Assignment 1: Solution

Denote $\sum_{i=1}^{n} s_i$ by t. Assume that for all $i \in \{1, \ldots, n\}$

$$s_i = \frac{d - \sum_{j \neq i} s_j}{2}$$

where d is some constant.

By adding the considered equations for all $i \in \{1, \ldots, n\}$ we obtain the equality

$$t = \frac{nd - (n-1)t}{2}$$

from which it follows that $2t = nd - (n-1)t$. So

$$t = \frac{nd}{n+1}$$

Further, for all $i \in \{1, \ldots, n\}$

$$\sum_{j \neq i} s_j = t - s_i,$$

so each original equation can be rewritten as

$$s_i = \frac{d - t + s_i}{2}$$

From this it follows that for all $i \in \{1, \ldots, n\}$

$$s_i = d - t,$$

that is all s_is are equal. Consequently for all $i \in \{1, \ldots, n\}$ $s_i = \frac{t}{n}$, that is

$$s_i = \frac{d}{n+1}$$

We obtain the solution to the first problem by using $d = 1$ and to the second problem by using $d = \frac{a-c}{b}$.