Dependence of the transition from Townsend to glow discharge on secondary emission

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In a recent paper, Šijačić and Ebert [Phys. Rev. E. **66**, 006410 (2002)] systematically studied the transition from Townsend to glow discharge, referring to older work by von Engel and M. Steenbeck [*Elektrische Gasentladungen. Ihre Physik und Technik* (Springer, Berlin 1934), Vol. II] up to Raizer [*Gas Discharge Physics* (Springer, Berlin, 1991)]. Šijačić and Ebert stated that this transition strongly depends on secondary emission γ from the cathode. We show here that the earlier results of von Engel and Raizer on the small current expansion about the Townsend limit actually are the limit of small γ of the Šijačić and Ebert expression, and that for larger γ the old and the Šijačić and Ebert new results vary by no more than a factor of 2. We discuss the γ dependence of the transition, which is rather strong for short gaps.

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In a recent article [1], the transition from Townsend to glow discharge was reinvestigated with analytical and numerical means. On the analytical side, a systematic, small current expansion about the Townsend limit was performed and it was stated:

"The result agrees qualitatively with the one given by Raizer [2] and Engel and Steenbeck [3]. In particular, the leading order correction is also of order $\alpha''(j/\mu)^2$. However, the explicit coefficient of j^2 differs: while the coefficient in [2,3] does not depend on γ at all, we find that the dependence on γ is essential, as the plot of F in Fig. 1 of [1] clearly indicates. In fact, within the relevant range of $10^{-6} \leqslant \gamma \leqslant 10^0$, this coefficient varies by almost four orders of magnitude. We remark that it indeed would be quite a surprising mathematical result if the Townsend limit itself would depend on γ, but the small current expansion about it would not."

Here, we remark that while the systematic calculation in Ref. [1] was correct, the interpretation and comparison to earlier work requires some correction.

To be precise, the model treated in [1–3] and by many other authors is a one-dimensional time independent Townsend or glow discharge characterized by the classical equations for electron and ion particle current $J_{e,+}$ and electric field E, given by

$$\partial_x J_e = |J_e|\overline{\alpha}(|E|), \quad \partial_x J_+ = |J_e|\overline{\alpha}(|E|), \tag{1}$$

$$\partial_x E = \frac{\mathrm{e}}{\epsilon_0}(n_+ - n_e), \tag{2}$$

$$J_e = -n_e\mu_e E, \quad J_+ = n_+\mu_+ E. \tag{3}$$

Impact ionization in the bulk of the discharge is given by the Townsend approximation

$$\overline{\alpha}(|E|) = \alpha_0 \, e^{-E_0/|E|}. \tag{4}$$

(In [1], the generalized case $\overline{\alpha}(|E|) = \alpha_0 \exp(-E_0/|E|)^s$ was treated.) Boundary conditions at the anode $(x=0)$ and for secondary emission at the cathode $(x=d)$ are

$$J_+(0) = 0, \quad |J_e(d)| = \gamma\, |J_+(d)|. \tag{5}$$

The discharge is characterized by the potential U and total electric current J, as

$$U = \int_0^d dx\, E(x), \quad J = \mathrm{e}(n_+\mu_+ + n_e\mu_e)E. \tag{6}$$

It is useful to introduce dimensionless voltage and current, as

$$u = \frac{U}{E_0/\alpha_0}, \quad \overline{j} = \frac{J}{\epsilon_0\alpha_0 E_0\ \mu_+ E_0}, \tag{7}$$

where $\overline{j} = j/\mu$ with the definition of j from [1]. It should be noted that only bulk gas parameters have been used as units; therefore, the dimensionless u and $\overline{j}$ are independent of γ.

Further dimensional analysis yields that the current-voltage characteristics $u=u(\overline{j})$ can depend on three parameters only; namely, on the dimensionless gap length $L=\alpha_0 d$, on the coefficient γ of secondary emission, and on the mobility ratio $\mu=\mu_+/\mu_e$. In practice, the dependence on the small parameter μ is almost negligibly weak [1]; therefore, $u=u(\overline{j}, L, \gamma)$. Here, the dimensionless gap length L is related to pd through $L=Apd$ as long as the coefficient α_0 is related to pressure as $\alpha_0 = Ap$.

How strongly does the characteristics $u=u(\overline{j}, L, \gamma)$ depend on γ? In [1], Šijačić and Ebert (SE) calculated the whole Townsend-to-glow regime numerically and derived, by expanding systematically in powers of current $\overline{j}$ about the Townsend limit, that

$$u = u_T - A_{SE}\, \overline{j}^{\,2} + O(\overline{j}^{\,3}), \tag{8}$$

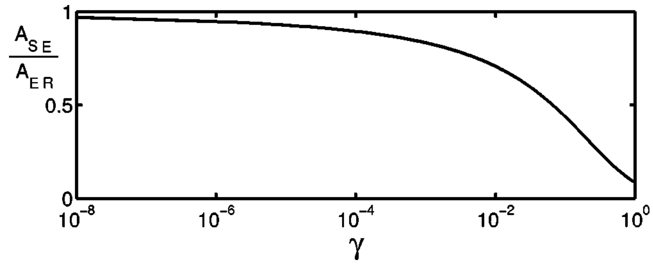

FIG. 1. The ratio A_{SE}/A_{ER} of the small current expansions by Šijačić and Ebert and by von Engel and Raizer as a function of γ.

$$A_{SE} = \frac{\mathcal{E}_T}{2}\frac{\alpha''}{\alpha'}\frac{F(\gamma,\mu)}{(\alpha\mathcal{E}_T)^3}, \tag{9}$$

$$\alpha(\mathcal{E}_T) = e^{-1/|\mathcal{E}_T|}, \tag{10}$$

which gave an excellent fit to the numerical solutions. Here,

$$F(\gamma,\mu) = \frac{L_\gamma^3}{12} + (1+\mu)(2 - L_\gamma - 2e^{-L_\gamma} - L_\gamma e^{-L_\gamma}) + (1+\mu)^2\left(\frac{1-e^{-2L_\gamma}}{2} - \frac{(1-e^{-L_\gamma})^2}{L_\gamma}\right),$$

$$L_\gamma = \ln\frac{1+\gamma}{\gamma}, \tag{11}$$

and $\mathcal{E}_T$ and u_T are field and potential in the Townsend limit of "vanishing" current, i.e., with breakdown values

$$\mathcal{E}_T = \frac{1}{\ln(L/L_\gamma)}, \quad u_T = \frac{L}{\ln(L/L_\gamma)}. \tag{12}$$

The minimal potential u_T is $L_\gamma e^1$, it is attained for gap length $L=L_\gamma e^1$ on the Paschen curve $u_T=u_T(L)$ [1–3].

In [1], it was argued that the coefficient A_{SE} in (8) strongly depends on γ due to the factor $F(\gamma,\mu)$ in (9). This factor $F(\gamma,\mu)$ indeed strongly depends on γ, for small γ actually in leading order like $L_\gamma^3/12$. (Note that there is a discrepancy between equation (50) in [1] for $F(\gamma,\mu)$ which is reproduced as Eq. (11) in the present paper, and the plot in Fig. 1 of [1] for $10^{-1}<\gamma<10^0$. Equation (50) in [1] is correct and the figure erroneous. $F(\gamma,\mu)$ actually varies by five orders of magnitude on $10^{-6}<\gamma<10^0$, not only by four.)

At this point, the question of how the remaining factors in A_{SE} depend on γ was omitted. In fact, the denominator $(\alpha\mathcal{E}_T)^3$ in (8) has in leading order the same strong dependence on γ, since

$$\frac{1}{(\alpha\mathcal{E}_T)^3} = \left[\frac{L}{L_\gamma}\ln(L/L\gamma)\right]^3, \tag{13}$$

according to the Townsend breakdown criterion $\alpha L = L_\gamma$; cf. (10)–(12). Therefore, the leading order dependence on L_γ^3 of the coefficient of $\overline{j^2}$ in (8) is cancelled and replaced by a dependence on L^3, while the term with α'' has the classical explicit form

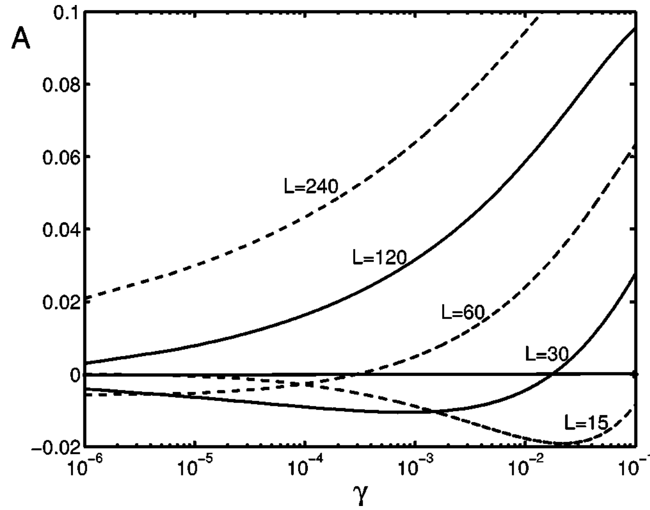

FIG. 2. The normalized coefficient $A=24\,A_{SE}/(L^3\ln^4 L)$ as a function of γ for gap lengths $L=Apd=15$, 30, 60, 120, 240 (dashed and solid lines with labels).

$$\frac{\mathcal{E}_T\ \alpha''}{2\alpha'} = \frac{1-2\mathcal{E}_T}{2\mathcal{E}_T} = \frac{\ln(L/L_\gamma)-2}{2}. \tag{14}$$

In [2,3], another small current expansion was derived from (1)–(3), assuming $n_+ \gg n_e$ and $n_+(x) \approx$ const. This approximation was criticized in [1], since it is in contradiction with the boundary condition (5); however, for very small γ, it is a good approximation in a large part of the gap. The resulting equations (8.8) and (8.10) from [2] read in the notation of the present paper

$$U = U_T - \frac{U_T}{48}\frac{1-2\mathcal{E}_T}{2\mathcal{E}_T}\left(\frac{J}{J_L}\right)^2, \tag{15}$$

$$J_L = \frac{\epsilon_0\mu_+ U_T^2}{2d^3}. \tag{16}$$

(Here, a misprint in [2] was corrected, namely, the missing factor U_T in the coefficient of J^2 in (15), is now included. Furthermore, the factor $1/(8\pi)$ in (8.8) is substituted by $\epsilon_0/2$ in (16), since we here write the Poisson equation (2) in MKS units rather than in Gaussian units; cf. (8.6) in [2].)

In (15), the physical current density J is compared to J_L. J_L is the current density at which deviations from the Townsend limit through space charges start to occur; it explicitly depends on γ through U_T (12).

Comparison of the results of Šijačić and Ebert (8) and of von Engel and Raizer (ER) (15) show that the coefficients $A_{SE,ER}$ in the expansion (8) are related as

$$A_{SE} = A_{ER}\frac{12\ F(\gamma,\mu)}{L_\gamma^3}, \quad A_{ER} = \frac{1-2\mathcal{E}_T}{2\mathcal{E}_T}\frac{L^3}{12\ \mathcal{E}_T^3}. \tag{17}$$

The coefficients A_{SE} and A_{ER} depend in the same way on L, and they are essentially independent of μ for realistic values of μ. Therefore, the ratio A_{SE}/A_{ER} depends only on γ as shown in Fig. 1. For $\gamma \to 0$, the ratio tends to unity. For a large range of γ values, the deviation is not too large, approaching a factor 0.44 for $\gamma = 10^{-1}$.

Figure 2 shows that the factor A_{SE} indeed strongly depends on γ for the given L.

The strong dependence of A_{SE} or A_{ER} on γ for a given short gap length L means that we can obtain both negative and positive differential resistance dU/dJ close to the Townsend limit for the same gap length. Therefore, the choice of γ is important since it can change the differential conductivity and therefore the stability of a Townsend discharge in a short gap.

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