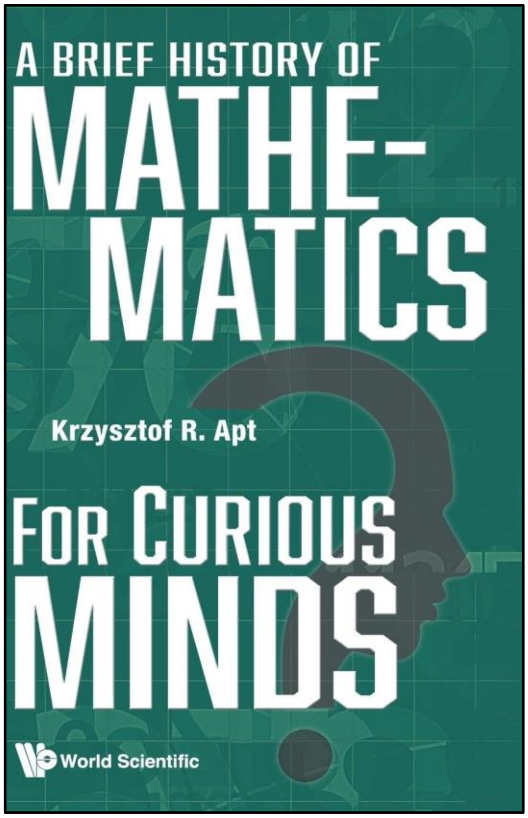
Book Review

A Brief History of Mathematics for Curious Minds

by Krzysztof R. Apt

Reviewed by Brian Monahan



A Brief History of Mathematics for Curious Minds
by Krzysztof R. Apt, World Scientific Pub., 2023, 210pp, **ISBN: 978-9811280443**

A senior member of the international formal methods community strongly recommended that *FACS FACTS* take a look at this book—and I'm very glad that he did! It is written by Krzysztof Apt, a professor emeritus at the University of Amsterdam, well known for his computer science work in program verification and constraint programming but also, more recently, at CWI, for his work in game theory and economic mechanism design.

The title declares what this book is all about. The part about "Curious Minds" hints that it may not be quite the somewhat dry and straightforward account of mathematical history that one might have otherwise expected. Indeed, not only is it delightfully packed full of all sorts of curious and surprising facts about the mathematicians who brought mathematics to light, but it fortunately also contains many short appendices (32 in all!) explaining various mathematical topics and theorems at an intermediate level. I can think of nothing more frustrating or tedious than a book discussing mathematics at some length – but then comprehensively neglecting to do any! Fortunately, that particular criticism *doesn't* apply here.

Although reflecting its historical development, it is the mathematics itself that drives the content of this book. From the above, you might be forgiven for thinking that the book is merely a historical compendium, running through the history of mathematics with some potted biographies of various notable mathematicians. Well, that remark is only occasionally true – yes, the book is historically organised, but it is also far from a series of "potted biographies". The author's approach is to generally find something unusual, notable and interesting to say about the various mathematicians being written about.

To begin with, in Chapter 1, we find some of the ancient origins of numbers and so on being discussed – but there are no individuals to speak of there. It is only in Chapter 2, focusing upon the Greeks, that we do come across particular individual mathematicians – but even there, more often than not, it is particular *communities* of mathematicians that are (all too) briefly discussed, focussing upon a diverse range of topic areas, such as geometry, astronomical calculations, the nature of Infinity, Greek number notations and so on. All of the well-known greats of Greek mathematics (spanning nearly 1000 years from the 6th century BCE onwards) are mentioned here – Euclid, Pythagoras, Archimedes, Ptolemy, Diophantus, beginning with Thales of Miletus (his theorem is discussed in Appendix 1).

Inevitably, the account here includes some non-mathematicians in addition to strict mathematicians – because mathematics wasn't entirely separated from other science-related pursuits; this only happened much later with the rise in applications and their technical complexity. A good case in point is the extraordinary mathematician and engineer *Heron of Alexandria*[30], who appears to have taught a wide range of subjects at the *Musaeum* in Alexandria, which included the famous Library. Mathematically, Heron is famous for his formula relating the semi-perimeter length and sides of a triangle to its area: if a, b, and c are the lengths of the sides:

$$\text{Area} = \sqrt{s(s-a)(s-b)(s-c)}$$

where s is half the perimeter, or $(a + b + c)/2$. What's more, he is credited with writing about how pure geometry can, via early surveying instruments like the *dioptra*[31], show how two teams can tunnel from opposite sides of a hill and ensure a close enough meeting in the middle. Quite remarkable.

But the Greeks weren't the only developers of mathematics in ancient times. Chapter 3 briefly discusses ancient Chinese and Indian mathematical developments. The author observes that Chinese mathematics focuses mostly on empirical matters, very different from the Greek focus on principles and deduction. Even so, there is evidence that Chinese mathematicians knew of and had demonstrated for themselves Pythagoras's theorem – but clearly far *later* than the Greeks had. Of course, Hindu mathematicians discovered and developed the Hindu-Arabic numeral notation. However, because of its wider ramifica-

[30] https://en.wikipedia.org/wiki/Hero_of_Alexandria
[31] https://en.wikipedia.org/wiki/Dioptra

tions, this topic is discussed in detail in the following chapter on Roman and medieval mathematics, spanning from the 5th century to the 15th.

The next three chapters cover the period from the 15th to the 19th century and briefly discuss, for example, the introduction of algebraic notation, complex numbers, the general solution of equations of third and fourth degree – but not the fifth, the birth and development of calculus, probability theory, and the emergence of more systematic algebraic notions such as Group and Galois theory, by many great mathematicians including such names as Fibonacci, the Bernoulli brothers, Euler and Gauss. That's a lot of mathematics!

Chapter 8 brings the topic up to date by discussing mathematics in the 20th and 21st centuries. Because of mathematics' extraordinary super-exponential growth, this discussion tends to be sketchy, as expected. Even so, computer science is briefly represented here by John Von Neumann, Alan Turing, Stephen Cook, Edsger Dijkstra, Donald Knuth, Tony Hoare, and Leonid Levin. Although both John Von Neumann and Alan Turing get a few paragraphs, the other mentions are far more perfunctory – for example, Tony Hoare is mentioned solely for *quicksort* – but not for Floyd-Hoare logic or, indeed, for CSP or UTP!

It is impossible to compactly mention all the topics and mathematicians that are all too briefly touched upon in this book – it is certainly an achievement to capture such a broad range of topics, albeit in somewhat restricted discussion. After all, this book is clearly the author's personal take on mathematics in a historical context overall. Consequently, this book is primarily about *classical* mathematics and tends to emphasise calculation over the conceptual. To that end, as far as I could tell, modern areas and approaches such as *lambda calculus, topology* and *category theory* were not even mentioned *anywhere.* For instance, although Henri Poincare is indeed substantially referred to in connection with the n-body dynamics problem and the stability of the Solar System, there was still no mention of topology, even when the famous Poincare conjecture is very briefly alluded to when discussing Gregori Perelmann's solution!

This book does not claim to be either comprehensive or definitive. However, it will no doubt prove useful as a reference guide to (classical) mathematics for the interested layperson who may have heard about some mathematical topic in another context. It is a shame that it doesn't extend to some of the more modern topics, but, of course, the author must finish somewhere! It is quite remarkable what has been packed into these 210 pages, with copious references, notes and even a smattering of colour photos and diagrams. For me, there is always the great charm in the 32 appendices discussing actual mathematics, i.e., what it's all *truly* about.