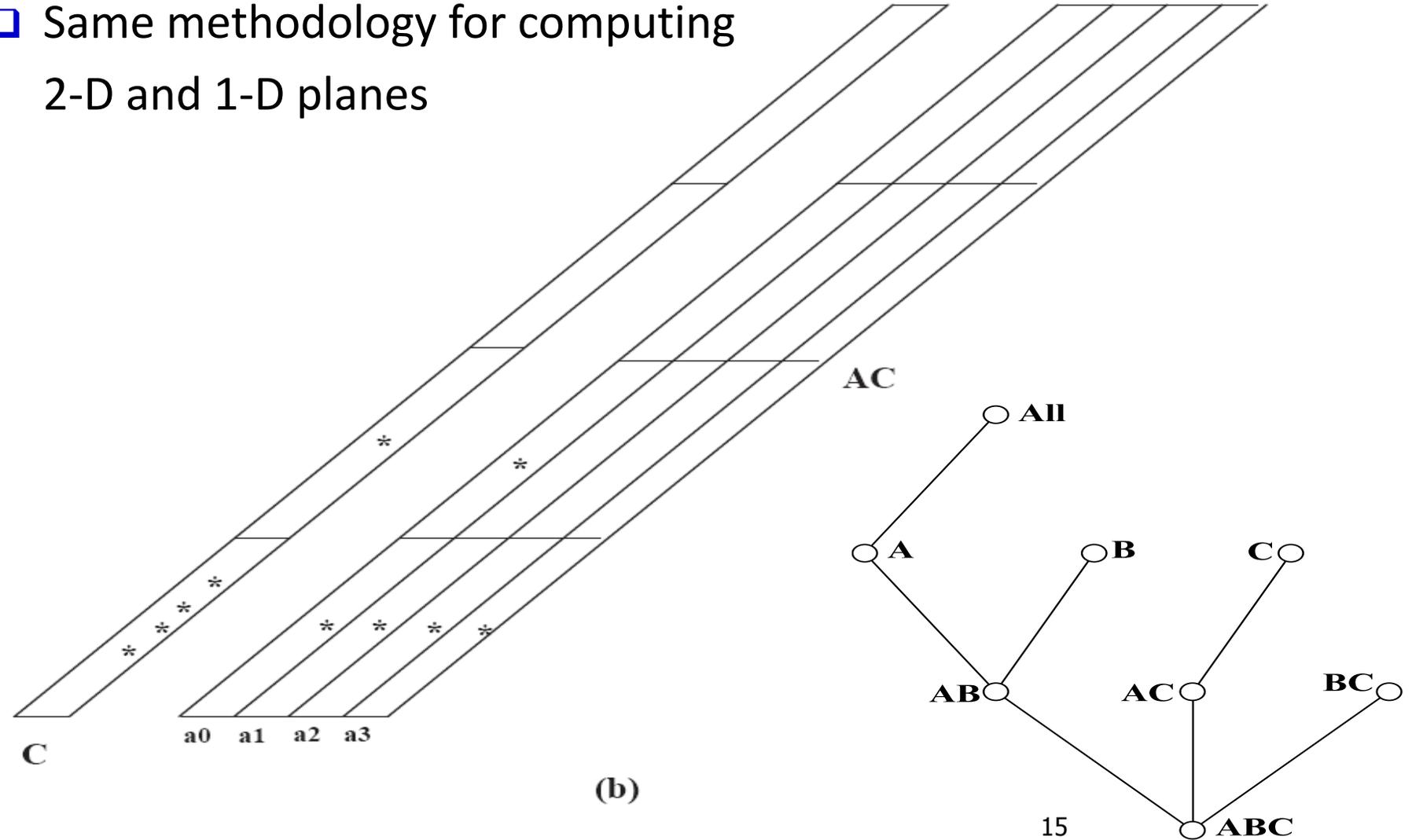
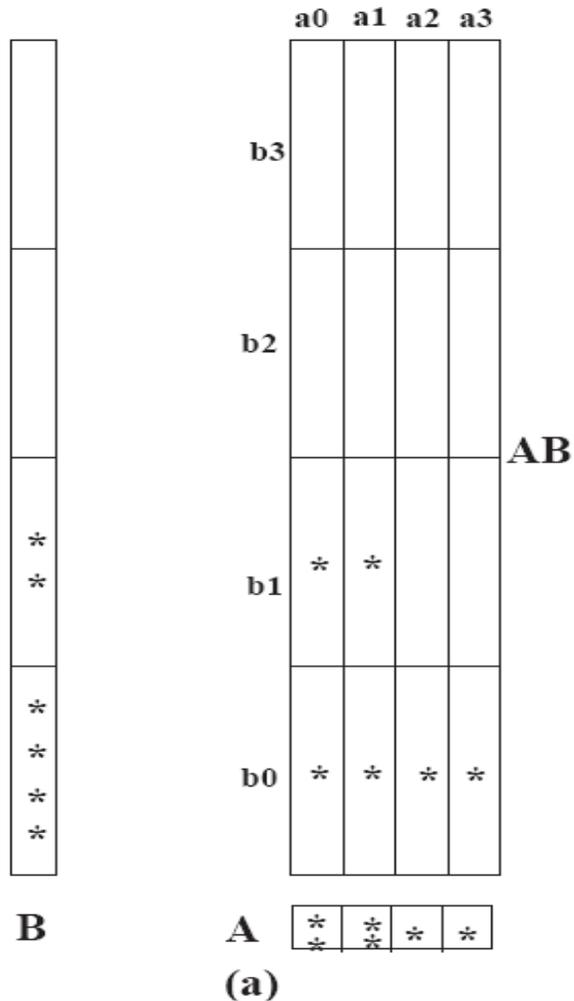
- How to minimize the memory requirement and reduced I/Os?



- Keep the smallest plane in main memory, fetch and compute only one chunk at a time for the largest plane
- The planes should be sorted and computed according to their size in ascending order

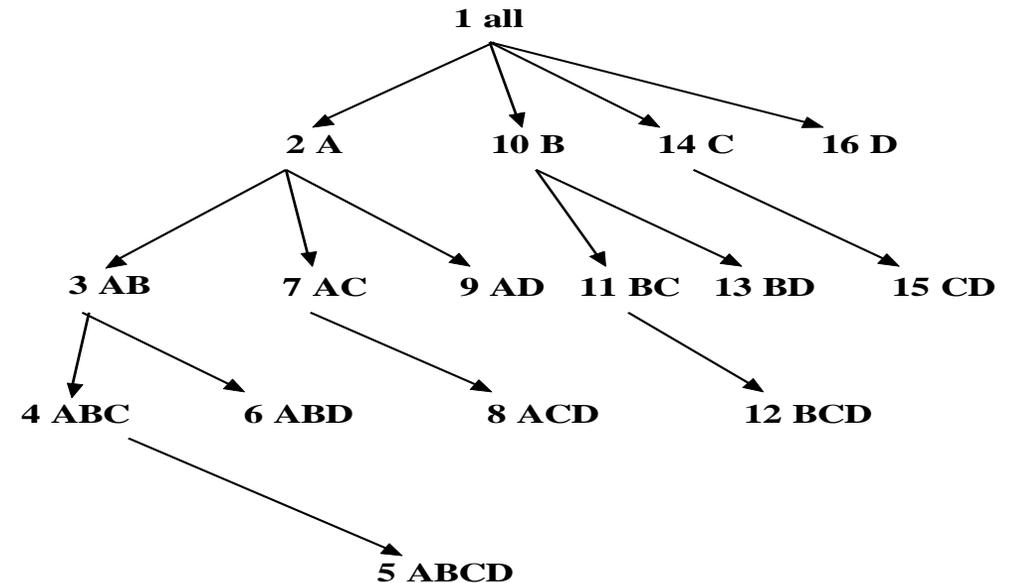
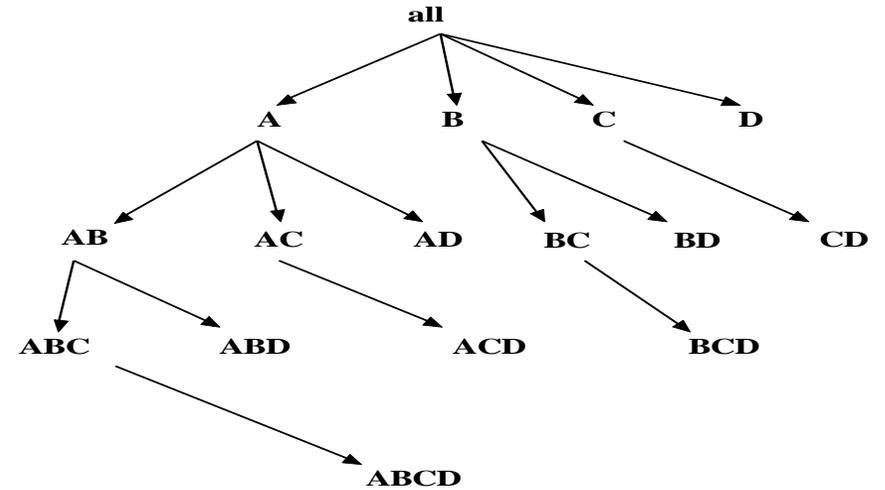
Multi-Way Array Aggregation (2-D to 1-D)

- Same methodology for computing 2-D and 1-D planes



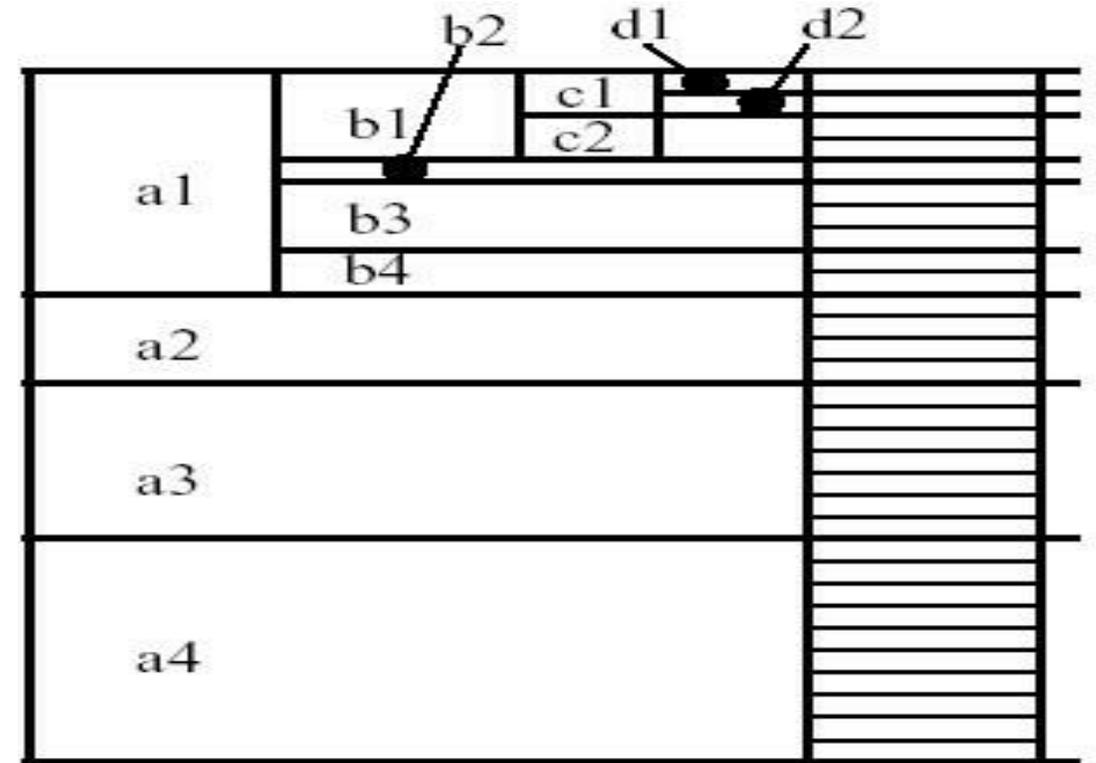
Cube Computation: Computing in Reverse Order

- ❑ BUC (Beyer & Ramakrishnan, SIGMOD'99)
BUC: acronym of Bottom-Up (cube) Computation
(Note: It is “top-down” in our view since we put Apex cuboid on the top!)
- ❑ Divides dimensions into partitions and facilitates iceberg pruning
 - ❑ If a partition does not satisfy min_sup , its descendants can be pruned
 - ❑ If $minsup = 1$ \Rightarrow compute full CUBE!
- ❑ No simultaneous aggregation



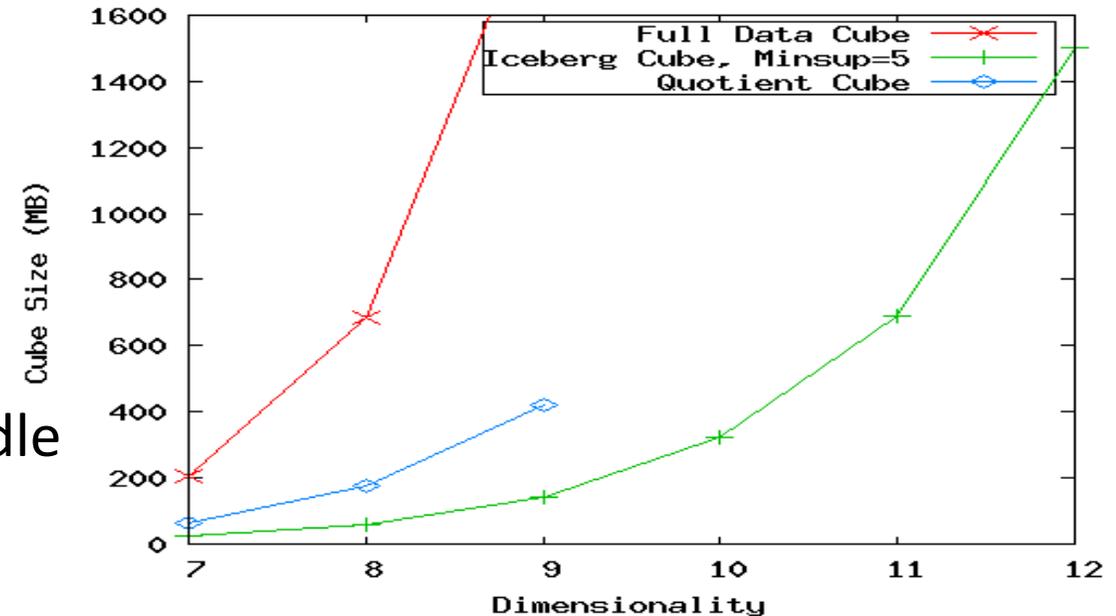
BUC: Partitioning and Aggregating

- ❑ Usually, entire data set cannot fit in main memory
- ❑ Sort *distinct* values
 - ❑ partition into blocks that fit
- ❑ Continue processing
- ❑ Optimizations
 - ❑ Partitioning
 - ❑ External Sorting, Hashing, Counting Sort
 - ❑ Ordering dimensions to encourage pruning
 - ❑ Cardinality, Skew, Correlation
 - ❑ Collapsing duplicates
 - ❑ Cannot do holistic aggregates anymore!



High-Dimensional OLAP?—The Curse of Dimensionality

- ❑ High-D OLAP: Needed in many applications
 - ❑ Science and engineering analysis
 - ❑ Bio-data analysis: thousands of genes
 - ❑ Statistical surveys: hundreds of variables
- ❑ None of the previous cubing method can handle high dimensionality!
 - ❑ Iceberg cube and compressed cubes: only delay the inevitable explosion
 - ❑ Full materialization: still significant overhead in accessing results on disk
- ❑ A shell-fragment approach: X. Li, J. Han, and H. Gonzalez, High-Dimensional OLAP: A Minimal Cubing Approach, VLDB'04



A curse of dimensionality: A database of 600k tuples. Each dimension has cardinality of 100 and *zipf* of 2.

Fast High-D OLAP with Minimal Cubing

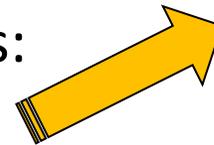
- ❑ Observation: OLAP occurs only on a small subset of dimensions at a time
- ❑ Semi-Online Computational Model
 - ❑ Partition the set of dimensions into **shell fragments**
 - ❑ Compute data cubes for each shell fragment while retaining **inverted indices** or **value-list indices**
 - ❑ Given the pre-computed **fragment cubes**, dynamically compute cube cells of the high-dimensional data cube *online*
- ❑ Major idea: Tradeoff between the amount of pre-computation and the speed of online computation
 - ❑ Reducing computing high-dimensional cube into precomputing a set of lower dimensional cubes
 - ❑ Online re-construction of original high-dimensional space
 - ❑ Lossless reduction

Computing a 5-D Cube with 2-Shell Fragments

- Example: Let the cube aggregation function be **count**

| <i>tid</i> | A | B | C | D | E |
|------------|----|----|----|----|----|
| 1 | a1 | b1 | c1 | d1 | e1 |
| 2 | a1 | b2 | c1 | d2 | e1 |
| 3 | a1 | b2 | c1 | d1 | e2 |
| 4 | a2 | b1 | c1 | d1 | e2 |
| 5 | a2 | b1 | c1 | d1 | e3 |

- Divide the 5-D table into 2 shell fragments:
 - (A, B, C) and (D, E)
- Build traditional invert index or RID list



| Attribute Value | TID List | List Size |
|-----------------|-----------|-----------|
| a1 | 1 2 3 | 3 |
| a2 | 4 5 | 2 |
| b1 | 1 4 5 | 3 |
| b2 | 2 3 | 2 |
| c1 | 1 2 3 4 5 | 5 |
| d1 | 1 3 4 5 | 4 |
| d2 | 2 | 1 |
| e1 | 1 2 | 2 |
| e2 | 3 4 | 2 |
| e3 | 5 | 1 |

Shell Fragment Cubes: Ideas

- ❑ Generalize the 1-D inverted indices to multi-dimensional ones in the data cube sense
- ❑ Compute all cuboids for data cubes ABC and DE while retaining the inverted indices
 - ❑ Ex. shell fragment cube ABC contains 7 cuboids:
 - ❑ A, B, C; AB, AC, BC; ABC
- ❑ This completes the offline computation

| Attribute Value | TID List | List Size |
|-----------------|-----------|-----------|
| a1 | 1 2 3 | 3 |
| a2 | 4 5 | 2 |
| b1 | 1 4 5 | 3 |
| b2 | 2 3 | 2 |
| c1 | 1 2 3 4 5 | 5 |
| d1 | 1 3 4 5 | 4 |
| d2 | 2 | 1 |
| e1 | 1 2 | 2 |
| e2 | 3 4 | 2 |
| e3 | 5 | 1 |

Shell-fragment AB



- ❑ ID_Measure Table
 - ❑ If measures other than count are present, store in *ID_measure* table separate from the shell fragments

| tid | count | sum |
|-----|-------|-----|
| 1 | 5 | 70 |
| 2 | 3 | 10 |
| 3 | 8 | 20 |
| 4 | 5 | 40 |
| 5 | 2 | 30 |

| Cell | Intersection | TID List | List Size |
|-------|--------------------|----------|-----------|
| a1 b1 | 1 2 3 \cap 1 4 5 | 1 | 1 |
| a1 b2 | 1 2 3 \cap 2 3 | 2 3 | 2 |
| a2 b1 | 4 5 \cap 1 4 5 | 4 5 | 2 |
| a2 b2 | 4 5 \cap 2 3 | ϕ | 0 |

Shell Fragment Cubes: Size and Design

- Given a database of T tuples, D dimensions, and F shell fragment size, the fragment cubes' space requirement is:

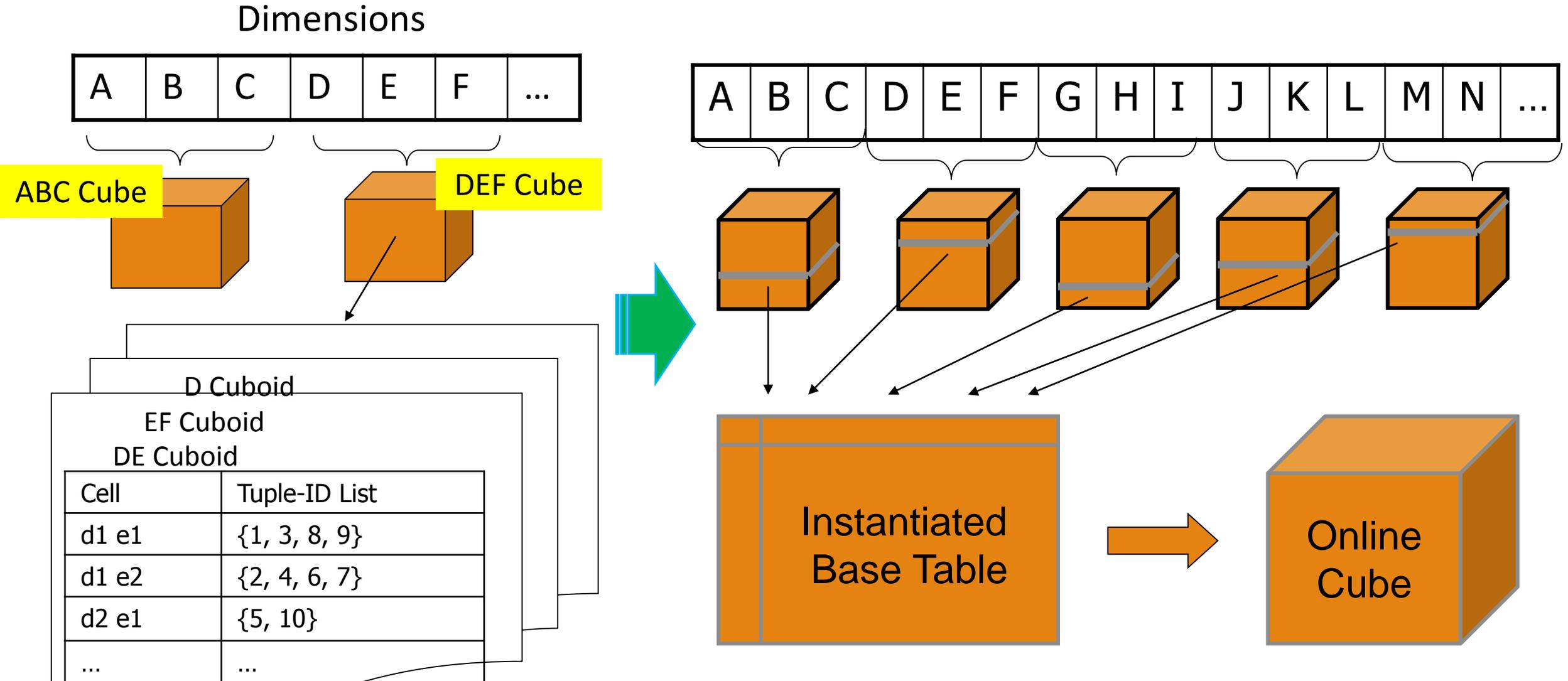
$$O\left(T \left\lceil \frac{D}{F} \right\rceil (2^F - 1)\right)$$

- For $F < 5$, the growth is sub-linear
- Shell fragments do not have to be disjoint
- Fragment groupings can be arbitrary to allow for maximum online performance
- Known common combinations (e.g., <city, state>) should be grouped together
- Shell fragment sizes can be adjusted for optimal balance between offline and online computation

| Attribute Value | TID List | List Size |
|-----------------|-----------|-----------|
| a1 | 1 2 3 | 3 |
| a2 | 4 5 | 2 |
| b1 | 1 4 5 | 3 |
| b2 | 2 3 | 2 |
| c1 | 1 2 3 4 5 | 5 |
| d1 | 1 3 4 5 | 4 |
| d2 | 2 | 1 |
| e1 | 1 2 | 2 |
| e2 | 3 4 | 2 |
| e3 | 5 | 1 |

| Cell | Intersection | TID List | List Size |
|-------|--------------------|----------|-----------|
| a1 b1 | 1 2 3 \cap 1 4 5 | 1 | 1 |
| a1 b2 | 1 2 3 \cap 2 3 | 2 3 | 2 |
| a2 b1 | 4 5 \cap 1 4 5 | 4 5 | 2 |
| a2 b2 | 4 5 \cap 2 3 | ϕ | 0 |

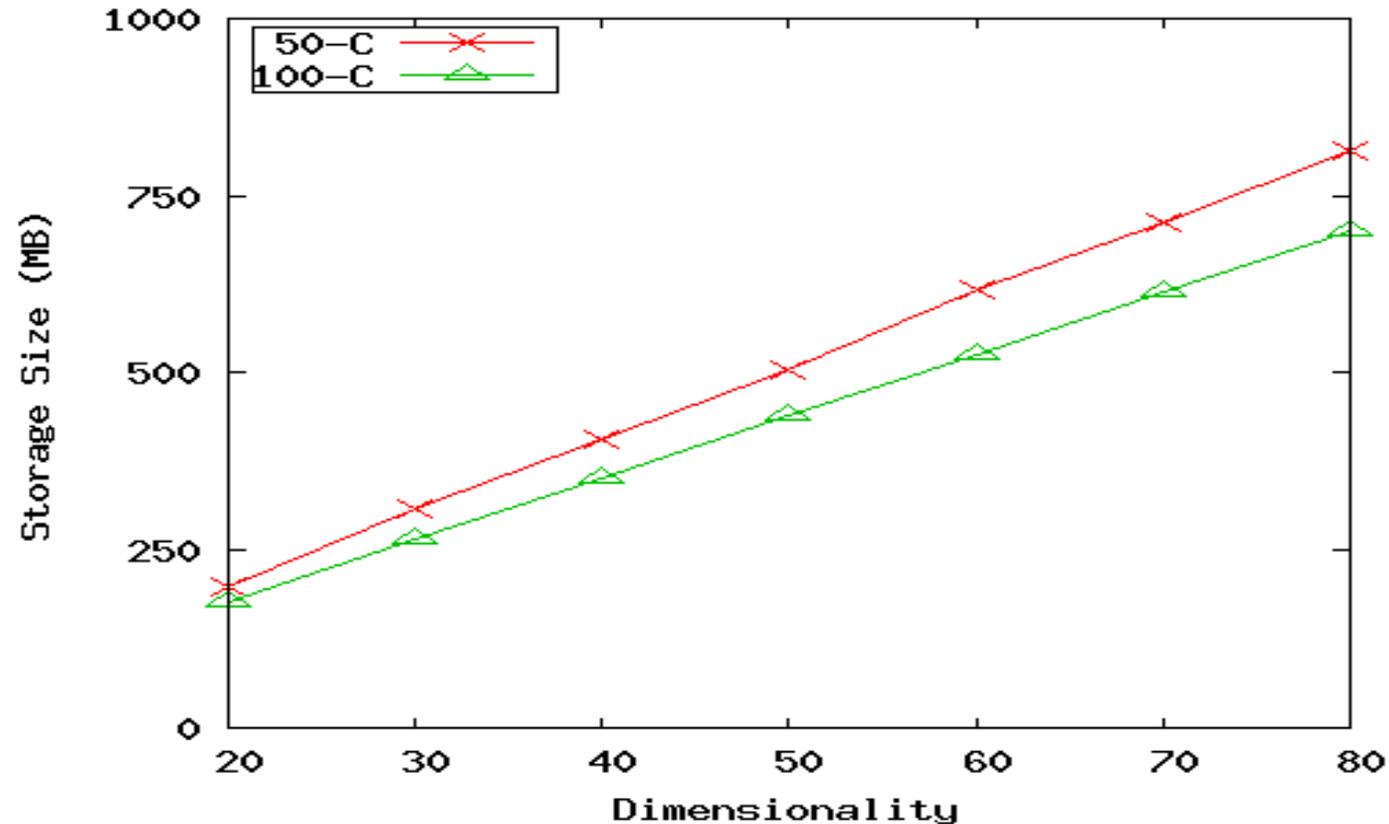
Use Frag-Shells for Online OLAP Query Computation



Online Query Computation with Shell-Fragments

- ❑ A query has the general form: $\langle a_1, a_2, \dots, a_n: M \rangle$
- ❑ Each a_i has 3 possible values (e.g., $\langle 3, ?, ?, *, 1: \text{count} \rangle$ returns a 2-D data cube)
 - ❑ Instantiated value
 - ❑ Aggregate * function
 - ❑ Inquire ? Function
- ❑ Method: Given the fragment cubes, process a query as follows
 - ❑ Divide the query into fragment, same as the shell-fragment
 - ❑ Fetch the corresponding TID list for each fragment from the fragment cube
 - ❑ Intersect the TID lists from each fragment to construct **instantiated base table**
 - ❑ Compute the data cube using the base table with any cubing algorithm

Experiment: Size vs. Dimensionality (50 and 100 cardinality)



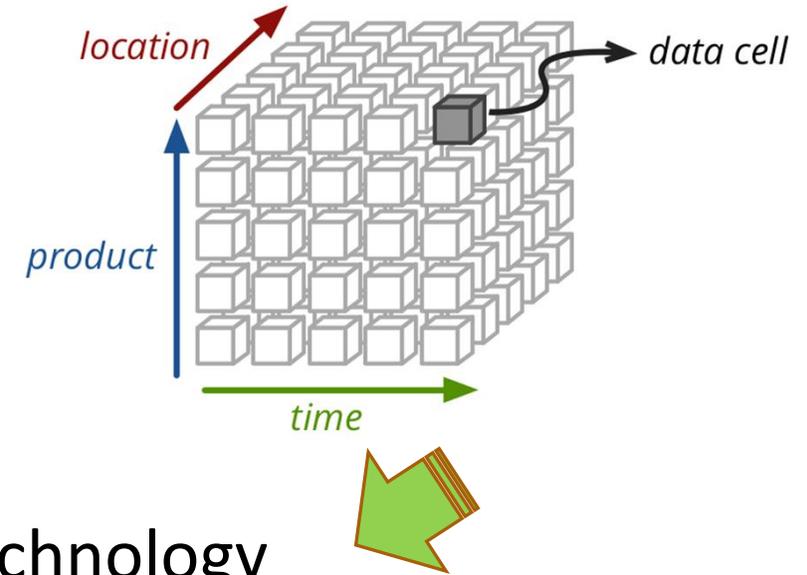
- (50-C): 10^6 tuples, 0 skew, 50 cardinality, fragment size 3
- (100-C): 10^6 tuples, 2 skew, 100 cardinality, fragment size 2

Experiments on real-world data

- UCI Forest CoverType data set
 - 54 dimensions, 581K tuples
 - Shell fragments of size 2 took 33 seconds and 325MB to compute
 - 3-D subquery with 1 instantiate D: 85ms~1.4 sec.
- Longitudinal Study of Vocational Rehab.
 - Data: 24 dimensions, 8818 tuples
 - Shell fragments of size 3 took 0.9 seconds and 60MB to compute
 - 5-D query with 0 instantiated D: 227ms~2.6 sec.

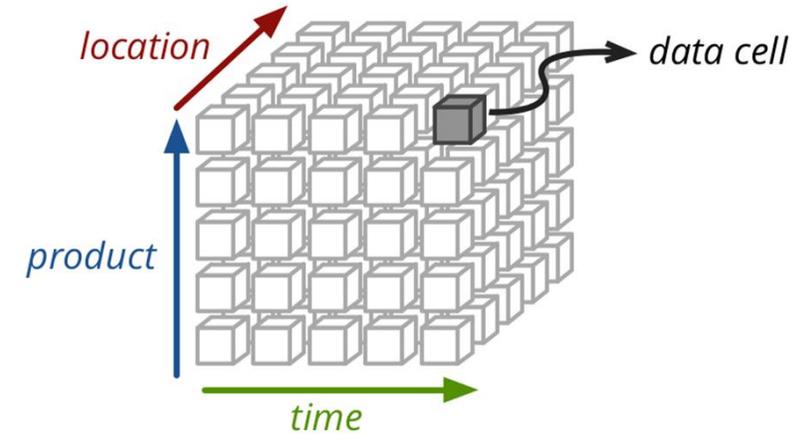
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Data Mining in Cube Space

- ❑ Data cube greatly increases the analysis bandwidth
- ❑ Four ways to interact OLAP-styled analysis and data mining
 - ❑ Using cube space to define data space for mining
 - ❑ Using OLAP queries to generate features and targets for mining, e.g., multi-feature cube
 - ❑ Using data-mining models as building blocks in a multi-step mining process, e.g., prediction cube
 - ❑ Using data-cube computation techniques to speed up repeated model construction
 - ❑ Cube-space data mining may require building a model for each candidate data space
 - ❑ Sharing computation across model-construction for different candidates may lead to efficient mining

Complex Aggregation at Multiple Granularities: Multi-Feature Cubes

- ❑ Multi-feature cubes (Ross, et al. 1998): Compute complex queries involving multiple dependent aggregates at multiple granularities
- ❑ Ex. Grouping by all subsets of {item, region, month}, find the maximum price in 2010 for each group, and the total sales among all maximum price tuples

select item, region, month, max(price), sum(R.sales)

from purchases

where year = 2010

cube by item, region, month: R

such that R.price = max(price)

- ❑ Continuing the last example, among the max price tuples, find the min and max shelf life, and find the fraction of the total sales due to tuple that have min shelf life within the set of all max price tuples

Discovery-Driven Exploration of Data Cubes

- ❑ Discovery-driven exploration of huge cube space (Sarawagi, et al.'98)
 - ❑ Effective navigation of large OLAP data cubes
 - ❑ pre-compute measures indicating exceptions, guide user in the data analysis, at all levels of aggregation
 - ❑ Exception: significantly different from the value anticipated, based on a statistical model
 - ❑ Visual cues such as background color are used to reflect the degree of exception of each cell
- ❑ Kinds of exceptions
 - ❑ SelfExp: surprise of cell relative to other cells at same level of aggregation
 - ❑ InExp: surprise beneath the cell
 - ❑ PathExp: surprise beneath cell for each drill-down path
- ❑ Computation of exception indicator can be overlapped with cube construction
 - ❑ Exceptions can be stored, indexed and retrieved like precomputed aggregates

Examples: Discovery-Driven Data Cubes

| | |
|--------|-----|
| item | all |
| region | all |

| Sum of sales | month | | | | | | | | | | | |
|--------------|-------|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|
| | Jan | Feb | Mar | Apr | May | Jun | Jul | Aug | Sep | Oct | Nov | Dec |
| Total | | 1% | -1% | 0% | 1% | 3% | -1 | -9% | -1% | 2% | -4% | 3% |

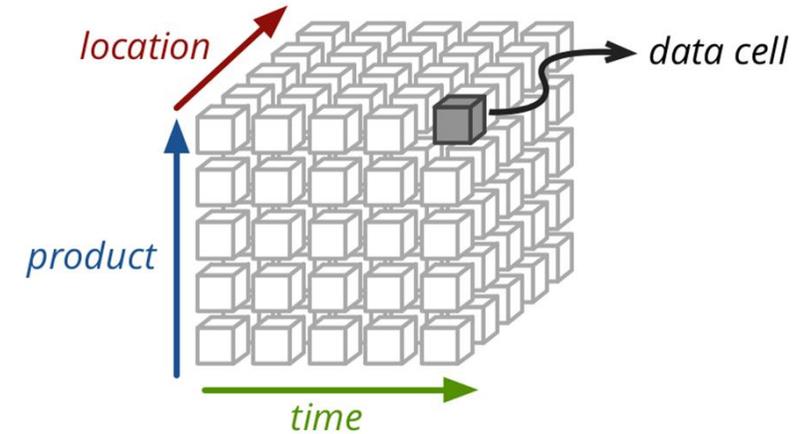
| Avg sales | month | | | | | | | | | | | |
|-------------------------|-------|-----|-----|-----|-----|-----|------|------|-----|------|------|------|
| | Jan | Feb | Mar | Apr | May | Jun | Jul | Aug | Sep | Oct | Nov | Dec |
| item | | | | | | | | | | | | |
| Sony b/w printer | | 9% | -8% | 2% | -5% | 14% | -4% | 0% | 41% | -13% | -15% | -11% |
| Sony color printer | | 0% | 0% | 3% | 2% | 4% | -10% | -13% | 0% | 4% | -6% | 4% |
| HP b/w printer | | -2% | 1% | 2% | 3% | 8% | 0% | -12% | -9% | 3% | -3% | 6% |
| HP color printer | | 0% | 0% | -2% | 1% | 0% | -1% | -7% | -2% | 1% | -5% | 1% |
| IBM home computer | | 1% | -2% | -1% | -1% | 3% | 3% | -10% | 4% | 1% | -4% | -1% |
| IBM laptop computer | | 0% | 0% | -1% | 3% | 4% | 2% | -10% | -2% | 0% | -9% | 3% |
| Toshiba home computer | | -2% | -5% | 1% | 1% | -1% | 1% | 5% | -3% | -5% | -1% | -1% |
| Toshiba laptop computer | | 1% | 0% | 3% | 0% | -2% | -2% | -5% | 3% | 2% | -1% | 0% |
| Logitech mouse | | 3% | -2% | -1% | 0% | 4% | 6% | -11% | 2% | 1% | -4% | 0% |
| Ergo-way mouse | | 0% | 0% | 2% | 3% | 1% | -2% | -2% | -5% | 0% | -5% | 8% |

| | |
|------|-------------------|
| item | IBM home computer |
|------|-------------------|

| Avg sales | month | | | | | | | | | | | |
|-----------|-------|-----|-----|-----|-----|-----|------|------|------|-----|-----|-----|
| | Jan | Feb | Mar | Apr | May | Jun | Jul | Aug | Sep | Oct | Nov | Dec |
| region | | | | | | | | | | | | |
| North | | -1% | -3% | -1% | 0% | 3% | 4% | -7% | 1% | 0% | -3% | -3% |
| South | | -1% | 1% | -9% | 6% | -1% | -39% | 9% | -34% | 4% | 1% | 7% |
| East | | -1% | -2% | 2% | -3% | 1% | 18% | -2% | 11% | -3% | -2% | -1% |
| West | | 4% | 0% | -1% | -3% | 5% | 1% | -18% | 8% | 5% | -8% | 1% |

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- ❑ Multidimensional Data Analysis in Cube Space
 - ❑ Multi-feature Cubes
 - ❑ Discovery-Driven Exploration of Data Cubes

Data Cube Technology: References (I)

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