Paradoxes in Social Networks with Multiple Products

Krzysztof R. Apt

CWI & and University of Amsterdam

Joint work with Evangelos Markakis and Sunil Simon

Paradox of Choice (B. Schwartz, 2005)

[*Gut Feelings*, G. Gigerenzer, 2008]

The more options one has, the more possibilities for experiencing conflict arise, and the more difficult it becomes to compare the options. There is a point where more options, products, and choices hurt both seller and consumer.

Plan

- Objective: To understand this paradox.
- Tools:
  - social networks with multiple products,
  - strategic games.

Recall: Social networks

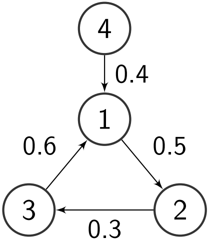
Essential components of our model

- Finite set of agents.
- Influence of "friends".
- Finite product set for each agent.
- Resistance level in (threshold for) adopting a product.

Recall: Social networks

Essential components of our model

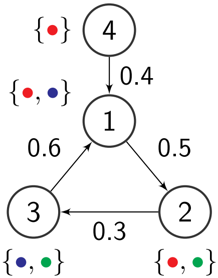
- Finite set of agents.
- Influence of "friends".
- Finite product set for each agent.
- Resistance level in (threshold for) adopting a product.



Recall: Social networks

Essential components of our model

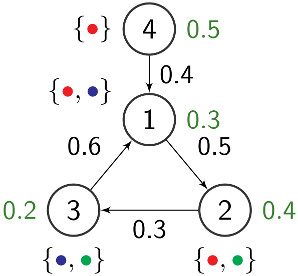
- Finite set of agents.
- Influence of "friends".
- Finite product set for each agent.
- Resistance level in (threshold for) adopting a product.



Recall: Social networks

Essential components of our model

- Finite set of agents.
- Influence of "friends".
- Finite product set for each agent.
- Resistance level in (threshold for) adopting a product.



The model

Social network [Apt, Markakis 2011]

- Weighted directed graph: $G = (V, \rightarrow, w)$, where
  V: a finite set of agents,
  $w_{ij} \in (0, 1]$: weight of the edge $i \rightarrow j$.
- Products: A finite set of products $\mathcal{P}$.
- Product assignment: $P : V \rightarrow 2^{\mathcal{P}} \setminus \{\emptyset\}$;
  assigns to each agent a non-empty set of products.
- Threshold function: $\theta(i, t) \in (0, 1]$, for each agent i and product $t \in P(i)$.

The associated strategic game

Interaction between agents: Each agent i can adopt a product from the set $P(i)$ or choose not to adopt any product (t_0).

Social network games

- Players: Agents in the network.
- Strategies: Set of strategies for player i is $P(i) \cup \{t_0\}$.
- Payoff: Fix $c > 0$.
  Given a joint strategy s and an agent i,

The associated strategic game

Interaction between agents: Each agent i can adopt a product from the set $P(i)$ or choose not to adopt any product (t_0).

Social network games

- Players: Agents in the network.
- Strategies: Set of strategies for player i is $P(i) \cup \{t_0\}$.
- Payoff: Fix $c > 0$.
  Given a joint strategy s and an agent i,
  - if $i \in source(\mathcal{S})$, $\quad p_i(s) = \begin{cases} 0 & \text{if } s_i = t_0 \\ c & \text{if } s_i \in P(i) \end{cases}$

The associated strategic game

Interaction between agents: Each agent i can adopt a product from the set $P(i)$ or choose not to adopt any product (t_0).

Social network games

- Players: Agents in the network.
- Strategies: Set of strategies for player i is $P(i) \cup \{t_0\}$.
- Payoff: Fix $c > 0$.
  Given a joint strategy s and an agent i,
  - if $i \in source(\mathcal{S})$, $p_i(s) = \begin{cases} 0 & \text{if } s_i = t_0 \\ c & \text{if } s_i \in P(i) \end{cases}$
  - if $i \notin source(\mathcal{S})$, $p_i(s) =$
    $\begin{cases} 0 & \text{if } s_i = t_0 \\ \sum_{j \in \mathcal{N}_i^t(s)} w_{ji} - \theta(i,t) & \text{if } s_i = t, \text{ for some } t \in P(i) \end{cases}$

  $\mathcal{N}_i^t(s)$: the set of neighbours of i who adopted in s the product t.

Vulnerability

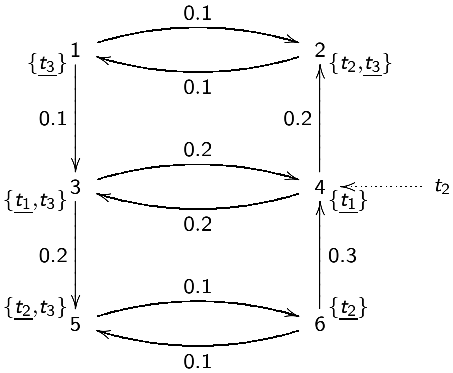
Given: a strategic game $(S_1, \ldots, S_n, p_1, \ldots, p_n)$.

- s and s': joint strategies.
  $s > s'$ if for all i, $p_i(s) > p_i(s')$.
- Social network $\mathcal{S}'$ is an expansion of $\mathcal{S}$ if it results from adding a product to the product set of a node in $\mathcal{S}$.
- We say then that $\mathcal{S}$ is a contraction of $\mathcal{S}'$.
- $\mathcal{S}$ is vulnerable if for some Nash equilibrium s in $\mathcal{G}(\mathcal{S})$, an expansion $\mathcal{S}'$ of $\mathcal{S}$ exists such that

  some improvement path in $\mathcal{G}(\mathcal{S}')$ leads from s to a Nash equilibrium s' in $\mathcal{G}(\mathcal{S}')$ such that $s > s'$.

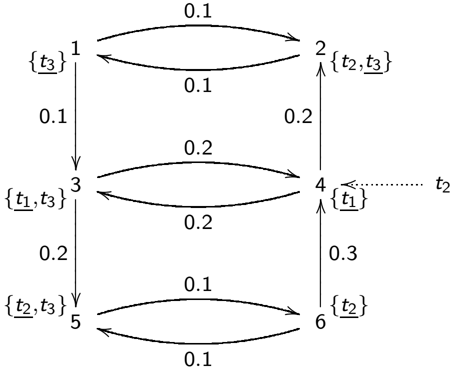
Vulnerable Networks (1)

Threshold θ is constant, $0 < \theta < 0.1$.



is a vulnerable network.

Vulnerable Networks (2)



Take the Nash equilibrium $(t_3, t_3, t_1, t_1, t_2, t_2)$ of the initial network.
The addition of product t_2 to node 4 triggers the best response improvement path

$$4 : t_2, 3 : t_3, 5 : t_3, 6 : t_0, 2 : t_2, 1 : t_0, 4 : t_0, 2 : t_0, 3 : t_0, 5 : t_0.$$

It ends in a Nash equilibrium in which each strategy equals t_0,

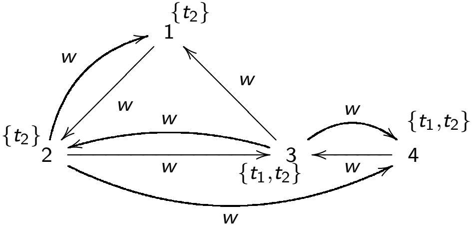
Inefficiency

- $\mathcal{S}$ is inefficient if for some Nash equilibrium s in $\mathcal{G}(\mathcal{S})$, a contraction $\mathcal{S}'$ of $\mathcal{S}$ exists such that

  each improvement path in $\mathcal{G}(\mathcal{S}')$ leads from s to a Nash equilibrium s' in $\mathcal{G}(\mathcal{S}')$ such that $s' > s$.

Inefficient Network (1)

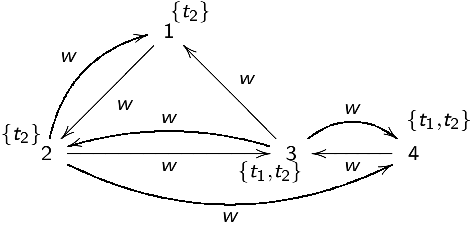
The weight of each edge is w.
Threshold θ is product independent.
For all nodes $w > \theta$.



is an inefficient network.

Inefficient Network (2)

$w > \theta$.



Take the Nash equilibrium (t_2, t_2, t_1, t_1) of the initial network with payoff equal to $w - \theta$ for all nodes.

Remove t_1 from the product set of node 3.

All improvement paths then lead to the Nash equilibrium (t_2, t_2, t_2, t_2) with payoff equal $2w - \theta$ for all nodes.

Example: $3 : t_2, 4 : t_0, 4 : t_2$.