Sequential Groves mechanisms for public project problems

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joint work with

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WHAT on earth is mechanism design? was the typical reaction to this year’s Nobel prize in economics, announced on October 15th. 

[...]

In fact, despite its dreary name, mechanism design is a hugely important area of economics, and underpins much of what dismal scientists do today. It goes to the heart of one of the biggest challenges in economics: how to arrange our economic interactions so that, when everyone behaves in a self-interested manner, the result is something we all like.

(Economist, 20 Oct. 2007)
Executive Summary

- Groves mechanisms allow us to implement desired decisions by imposing on the players taxes.
- In Groves mechanisms truth-telling is a dominant strategy.
- In Groves mechanisms for the public project problems taxes $\neq 0$ are unavoidable.
- So we study the sequential Groves mechanisms.
- We show that other dominant strategies than truth-telling may then exist.
- This can be used to minimize taxes in public projects problems.
"Perhaps this will refresh your memory."
Decision problems

Assume

- players 1, . . ., n,
- set of decisions $D$,
- for each player a set of types $\Theta_i$ and a utility function

$$v_i : D \times \Theta_i \rightarrow \mathbb{R}$$

that he wants to maximize.

**Decision rule:** a function $f : \Theta_1 \times \cdots \times \Theta_n \rightarrow D$.

We call

$$(D, \Theta_1, \ldots, \Theta_n, v_1, \ldots, v_n, f)$$

a decision problem.
Decisions, Decisions, . . .

One studies the following sequence of events:

1. each player \( i \) receives type \( \theta_i \),
2. each player \( i \) announces to the central planner a type \( \theta'_i \),
3. the central planner takes the decision \( d := f(\theta'_1, \ldots, \theta'_n) \), and communicates it to each player,
4. the resulting utility for player \( i \) is then \( v_i(d, \theta_i) \).

Problem to solve: Each player \( i \) wants to manipulate the choice of \( d \in D \) so that his utility \( v_i(d, \theta_i) \) is maximized.
A decision rule $f$ is called

- **strategy-proof (or incentive compatible)** if for all $\theta \in \Theta$, $i \in \{1, \ldots, n\}$ and $\theta'_i \in \Theta$

$$v_i(f(\theta_i, \theta_{-i}), \theta_i) \geq v_i(f(\theta'_i, \theta_{-i}), \theta_i).$$

- **efficient** if for all $\theta \in \Theta$ and $d' \in D$

$$\sum_{i=1}^{n} v_i(f(\theta), \theta_i) \geq \sum_{i=1}^{n} v_i(d', \theta_i).$$
Central authority takes the decision \( d := f(\theta') \) and computes the sequence of taxes \( t_1(\theta'), \ldots, t_n(\theta') \), where

\[
t_i(\theta') := \sum_{j \neq i} v_j(f(\theta'), \theta'_j) + h_i(\theta'_{-i}),
\]

\[
h_i : \Theta_{-i} \rightarrow \mathbb{R}
\]

is an arbitrary function.

player’s \( i \) modified utility is

\[
u_i((f, t)(\theta'_i, \theta_{-i}), \theta_i) := v_i(f(\theta'_i, \theta_{-i}), \theta_i) + t_i(\theta'_i, \theta_{-i}).
\]

Intuition:

\[
\sum_{j \neq i} v_j(f(\theta'), \theta'_j)
\]

is the society benefit with \( i \) excluded from decision \( f(\theta') \).

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Groves Mechanisms, ctd

- **Groves Theorem**
  Suppose \( f \) is efficient. Then in each Groves mechanism \((f, t)\) is strategy-proof.

- Groves mechanism with the tax function \( t := (t_1, \ldots, t_n) \) is
  - balanced if \( \sum_{i=1}^{n} t_i(\theta) = 0 \) for all \( \theta \),
  - feasible if \( \sum_{i=1}^{n} t_i(\theta) \leq 0 \) for all \( \theta \),
  - pay only if \( t_i(\theta) \leq 0 \) for all \( \theta \) and all \( i \in \{1, \ldots, n\} \).
Special Case: VCG Mechanism

- One uses $h_i(\theta_{-i}) := -\max_{d \in D} \sum_{j \neq i} v_j(d, \theta'_j)$.

- So $t_i(\theta') := \sum_{j \neq i} v_j(f(\theta'), \theta'_j) - \max_{d \in D} \sum_{j \neq i} v_j(d, \theta'_j)$.

- Intuition:

  - $\sum_{j \neq i} v_j(f(\theta'), \theta'_j)$ is the society benefit from $f(\theta')$ with $i$ excluded,

  - $\max_{d \in D} \sum_{j \neq i} v_j(d, \theta'_j)$ is the maximal society benefit from $f(\theta')$ with $i$ excluded.

- Note If with player $i$ excluded better decision can be taken, $i$ is pivotal. His tax is then $\neq 0$.

- Note VCG mechanism is pay only (so feasible).
Example

Public project problem:

- $D = \{0, 1\},$
- $[0, c] \subseteq \Theta_i \subseteq \mathbb{R_+},$
- cost of building the bridge: $c,$
- $v_i(d, \theta_i) := d(\theta_i - \frac{c}{n}),$
- $f(\theta) := \begin{cases} 
1 & \text{if } \sum_{i=1}^{n} \theta_i \geq c \\
0 & \text{otherwise}
\end{cases}$
- **Note** $f$ is efficient.
Example, ctd

Suppose \( c = 300 \) and \( n = 3 \).

<table>
<thead>
<tr>
<th>player</th>
<th>value</th>
<th>set of types ((\Theta_i))</th>
<th>tax</th>
<th>( u_i )</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>60</td>
<td>( \mathbb{R}_+ )</td>
<td>-20</td>
<td>-20</td>
</tr>
<tr>
<td>B</td>
<td>70</td>
<td>( \mathbb{R}_+ )</td>
<td>-10</td>
<td>-10</td>
</tr>
<tr>
<td>C</td>
<td>150</td>
<td>( \mathbb{R}_+ )</td>
<td>0</td>
<td>0</td>
</tr>
</tbody>
</table>

The project does not get through \((d = 0)\) since \( 60 + 70 + 150 < 300 \).

Yet both A and B have to pay a tax.
Optimality of VCG Mechanism

Groves mechanism $h'$ is strictly better than $h$ if

- for all $\theta \in \Theta$
  $h'$ generates a larger society benefit than $h$,

- for some $\theta \in \Theta$
  $h'$ generates a strictly larger society benefit than $h$. 
Optimality of VCG Mechanism, ctd

**Theorem**
In the public project problem for no $c > 0$ and $n \geq 2$ a feasible Groves mechanism exists that is strictly better than VCG mechanism.

**Corollary**
For no $c > 0$ and $n \geq 2$ a balanced Groves mechanism exists for the public project problem.
The players are randomly ordered. The following revised sequence of events then takes place:

1. each player $i$ receives type $\theta_i$,
2. each player $i$ in turn announces to the other players a type $\theta_i'$,
3. each player takes the decision $d := f(\theta_1', \ldots, \theta_n')$.

**Crucial difference:** Now each player announces his type after he saw the types of earlier players.
Sequential Mechanisms: Dominant Strategies

Set-up: sequential version of Example.

Theorem

\[ s_i(\theta_1, \ldots, \theta_i) := \begin{cases} 
\theta_i & \text{if } \sum_{j=1}^{i} \theta_j < c \text{ and } i < n, \\
0 & \text{if } \sum_{j=1}^{i} \theta_j < c \text{ and } i = n, \\
c & \text{if } \sum_{j=1}^{i} \theta_j \geq c. 
\end{cases} \]

is a dominant strategy for player \(i\).

Strategy \(s_i(\cdot)\) of player \(i\) minimizes the tax of every other player (subject to some mild conditions).
Back to our Example

<table>
<thead>
<tr>
<th>ordering</th>
<th>$t_A$</th>
<th>$t_B$</th>
<th>$t_C$</th>
</tr>
</thead>
<tbody>
<tr>
<td>A B C</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>A C B</td>
<td>0</td>
<td>−10</td>
<td>0</td>
</tr>
<tr>
<td>B A C</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>B C A</td>
<td>−20</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>C A B</td>
<td>0</td>
<td>−10</td>
<td>0</td>
</tr>
<tr>
<td>C B A</td>
<td>−20</td>
<td>0</td>
<td>0</td>
</tr>
</tbody>
</table>

Conclusions
In each ordering at least one player pays smaller taxes.
In 2 orderings taxes are 0.
Can we Reduce Taxes to 0?

Theorem

Assume each player follows $s_i(\cdot)$. For all $c > 0$, $n \geq 2$ and $\theta_1, \ldots, \theta_n$ for some permutation of players all taxes equal 0.

Proof idea

- Not all players can be pivotal.
- Put a pivotal player at the end.
Conclusions

“Bang! Bang! Bang!”
Advantages of sequential mechanism design

- In sequential mechanism design other dominant strategies may exist than truth-telling.
- Such strategies can be used to minimize taxes.
- Cooperative aspects can be incorporated.

Maxim: First take care of your benefit and then of the benefit of others.

Dalai Lama: *The intelligent way to be selfish is to work for the welfare of others.*

(Bowles ’04)

Applicable to various forms of financing of public projects.