Strategic Games - Assignment 4

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Exercise. Find a 2 by 2 strictly competitive game such that its mixed extension is not a strictly competitive game.

Define the game $G = (S_1, S_2, p_1, p_2)$ by

	C	D
A	4,0	0,4
B	1,2	3,1

We will decide for $i = 1, 2$ and $s, s' \in S$ what the relation between $p_i(s)$ and $p_i(s')$ is. We have

$p_1(AC) = 4 > 1 = p_1(BC)$	$p_2(AC) = 0 < 2 = p_2(BC)$
$p_1(AC) = 4 > 0 = p_1(AD)$	$p_2(AC) = 0 < 4 = p_2(AD)$
$p_1(AC) = 4 > 3 = p_1(BD)$	$p_2(AC) = 0 < 1 = p_2(BD)$
$p_1(BC) = 1 > 0 = p_1(AD)$	$p_2(BC) = 2 < 4 = p_2(AD)$
$p_1(BC) = 1 < 3 = p_1(BD)$	$p_2(BC) = 2 > 1 = p_2(BD)$
$p_1(AD) = 0 < 3 = p_1(BD)$	$p_2(AD) = 4 > 1 = p_2(BD)$

Hence, G is strictly competitive.

Let $H = (M_1, M_2, q_1, q_2)$ be the mixed extension of G. Define the mixed strategy m_2 that maps both C and D to $\frac{1}{2}$. Let m, m' be joint mixed strategies defined by $m = (A, m_2)$, $m' = (B, m_2)$. Then

$$\begin{aligned}
q_1(m) &= \sum_{s \in S} m(s) \cdot p_1(s) \\
&= m_2(C) \cdot p_1(AC) + m_2(D) \cdot p_1(AD) \\
&= \tfrac{1}{2}(4 + 0) = 2 \\
q_1(m') &= \sum_{s \in S} m'(s) \cdot p_1(s) \\
&= m_2(C) \cdot p_1(BC) + m_2(D) \cdot p_1(BD) \\
&= \tfrac{1}{2}(1 + 3) = 2.
\end{aligned}$$

So $q_1(m) \geq q_1(m')$. We also see that

$$\begin{aligned}
q_2(m) &= \sum_{s \in S} m(s) \cdot p_2(s) \\
&= m_2(C) \cdot p_2(AC) + m_2(D) \cdot p_2(AD) \\
&= \tfrac{1}{2}(0+4) = 2 \\
q_2(m') &= \sum_{s \in S} m'(s) \cdot p_2(s) \\
&= m_2(C) \cdot p_2(BC) + m_2(D) \cdot p_2(BD) \\
&= \tfrac{1}{2}(2+1) = \tfrac{3}{2}.
\end{aligned}$$

Hence, $q_2(m) > q_2(m')$. We conclude that H is not strictly competitive.