Assignment 5

Consider the network given in Figure 1. The delays on the road segments are either constant (4 or 5) or equal to the number of drivers who chose the segment (denoted by T).

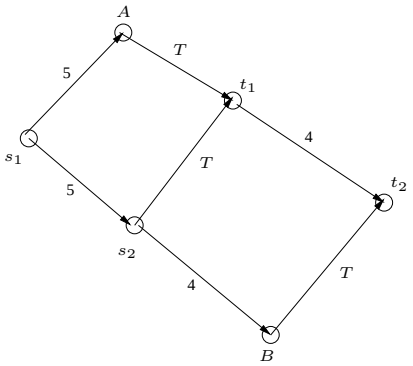

Figure 1: A network

There are 6 drivers who need to choose a road from s_1 to t_1 and 6 drivers who need to choose a road from s_2 to t_2. So each of the drivers in the first set has two strategies, corresponding respectively to the roads $s_1 \to A \to t_1$ and $s_1 \to s_2 \to t_1$, while each of the drivers in the second set has two strategies, corresponding respectively to the roads $s_2 \to t_1 \to t_2$ and $s_2 \to B \to t_2$.

Consider a joint strategy. Denote by

- T_1 the number of drivers who took the road $s_1 \to A \to t_1$,
- T_2 the number of drivers who took the road $s_1 \to s_2 \to t_1$,
- T_3 the number of drivers who took the road $s_2 \to t_1 \to t_2$,
- T_4 the number of drivers who took the road $s_2 \to B \to t_2$.

By assumption we have

$$T_1 + T_2 = 6,\ T_3 + T_4 = 6.$$

(i) Write the conditions on T_1, T_2, T_3, T_4 that determine that a joint strategy is a Nash equilibrium.

(ii) Write the expression in T_1, T_2, T_3, T_4 that defines the social cost of a joint strategy.

Suppose now that one adds to the network a road $t_1 \to B$ with delay 0. The resulting network is drawn in Figure 2.

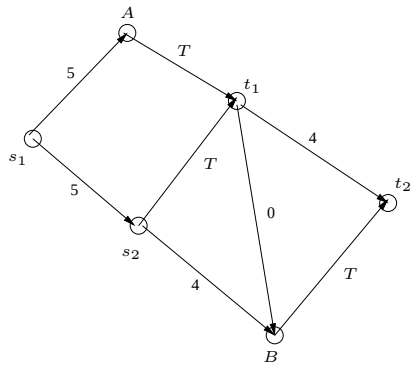

Figure 2: The new network

The drivers who need to choose a road from s_2 to t_2 have then three strategies. Given a joint strategy we denote now by

- T_5 the number of drivers who took the road $s_2 \to t_1 \to B \to t_2$,

and define T_1, T_2, T_3 and T_4 as before.
(iii) Write the conditions on T_1, T_2, T_3, T_4, T_5 that determine that a joint strategy is a Nash equilibrium.

(iv) Write the expression in T_1, T_2, T_3, T_4, T_5 that defines the social cost of a joint strategy.