Strategic Games – Assignment 3

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Domination in 2-turn Prisoner's Dilemma

In basic Prisoner's Dilemma, there are 2 players, each one having two possible action: Cooperate (C) or Defect (D). The payoff table is as follows:

	C	D
C	2,2	0,3
D	3,0	1,1

In this setting, strategy C is strictly dominated by D, because the payoff for D is always higher than the payoff for C given all fixed strategies of the other player. In the 2-turn case, the game gets more complicated because the action of the other player in the first turn can be taken into account for the second turn. This results in 8 possible strategies for each player: there are two options for the first round (simply C or D) and then, there are two options for each possible first move of the other player, making 4 possible strategies for the second turn. This is a simplification of representing the second move on the possible outcomes of the first turn (4 possibilities), which is possible because the possible outcomes are already limited by the first move of the first player.

The representation of a strategy therefore is a triplet (e.g. (C,D,C)) where each element is either C or D, the first element representing the first turn (Cooperate in the example) the second element representing the second turn in case the other player played Cooperate in the first round (Defect) and the third element rperesents the strategy for the second turn in case the other player plays Defect in the first round. This results in the following table, where the payoff is being decided by the strategy for the second round that coincides with the strategy of the first round of the other player:

	CCC	CCD	CDC	CDD	DCC	DCD	DDC	DDD
CCC	4,4	4,4	2,5	2,5	2,5	2,5	0,6	0,6
CCD	4,4	4,4	2,5	2,5	3,3	3,3	1,4	1,4
CDC	5,2	5,2	3,3	3,3	2,5	2,5	0,6	0,6
CDD	5,2	5,2	3,3	3,3	3,3	3,3	1,4	1,4
DCC	5,2	3,3	5,2	3,3	3,3	1,4	3,3	1,4
DCD	5,2	3,3	5,2	3,3	4,1	2,2	4,1	2,2
DDC	6,0	4,1	6,0	4,1	3,3	1,4	3,3	1,4
DDD	6,0	4,1	6,0	4,1	4,1	2,2	4,1	2,2

From this table, IESDS and IEWDS can be used to respectively eliminate strictly and weakly dominated strategies. This way, it becomes clear that CCC and DCC are strictly dominated, resulting in the following table:

	CCD	CDC	CDD	DCD	DDC	DDD
CCD	4,4	2,5	2,5	3,3	1,4	1,4
CDC	5,2	3,3	3,3	2,5	0,6	0,6
CDD	5,2	3,3	3,3	3,3	1,4	1,4
DCD	3,3	5,2	3,3	2,2	4,1	2,2
DDC	4,1	6,0	4,1	1,4	3,3	1,4
DDD	4,1	6,0	4,1	2,2	4,1	2,2

No more steps can be taken with IESDS, so CCC and DCC are the only strictly dominated strategies.

Using IEWDS, Using the same table (a strictly dominated strategy is also a weakly dominated strategy by definition), more strategies can be eliminated. CCD and CDC are weakly dominated by CDD, and so are DCD and DDC w.r.t. DDD, therefore the table can be reduced to:

	CDD	DDD
CDD	3,3	1,4
DDD	4,1	2,2

In this case, CDD, continuing IEWDS, CDD is dominated by DDD, and therefore the only non-dominated strategy is DDD, with a payoff of 2.

In conclusion: cooperating in the second turn no matter what is strictly dominated, cooperating at all in the second turn is weakly dominated, just like cooperating in the first round, when cooperation in the second round is eliminated. Defecting is the only non-dominated strategy.