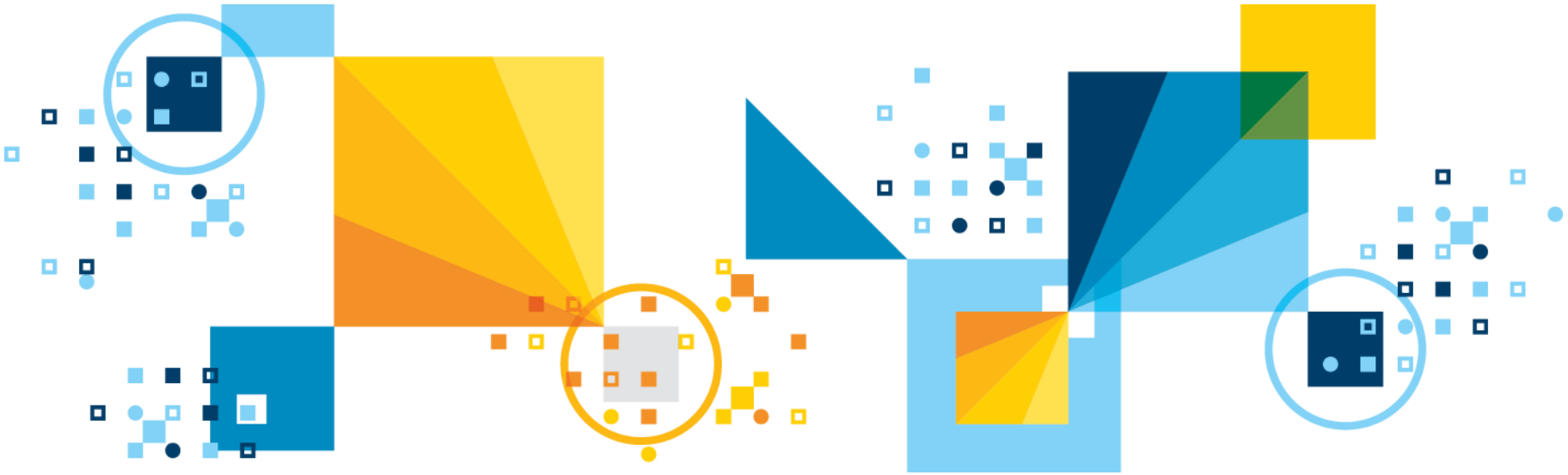
Andrea Tramontani
CPLEX Optimization, IBM
CWI, Amsterdam, June 12, 2018

Heuristics in Commercial MIP Solvers

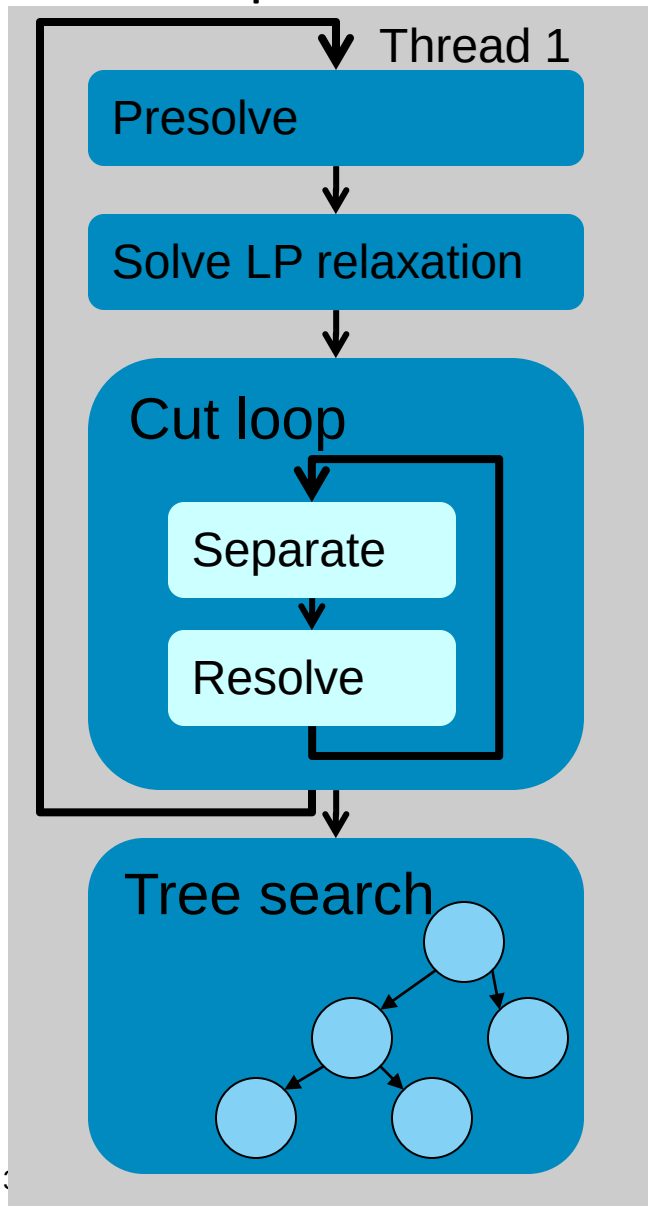
Part I (Heuristics in IBM CPLEX)



Agenda

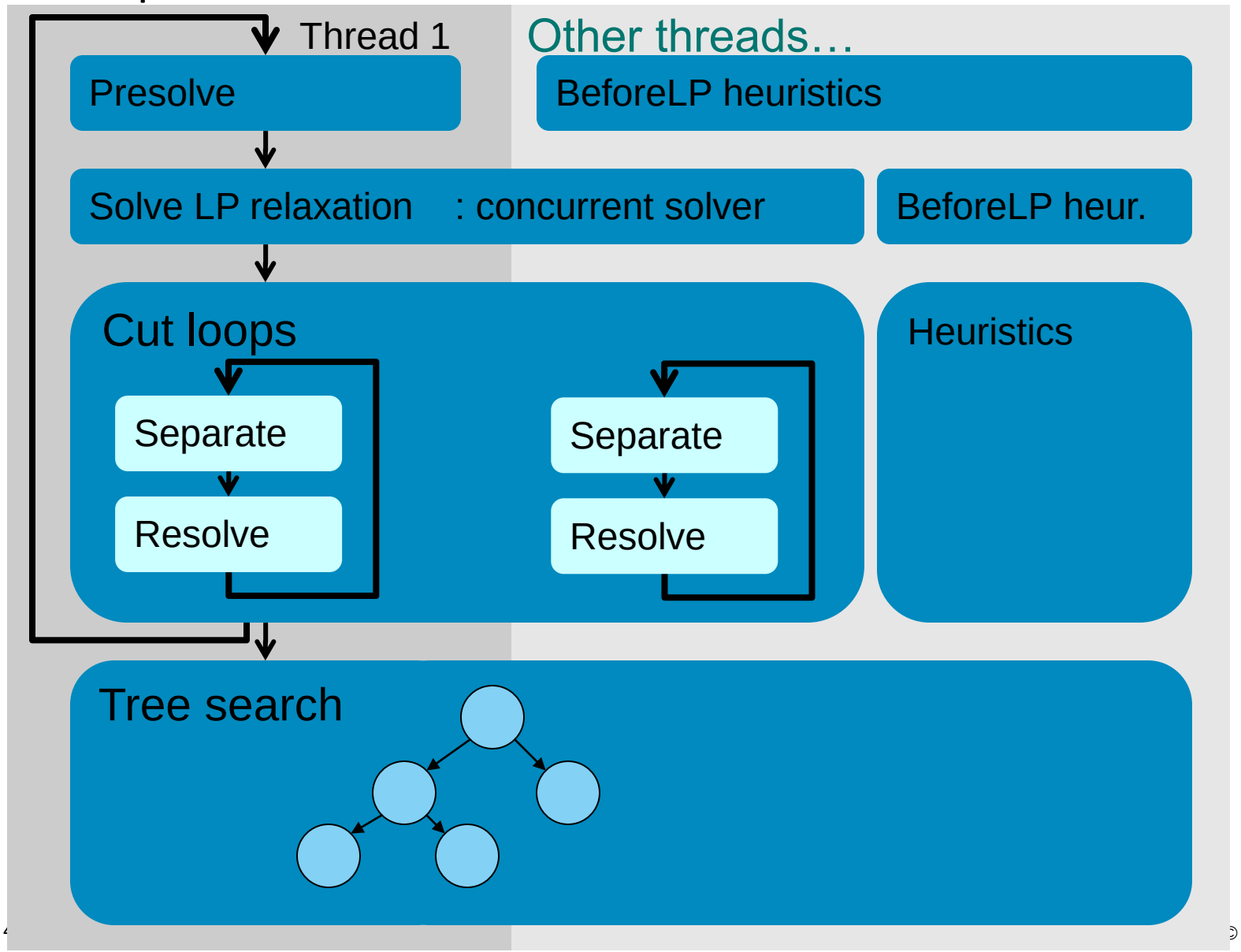
- CPLEX Branch-and-Bound (B&B)
- Primal heuristics in CPLEX
  - Overview and classification
  - Some examples
    - Diving heuristics
    - SubMIP heuristics
    - Tabu search heuristic for 0-1 IPs
  - Heuristic manager
- Performance analysis
  - Performance impact of heuristics in solving problems to optimality

CPLEX sequential MIP solver



$$(MIP) \quad Minimize \quad z = c^T x$$
$$Subject\ to \quad Ax = b$$
$$l \le x \le u$$
$$\text{some or all } x_j \text{ integer}$$

CPLEX parallel MIP Solver



Primal heuristics in one slide

- What are they?
  - **Incomplete methods** that simply look for feasible solutions
  - No guarantees of any kind
  - Not always predictable (short) running time
    - Must run within a **controlled environment**

- Why do we need them?
  - Prove feasibility of the model
  - **Speed up search**
    - Primal bound needed for pruning and reduced cost fixing
  - Sometimes good enough for **practical purposes**
    - Often optimization is stopped when gap is small enough (not yet 0)
    - Fundamental when problems can't be solved to optimality (e.g., optimality proof is unpractical or strict work limits are imposed)

Starting heuristics vs. improving heuristics

- Starting heuristics
  - Do not need any feasible solution available
    - But can implicitly exploit an UB if available (e.g., diving heuristics)
  - Relevant examples
    - Before LP heuristics
    - Rounding heuristics
    - Tabu search for 0-1 IPs (inspired to WalkSat paradigm)
    - **Diving heuristics**
    - Feasibility pump (Fischetti et al., 2003)

- Improving heuristics
  - Explore neighborhoods of feasible solutions to improve on the incumbent solution
  - Relevant examples
    - Tabu search for 0-1 IPs (inspired to WalkSat paradigm)
    - **RINS (Danna et al., 2005)**
    - Genetic Algorithm (Rothberg, 2007)
    - Local branching (Fischetti and Lodi, 2003)

Before LP heuristics vs. after LP heuristics

- Before LP heuristics
  - Run sequentially before solving the first LP relaxation or concurrently to the first LP solve
  - **Cannot take advantage from the knowledge of an LP solution**
  - Almost all of them are starting heuristics
  - Sequential heuristics
    - Must be cheap (**very strict work limit**)
  - Concurrent heuristics
    - Do not need strict work limit (**killed anyway when the first LP solve is done**)
  - Relevant examples
    - Fix and propagate (no LP solves)
    - Simple local search (no LP solves)
    - Tabu search for 0-1 IPs (inspired to WalkSat paradigm)
    - Fix and solve subMIP
    - Zero-objective subMIP

Before LP heuristics vs. after LP heuristics (Cont. d)

- After LP heuristics
  - Can take advantage from the **knowledge of an LP solution**
  - Allowed to be **more expensive**
    - Can solve one or more LP of same size of the full LP relaxation
  - Relevant examples
    - Rounding heuristics
    - Tabu search for 0-1 IPs
    - Diving heuristics
    - RINS (Danna et al., 2005)
    - Genetic Algorithm
    - Zero-objective subMIP

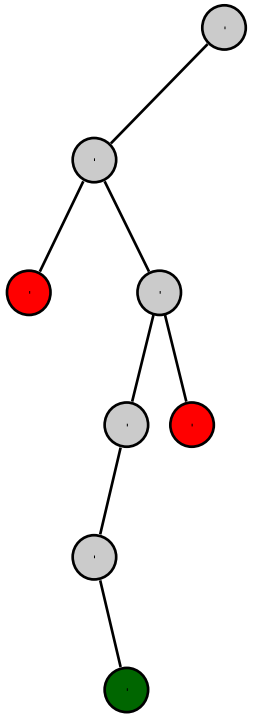
General purpose vs. special purpose heuristics

- General purpose heuristics
  - Do not need the problem to have any specific structure

- Special purpose heuristics
  - **Tabu search for 0-1 IPs**
  - Specialized heuristics for **set covering/partitioning** problems
  - Specialized B&B for problems with a big **set packing** components
    - No LP solve
    - Branching, node selection, constraint propagation entirely based on clique table
    - Inspired to Rapid Learning techniques (Berthold et al., 2010)

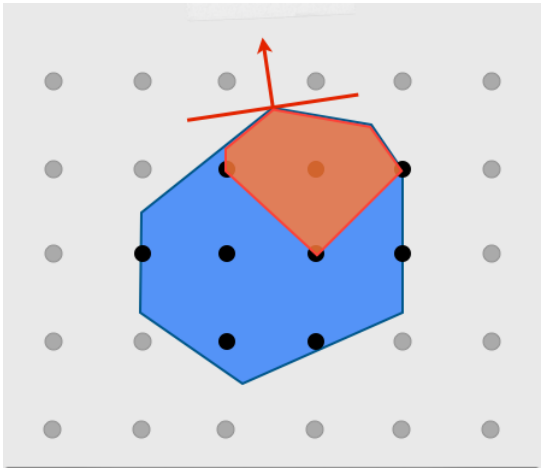
Heuristic catalog – Diving heuristics

- Given the fractional solution *x* of a certain B&B node
  - **Simulate Depth First Search** (DFS) with a special "branching" strategy:
    - Change variable bound(s)
    - Propagate constraints
    - Resolve LP
  - **Different variants** (from cheap to expensive) implemented:
    - Alternative strategies/scores to select the variable bound(s) to change
    - Different frequencies to resolve the LPs after propagation
    - Different level of backtracking allowed
- Very cheap version (no LP solve) applied **also as before LP heuristic**
- Heavily applied at the **root node** and in the **search tree**



Heuristic catalog – SubMIP heuristics

- RINS (Danna et al., 2005)
  - Explore a neighborhood given by the incumbent solution and the current fractional solution
  - **Different variants** implemented (from cheap to expensive)
  - Applied at the **root node** and in the **search tree**
- Genetic Algorithm (GA) (Rothberg, 2007)
  - Generate (several) partial solution vectors by applying crossover and mutation operators to (some of) the feasible solutions available in the pool
  - For each partial solution vector *v*
    - Fix variables to the values of *v*
    - Solve the corresponding subMIP
  - Very expensive
    - **Sporadically applied** during tree search in default setting
    - Bulk of **solution polishing** feature



- Zero-objective subMIP
  - Remove objective function and solve subMIP
  - Can be expensive, but also applied as beforeLP heuristic (with proper work limits)

Heuristic catalog – Tabu Search (TS) heuristic for 0-1 IPs

```
Guess an infeasible 0-1 vector x;
Compute score s(x) that measures infeasibility of x;
while (x not feasible && work limit not reached) {
  Select variable x_j to flip in x
     to obtain x' with smallest possible s(x');
  Flip x_j and mark the flip as tabu;
  Update x and s(x);
}
```

- Different types of **randomization** applied:
  - Tabu tenure of each flip/move
  - Subset of flips/moves that are evaluated at every iterations
- Different ways to select the initial x vector:
  - A random vector
  - The incumbent solution vector (when used as improving heuristic)
  - ~~Rounding of solution of LP relaxation~~ (sounds natural, but **not done**)
- Called as **beforeLP heuristic** and at the **root node**

Heuristic manager

- CPLEX implements around 50 primal heuristics
  - Different variants of the same heuristic are considered as different ones

- The heuristic manager
  - Keeps **statistics** for every heuristic
    - #calls, success rate, deterministic time spent, …
  - Decides
    - **Frequency of call** for every heuristic
    - **Work limit** for every heuristic call
  - Main goals:
    - Ensure some diversification
    - Favor heuristics that appear to be more effective
    - Make sure that, in the long run, the overall time spent in all heuristics is at most a certain fraction of the whole running time

Performance Analysis

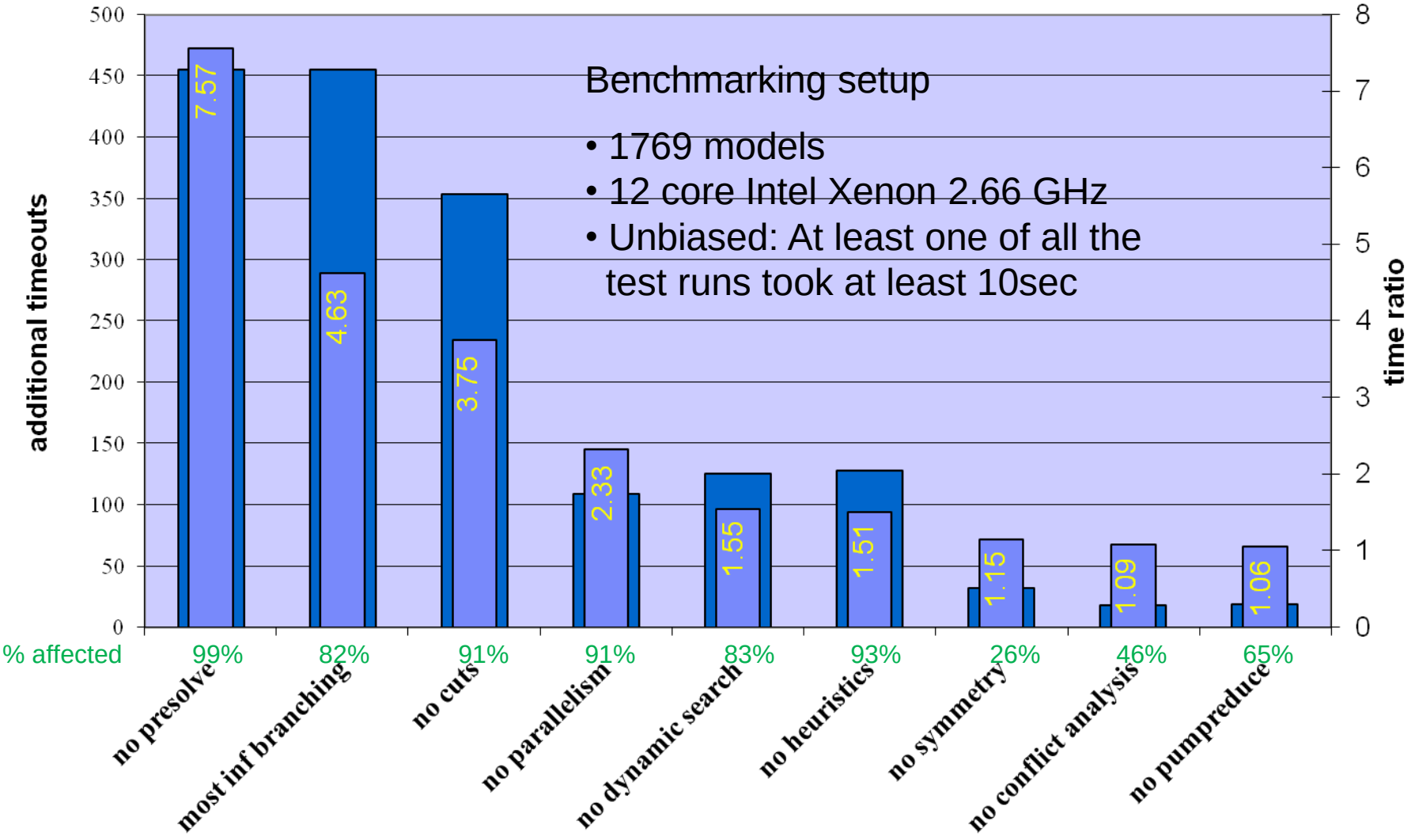
Main building blocks: Measuring performance impact

- How important is each component?
  Compare runs with feature turned on and off
  - Solution time degradation (geometric mean)
  - # of solved models
    - Essential or just speedup?
  - Number of affected models
    - General of problem specific?

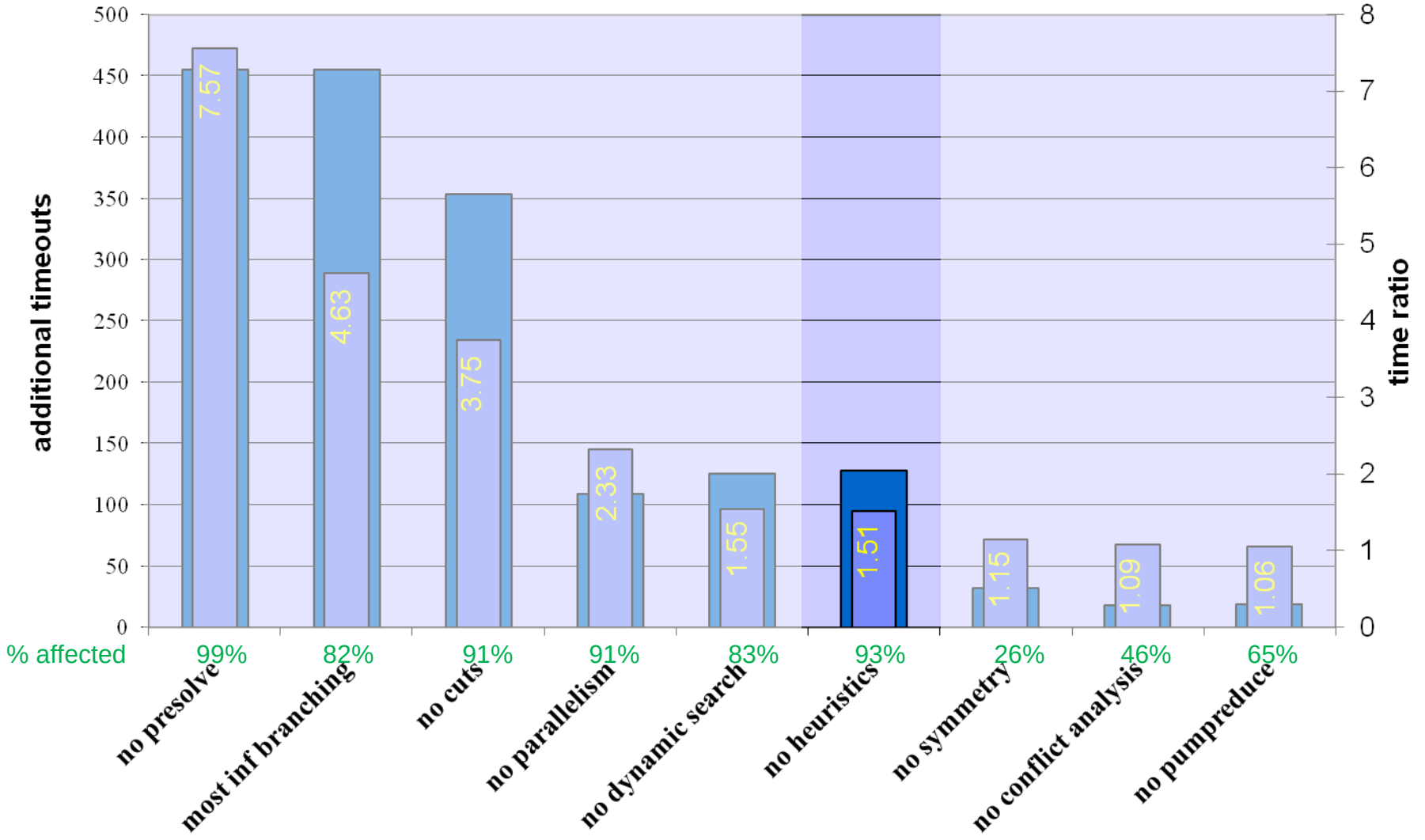
- Experiments conducted with CPLEX 12.5.0 (2012)
  - Several features not available yet, e.g.,
    - L&P cuts, Parallel cut loop, TS heuristic for 0-1 IPs, other special purpose heuristics, …

- More detailed analysis in:
  T. Achterberg and R. Wunderling, “Mixed Integer Programming: Analyzing 12 Years of Progress”, in: Jünger and Reinelt (eds.) Facets of Combinatorial Optimization, Festschrift for Martin Grötschel, pp.449-481, Springer, Berlin-Heidelberg (2013)

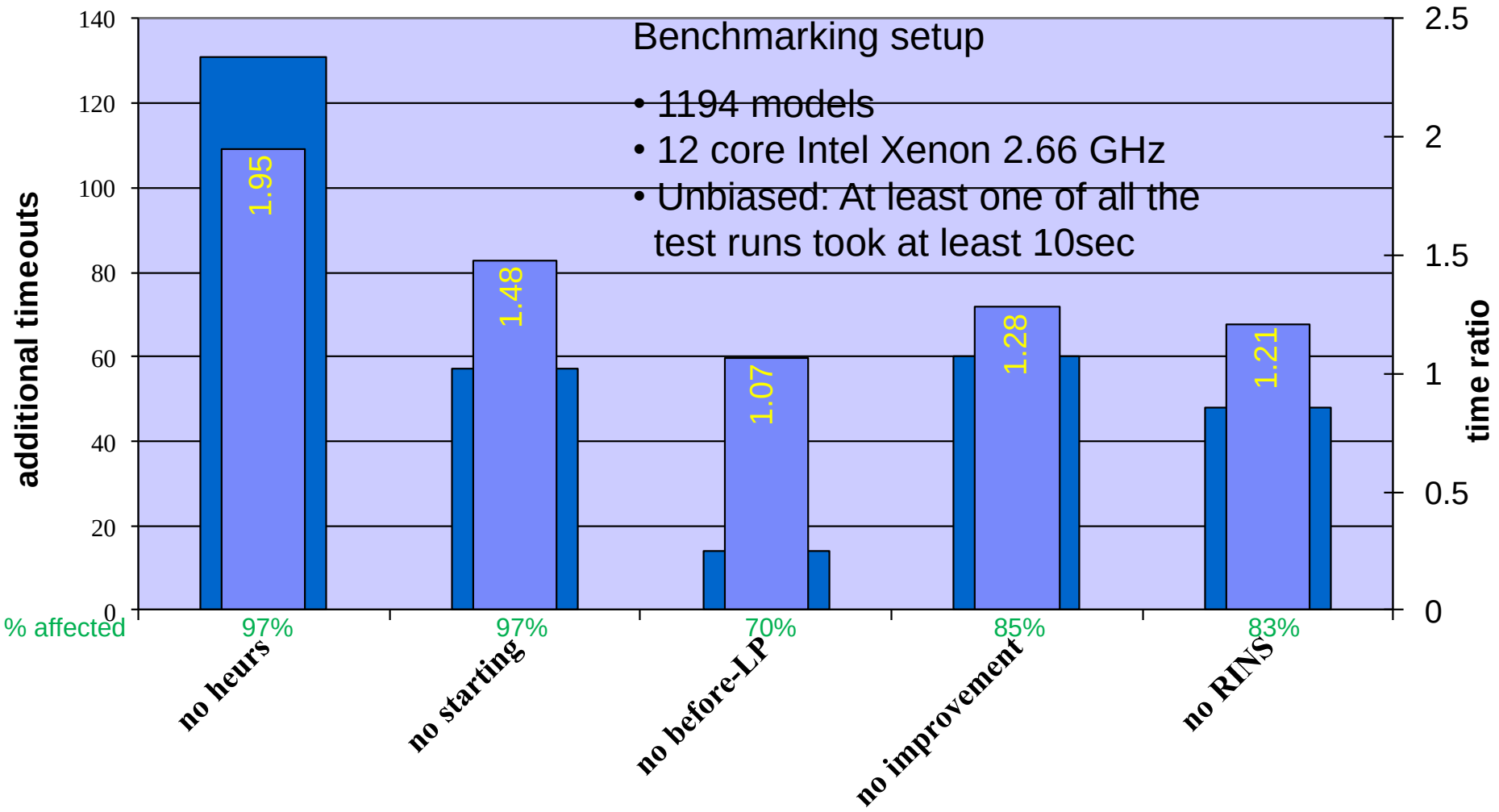
Component Impact CPLEX 12.5.0 – Summary



Component Impact CPLEX 12.5.0 – Summary



Component Impact CPLEX 12.5.0 – Primal heuristics



Legal Disclaimer