The Heidelberg Laureate Forum on the Moving Frontier Between Mathematics and Computer Science

Young and early-career researchers at the 2016 Heidelberg Laureate Forum discuss how the frontier between mathematics and computer science is shifting, what the future promises, and the implications the frontier's shape and dynamics will have on both fields.

By Edmon Begoli, Vincent Schlegel, Sir Michael Atiyah, Praise Adeyemo, and Tim Baarslag

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In 1936, British mathematician Alan Turing published his famous paper, "On Computable Numbers, with an Application to the *Entscheidungsproblem*," in which he considered the notion of a machine capable of computing anything that is computable. As a result of Turing's work, the field of computer science was born directly out of mathematical theory. Since then, the two disciplines have developed on their own, in predominantly different directions. Here, we summarize a discussion among young researchers in mathematics and computer science at the Heidelberg Laureate Forum (HLF) at Heidelberg University, Heidelberg, Germany, on the frontier between these two fundamental areas of research.

The HLF is an annual event that brings together prestigious laureates of computer science and mathematics (winners of Fields, Abel, Nevanlinna, and Turing awards) to meet with the next generation of computer scientists and mathematicians from around the world.

The HLF is intended to bridge the generational divide and help young scientists meet and be inspired by the most accomplished living scientists in their respective fields. To foster conversation and inspire future developments, at the fourth HLF in 2016, 200 of the most promising young and early career researchers were mentored by some of the most accomplished scientists of the day (see the sidebar "Excited About the Heidelberg Laureate Forum?").

To explore topics of relevance to both mathematicians and computer scientists, the HLF organized a series of workshops mentored by the laureates. Here, we report on the outcomes of the workshop "The Moving Frontier between Mathematics and Computer Science" organized by Edmon Begoli and Vincent Schlegel and mentored by Sir Michael Atiyah, recipient of both the Fields Medal and Abel Prize in mathematics (see Figure 1).





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Figure 1. The workshop.

Figure 2. Group photo of workshop participants with Sir Michael Atiyah.

DISCUSSION

Computer scientists at the cutting edge of developments in data science, machine learning, and algorithm verification find themselves in need of an increasingly varied mathematical toolbox. On the other hand, computer science has gradually been leaving an indelible mark on modern mathematics, most notably through the computer-assisted proof of the Four Color Theorem. With these developments in mind, HLF participants discussed:

Frontier. How the frontier between mathematics and computer science has shifted in the past few decades;

Enrichment. The ways these distinct disciplines can be enriched by each other; and

Challenges. The most significant challenges at the intersection of mathematics and computer science.

Following are the main observations from that discussion.

A SHIFT IN AUTONOMY

The main theme of the HLF was the fast-developing field of artificial intelligence. A major trend in current computer science research is the development and application of machine learning algorithms, processes that can learn from and make predictions about data. While the basic architectures of machine learning algorithms are well understood by computer scientists, they have not been deeply understood from a mathematical perspective. There is a crucial lack of mathematical understanding about the formal properties of machine learning algorithms as a whole. Indeed, machine learning methods in general, particularly in the sub-area of deep learning, are most often verified empirically due to the lack of an underlying theoretical framework.

The impressive empirical success of machine learning methods is the main factor behind their popularity in real-world applications. Yet their increasingly widespread implementation is fundamentally at odds with the lack of theoretical understanding and leads to an autonomy problem. Indeed, the better our machine learning algorithms become, the more we are able to rely on them as computational black boxes. In this way, the algorithms themselves attain autonomy since we, as users, are unable to account for their predictions and inferences. This lack of accountability can have significant consequences when machine learning is used in sensitive contexts (such as automated driving, automated trading, and information filtering). In order to address them, it is imperative that we gain a deeper mathematical understanding of machine learning algorithms. We need much more theoretical input to understand how to design responsible AI machines and human-in-the-loop system architectures.

RESURGENCE OF MATHEMATICS IN COMPUTER SCIENCE

Mathematical fluency is increasingly important in modern computer science. With the emergence of "big data" science and machine learning as go-to methods for a host of commercial applications, the demand for computer scientists and programmers with the requisite mathematical training is also increasing. The current theoretical shortcomings in these areas provide mathematicians with a slew of engaging new problems. Just as computer scientists are increasingly reliant on mathematicians, mathematicians can look to computer scientists for inspiration and new research directions.

UTILITY AND UBIQUITY OF PROGRAMMING

The rapid pace of computerization in recent years has led everyone to recognize the need for a computer-literate and computer-savvy populace. As a result, courses in programming languages are finding their way worldwide into many high school curricula. Not even mathematicians are immune from this trend, as computational methods become increasingly popular in the field. Indeed, beyond the traditionally computationally intensive areas of mathematics (such as statistics and applied fluid dynamics), the development of proof assistant programs provides mathematicians with a new way to develop and verify proofs via human-computer collaboration. These developments have culminated in the UniMath project (https://github.com/UniMath/UniMath), a joint effort by mathematicians and computer scientists to formalize foundations for mathematics in a way that is computer-verifiable.

Independent of proof assistants, computer technology is finding its way into even the purest areas of mathematics. A particularly good example is the field of algebraic geometry, which has recently seen a shift toward using computational techniques to solve certain problems. For instance, computer algorithms play a vital role in studying and calculating invariants of varieties (such as Hilbert polynomials). In this way, programming skills are quickly becoming indispensable to mathematicians in a variety of fields.

RECOMMENDATIONS

Mathematics and computer science are often viewed as two independent and disjoint disciplines. Our discussion at the HLF revealed a great deal of evidence to the contrary. There are a great many challenging problems that belong to both computer science and mathematics, and tackling them will require bright minds with knowledge from both areas.

While numerous universities around the world offer joint degrees in mathematics and computer science, these areas are often run independently and in separate departments. In order to foster interdisciplinary collaborations, computer scientists ought to learn much more of the core mathematical subjects than they currently do. This will go a long way toward giving them a deeper understanding of the mathematical foundations of their own discipline, and also give them the ability to communicate much more effectively with mathematicians about difficult theoretical problems in their field.

There are many ways mathematicians can benefit from a deeper exposure to computer science. In order to maximize the benefit from computer science as a source of interesting research problems, as well as computational problem-solving techniques, we recommend mathematicians receive a comprehensive education in the basic areas of computer science. Topics might include fundamental programming skills, awareness of good programming practice, analysis design, and machine learning (see Figure 2).

CONCLUSION

The frontier between computer science and mathematics is moving, and will continue to move. This is a natural byproduct of the evolution of these two closely related fields. There are numerous exciting theoretical challenges at the intersection of mathematics and computer science. Yet there are also many practical aspects that require attention and increased awareness of the other discipline that will undoubtedly make the research careers of mathematicians and computer scientists more interesting and productive. Both disciplines have much to learn from each other. Mathematicians will benefit from computational methods, tools, and resources that aid in theoretical research. Computer scientists need the rigor and the theoretical insights that mathematics has to offer. In both cases, the future of the two fields will bring exciting developments within the separate disciplines, but especially at the ever-changing frontier between them.

Excited About the Heidelberg Laureate Forum?

Are you interested in participating in the HLF and meeting your heroes in computer science and mathematics? Anyone, from undergraduate students to Ph.D. candidates and postdocs, can apply to attend. The HLF's Scientific Committee will make the final selection based on your CV, scientific achievements, letters of recommendation, and motivation letter. Please see http://www.heidelberg-laureate-forum.org/ for more information on how to apply.

Biographies

Edmon Begoli, Ph.D., is chief data architect and senior scientist in the Computational Sciences and Engineering Division at the Oak Ridge National Laboratory. After years of practice as a software architect and software engineer, he transitioned into applied computer science research. His current area of research is high-performance data analysis platforms for life and medical sciences. He holds undergraduate, graduate, and doctoral degrees in computer science from the University of Colorado-Boulder and the University of Tennessee-Knoxville.

Vincent Schlegel is a Ph.D. candidate in pure mathematics at the Universität Zürich in Switzerland. He holds an M.Phil. in pure mathematics from the University of Adelaide, Australia, and is working to understand the higher mathematical structure of string theory and quantum physics. In his spare time, he enjoys reading, debating economics and philosophy, and skiing and hiking in the Swiss Alps.

Sir Michael Atiyah is a British mathematician best known for his unifying work on K-theory and the Atiyah-Singer Index Theorem. He was awarded the Fields Medal in 1966 and, together with Isadore Singer, received the 2004 Abel Prize. Sir Michael's work draws on many different areas of mathematics and provides a powerful tool for building bridges between mathematics and theoretical physics. He received a knighthood in 1983, the Order of Merit in 1992, and served as president of the Royal Society from 1990 to 1995.

Praise Adeyemo, Ph.D., holds a Fields-Perimeter Africa Postdoctoral Fellowship at the Fields Institute, University of Toronto, Canada. He is a faculty member in the Department of Mathematics at the University of Ibadan, Nigeria. His research is in the broad area of algebraic geometry, with specific interest in the cohomology theories of flag manifolds.

Tim Baarslag is a researcher at the Centrum Wiskunde & Informatica, the national research institute for Mathematics and Computer Science in the Netherlands. He is a visiting assistant professor at Nagoya University of Technology, Nagoya, Japan, and visiting fellow at the University of Southampton, Southampton, England. He obtained his Ph.D. from Delft University of Technology in 2014 on the topic of joint decision making. His current research focus is on coordination and cooperation, negotiation, preference elicitation, and decision making under uncertainty.