Correction to Theorem 12.1.1

In [BCN], Theorem 12.1.1 the existence of a certain association scheme is claimed, and details are given for $n = 3$. As Frédéric Vanhove (pers.comm., Sept. 2013) observed, things are slightly different for odd $n \geq 5$.

Let q be a power of 2, and $n \geq 3$. Let V be an n-dimensional vector space over $\mathbb{F}_q$ provided with a nondegenerate quadratic form Q. If n is odd, there will be a nucleus $N = V^\perp$.

We construct an association scheme with point set X, where X is the set of projective points not on the quadric Q and (for odd n) distinct from N. For $n = 3$ and for even n, the relations will be R_0, R_1, R_2, R_3 where

$$\begin{aligned}
R_0 &= \{(x,x) \mid x \in X\}, \text{ the identity relation;}\\
R_1 &= \{(x,y) \mid x+y \text{ is a hyperbolic line (secant)}\};\\
R_2 &= \{(x,y) \mid x+y \text{ is an elliptic line (exterior line)}\};\\
R_3 &= \{(x,y) \mid x+y \text{ is a tangent}\}.
\end{aligned}$$

For odd n, $n \geq 5$, it is necessary to distinguish R_{3a} and R_{3n}, defined by

$$\begin{aligned}
R_{3a} &= \{(x,y) \mid x+y \text{ is a tangent not on } N\};\\
R_{3n} &= \{(x,y) \mid x+y \text{ is a tangent on } N\}.
\end{aligned}$$

For $q = 2$ a hyperbolic line contains only one nonisotropic point, so that R_1 is empty.

Theorem 12.1.1 (corrected)
(i) $(X, \{R_0, R_1, R_2, R_3\})$ *is an association scheme for even* $n = 2m \geq 4$. *It has eigenmatrix*

$$P = \begin{pmatrix}
1 & \frac{1}{2}q^{m-1}(q^{m-1}+\varepsilon)(q-2) & \frac{1}{2}q^m(q^{m-1}-\varepsilon) & q^{2m-2}-1\\
1 & \frac{1}{2}\varepsilon q^{m-2}(q+1)(q-2) & -\frac{1}{2}\varepsilon q^{m-1}(q-1) & \varepsilon q^{m-2}-1\\
1 & 0 & \varepsilon q^{m-1} & -\varepsilon q^{m-1}-1\\
1 & -\varepsilon q^{m-1} & 0 & \varepsilon q^{m-1}-1
\end{pmatrix}$$

and multiplicities 1, $q^2(q^{n-2}-1)/(q^2-1)$, $\frac{1}{2}q(q^{m-1}-\varepsilon)(q^m-\varepsilon)/(q+1)$, $\frac{1}{2}(q-2)(q^{m-1}+\varepsilon)(q^m-\varepsilon)/(q-1)$.

(ii) $(X, \{R_0, R_1, R_2, R_{3a}, R_{3n}\})$ *is an association scheme for odd* $n = 2m+1 \geq 3$. *It has eigenmatrix*

$$P = \begin{pmatrix}
1 & \frac{1}{2}q^{2m-1}(q-2) & \frac{1}{2}q^{2m} & q(q^{2m-2}-1) & q-2\\
1 & \frac{1}{2}q^{m-1}(q-2) & \frac{1}{2}q^m & -(q^{m-1}+1)(q-1) & q-2\\
1 & -\frac{1}{2}q^{m-1}(q-2) & -\frac{1}{2}q^m & (q^{m-1}-1)(q-1) & q-2\\
1 & \frac{1}{2}q^m & -\frac{1}{2}q^m & 0 & -1\\
1 & -\frac{1}{2}q^m & \frac{1}{2}q^m & 0 & -1
\end{pmatrix}$$

and multiplicities 1, $\frac{1}{2}q(q^m+1)(q^{m-1}-1)/(q-1)$, $\frac{1}{2}q(q^m-1)(q^{m-1}+1)/(q-1)$, $\frac{1}{2}(q-2)(q^{2m}-1)/(q-1)$ *(twice).*

When $n = 3$, the relation R_{3a} is empty, and the second eigenspace is absent.