Exercise 1 (Max of Gaussians)
Let $X_1, \ldots, X_n$ be i.i.d. $N(0,1)$. Prove that

$$\mathbb{E}[\max_{i \in [n]} |X_i|] \geq c\sqrt{\log n}$$

for a universal constant $c > 0$.

Exercise 2 (Gaussian Mass and Origin Containment)
Let $K \subseteq \mathbb{R}^n$ be a convex body such that $\gamma_n(K) \geq 1/2$. Show that $0 \in K$.
(Hint: Separator theorem.)

Exercise 3 (Discrepancy Lower Bound)
Let $H \in \mathbb{R}^{n \times n}$, $n = 2^k$, denote the Hadamard matrix. Indexing the rows and columns of H by vectors in $\{0,1\}^k$, one may define H so that $H_{x,y} = (-1)^{\langle x,y \rangle}$, $\forall x, y \in \{0,1\}^k$. Prove that

$$\min_{z \in \{-1,1\}^n} \|Hz\|_\infty \geq \sqrt{n}.$$

(Hint: Show that columns of H are orthogonal and then relate ℓ_2 and ℓ_∞ norm.)

Exercise 4 (Marginals of Convex Bodies)
Let $K \subseteq \mathbb{R}^n$ be a convex body. Let $f : \mathbb{R} \to \mathbb{R}_+$ be defined by $f(t) = \mathrm{vol}_{n-1}(\{x \in \mathbb{R}^{n-1} : (t,x) \in K\})$.

1. Prove that $f^{1/(n-1)}$ is a concave function.
   (Hint: Brunn-Minkowski.)
2. Let $S = \{t \in \mathbb{R} : \exists x \in \mathbb{R}^{n-1}, (t,x) \in K\}$ and $b = \max\{f(t) : t \in S\}$. Show that

   $$\mathrm{vol}_1(S)b/n \leq \mathrm{vol}_n(K) \geq \mathrm{vol}_1(S)b.$$

   (Hint: Base $\times$ Height.)
3. Assume that K is symmetric and let $M = \sup\{x_1 : x \in K\}$. Show that

   $$M/4 \leq \mathbb{E}_{X \sim \mathrm{unif}(K)}[|X_1|^n]^{1/n} \leq M$$

   where $\mathrm{unif}(K)$ denotes the uniform measure on K.
   (Hint: Lower bound $\Pr[|X_1| \geq M/2]$ and use Markov's inequality.)