Heuristics in Commercial MIP Solvers
Part I (Heuristics in IBM CPLEX)
Agenda

- CPLEX Branch-and-Bound (B&B)

- Primal heuristics in CPLEX
  - Overview and classification
  - Some examples
    - Diving heuristics
    - SubMIP heuristics
    - Tabu search heuristic for 0-1 IPs
  - Heuristic manager

- Performance analysis
  - Performance impact of heuristics in solving problems to optimality
(MIP) Minimize $z = c^T x$

Subject to $Ax = b$

$l \leq x \leq u$

some or all $x_i$ integer
**CPLEX parallel MIP Solver**

- **Thread 1**
  - Presolve
  - Solve LP relaxation: concurrent solver
  - Cut loops
    - Separate
    - Resolve
  - Heuristics
    - Separate
    - Resolve
  - Tree search

- **Other threads...**
  - BeforeLP heuristics
Primal heuristics in one slide

- What are they?
  - **Incomplete methods** that simply look for feasible solutions
  - No guarantees of any kind
  - Not always predictable (short) running time
    - Must run within a **controlled environment**

- Why do we need them?
  - Prove feasibility of the model
  - **Speed up search**
    - Primal bound needed for pruning and reduced cost fixing
  - Sometimes good enough for **practical purposes**
    - Often optimization is stopped when gap is small enough (not yet 0)
    - Fundamental when problems can’t be solved to optimality (e.g., optimality proof is unpractical or strict work limits are imposed)
Starting heuristics vs. improving heuristics

- **Starting heuristics**
  - Do not need any feasible solution available
    - But can implicitly exploit an UB if available (e.g., diving heuristics)
  - Relevant examples
    - Before LP heuristics
    - Rounding heuristics
    - Tabu search for 0-1 IPs (inspired to WalkSat paradigm)
    - **Diving heuristics**
    - Feasibility pump (Fischetti et al., 2003)

- **Improving heuristics**
  - Explore neighborhoods of feasible solutions to improve on the incumbent solution
  - Relevant examples
    - Tabu search for 0-1 IPs (inspired to WalkSat paradigm)
    - **RINS (Danna et al., 2005)**
    - Genetic Algorithm (Rothberg, 2007)
    - Local branching (Fischetti and Lodi, 2003)
Before LP heuristics vs. after LP heuristics

- **Before LP heuristics**
  - Run sequentially before solving the first LP relaxation or concurrently to the first LP solve
  - **Cannot take advantage from the knowledge of an LP solution**
  - Almost all of them are starting heuristics
  - Sequential heuristics
    - Must be cheap (**very strict work limit**)
  - Concurrent heuristics
    - Do not need strict work limit (**killed anyway when the first LP solve is done**)
  - Relevant examples
    - Fix and propagate (no LP solves)
    - Simple local search (no LP solves)
    - Tabu search for 0-1 IPs (inspired to WalkSat paradigm)
    - Fix and solve subMIP
    - Zero-objective subMIP
Before LP heuristics vs. after LP heuristics (Cont. d)

- After LP heuristics
  - Can take advantage from the **knowledge of an LP solution**
  - Allowed to be more expensive
    - Can solve one or more LP of same size of the full LP relaxation
  - Relevant examples
    - Rounding heuristics
    - Tabu search for 0-1 IPs
    - Diving heuristics
    - RINS (Danna et al., 2005)
    - Genetic Algorithm
    - Zero-objective subMIP
General purpose vs. special purpose heuristics

- General purpose heuristics
  - Do not need the problem to have any specific structure

- Special purpose heuristics
  - Tabu search for 0-1 IPs
  - Specialized heuristics for set covering/partitioning problems
  - Specialized B&B for problems with a big set packing components
    - No LP solve
    - Branching, node selection, constraint propagation entirely based on clique table
    - Inspired to Rapid Learning techniques (Berthold et al., 2010)
Heuristic catalog – Diving heuristics

- Given the fractional solution $x$ of a certain B&B node
  - **Simulate Depth First Search** (DFS) with a special “branching” strategy:
    - Change variable bound(s)
    - Propagate constraints
    - Resolve LP
  - **Different variants** (from cheap to expensive) implemented:
    - Alternative strategies/scores to select the variable bound(s) to change
    - Different frequencies to resolve the LPs after propagation
    - Different level of backtracking allowed

- Very cheap version (no LP solve) applied **also as before LP heuristic**

- Heavily applied at the **root node** and in the **search tree**
Heuristic catalog – SubMIP heuristics

- **RINS (Danna et al., 2005)**
  - Explore a neighborhood given by the incumbent solution and the current fractional solution
  - **Different variants** implemented (from cheap to expensive)
  - Applied at the root node and in the search tree

- **Genetic Algorithm (GA) (Rothberg, 2007)**
  - Generate (several) partial solution vectors by applying crossover and mutation operators to (some of) the feasible solutions available in the pool
  - For each partial solution vector $v$
    - Fix variables to the values of $v$
    - Solve the corresponding subMIP
  - Very expensive
    - **Sporadically applied** during tree search in default setting
    - Bulk of solution polishing feature

- **Zero-objective subMIP**
  - Remove objective function and solve subMIP
  - Can be expensive, but also applied as beforeLP heuristic (with proper work limits)
Heuristic catalog – Tabu Search (TS) heuristic for 0-1 IPs

Guess an infeasible 0-1 vector $x$;
Compute score $s(x)$ that measures infeasibility of $x$;
while (x not feasible && work limit not reached) {
    Select variable $x_j$ to flip in $x$
    to obtain $x'$ with smallest possible $s(x')$;
    Flip $x_j$ and mark the flip as tabu;
    Update $x$ and $s(x)$;
}

- Different types of **randomization** applied:
  - Tabu tenure of each flip/move
  - Subset of flips/moves that are evaluated at every iterations
- Different ways to select the initial $x$ vector:
  - A random vector
  - The incumbent solution vector (when used as improving heuristic)
  - Rounding of solution of LP relaxation (sounds natural, but **not done**)
- Called as **beforeLP heuristic** and at the **root node**
Heuristic manager

- CPLEX implements around 50 primal heuristics
  - Different variants of the same heuristic are considered as different ones

- The heuristic manager
  - Keeps **statistics** for every heuristic
    - #calls, success rate, deterministic time spent, ...
  - Decides
    - **Frequency of call** for every heuristic
    - **Work limit** for every heuristic call
  - Main goals:
    - Ensure some diversification
    - Favor heuristics that appear to be more effective
    - Make sure that, in the long run, the overall time spent in all heuristics is at most a certain fraction of the whole running time
Performance Analysis
Main building blocks: Measuring performance impact

- How important is each component?
  Compare runs with feature turned on and off
  - Solution time degradation (geometric mean)
  - # of solved models
    - Essential or just speedup?
  - Number of affected models
    - General of problem specific?

- Experiments conducted with CPLEX 12.5.0 (2012)
  - Several features not available yet, e.g.,
    - L&P cuts, Parallel cut loop, TS heuristic for 0-1 IPs, other special purpose heuristics, ...

- More detailed analysis in:
Component Impact CPLEX 12.5.0 – Summary

Benchmarking setup

- 1769 models
- 12 core Intel Xenon 2.66 GHz
- Unbiased: At least one of all the test runs took at least 10sec

<table>
<thead>
<tr>
<th>% affected</th>
<th>Time Ratio</th>
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<tbody>
<tr>
<td>no presolve</td>
<td>7.57</td>
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<tr>
<td>most inf branching</td>
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<td>no cuts</td>
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<td>no conflict analysis</td>
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<tr>
<td>no pmpreduce</td>
<td>1.08</td>
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</tbody>
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Component Impact CPLEX 12.5.0 – Summary

- No presolve: 7.57
- Most inf branching: 4.63
- No cuts: 3.75
- No parallelism: 2.33
- No dynamic search: 1.55
- No heuristics: 1.51
- No symmetry: 1.15
- No conflict analysis: 1.08
- No pmpreduce: 1.08

% affected:
- 99%
- 82%
- 91%
- 91%
- 83%
- 93%
- 26%
- 46%
- 65%
Component Impact CPLEX 12.5.0 – Primal heuristics

Benchmarking setup

- 1194 models
- 12 core Intel Xenon 2.66 GHz
- Unbiased: At least one of all the test runs took at least 10sec

% affected

- 97% no hours
- 97% no starting
- 70% no before-LP
- 85% no improvement
- 83% no RINS

Additional timeouts

Time ratio

- 1.95
- 1.48
- 1.07
- 1.28
- 1.21

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