Topic No. 4 (dominantly), for methods also 13, 14, 17

Spontaneous Branching of Anode-Directed Discharge Streamers: Conformal Analysis and Numerical Results

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We recently have identified a simple mechanism of spontaneous streamer branching [1, 2]. Here we discuss when this instability occurs, we present new numerical results, and we present a reduced model in which the instability can be studied analytically.

1. Observations and minimal model

Discharge streamers appear in the initial stages of natural sparking as well as in many plasma reactors. Streamers also play a role in the recently observed upwards sparking from clouds towards the ionosphere [3]. Frequently streamers do branch.
It is generally accepted that the properties of streamers are determined by space charge effects. Space charges enhance the field at the rapidly propagating head of the ionized streamer channel and create an active ionization zone that now can be visualized by fast CCD cameras [4].

We investigate the **minimal model** for anode-directed streamers in a non-attaching and non-ionized gas. It is a "fluid model" with impact ionization in local field approximation, with drift and diffusion of charged particles between abundant neutral particles, and with space charge effects through the Coulomb equation of electrostatics. In dimensionless units [1, 2], the model for electron density σ, ion density ρ and electric potential Φ has the form:

$$\begin{aligned}
\partial_t\,\sigma - \nabla\cdot(\sigma\,\mathbf{E} + D\,\nabla\sigma) &= \sigma\,|\mathbf{E}|\,\alpha(|\mathbf{E}|)\ ,\\
\partial_t\,\rho &= \sigma\,|\mathbf{E}|\,\alpha(|\mathbf{E}|)\ ,\\
\rho-\sigma = \nabla\cdot\mathbf{E} \quad , \quad \mathbf{E} &= -\nabla\Phi\ ,
\end{aligned}$$

where $\alpha(|\mathbf{E}|)$ in our numerical work is approximated by the Townsend expression $e^{-1/|\mathbf{E}|}$.

2. Two transitions, the Firsov limit

The avalanche-to-streamer-transition is a classical concept, it occurs when the space charge $\rho-\sigma$ is not negligible anymore. In the streamer phase, the interior of the ionized channel is screened from the externally applied field, the field at the active head is enhanced. This field enhancement makes the streamer propagate more rapidly than the avalanche.
In our recent numerical evaluation of the minimal model [1, 2], we applied an external field twice as high as previous authors [5]. Ionization gradients were much steeper and the field enhancement much stronger, and the streamer splitted spontaneously. We interprete this as a second "transition": the streamer head accumulates enough charge to approach the limit of "ideal conductivity" according to the old concept of Lozansky and Firsov [6]. However, the simple parabolic front shape solution [6] found for the "ideally conducting" streamer at the time is just one possible solution of the problem. Actually, we show that the head dynamics of a streamer in this limit can be surprisingly rich, and that branching is a generic instability:

3. Head dynamics of Firsov-streamers

3.1 Numerical results, adaptive grid

Spontaneous streamer branching was found in the numerical studies of M. Arrayás, A. Rocco, W. Hundsdorfer and U. Ebert [1, 2]. This work is presently being extended by C. Montijn, W. Hundsdorfer and U. Ebert. Specifically, an adaptive grid for the ionized regions has now been implemented which allows for finer grid spacing and more rapid calculations. The numerical discretization has been reinvestigated, and a larger parameter range is presently being tested. We will present our state of the art.

3.2 Analytical approach, conformal mapping

The simplest sketch of the dynamics of our Firsov streamers consists of the following building blocks: the interior of the streamer is equipotential, its boundary moves everywhere with a local velocity $\mathbf{v} = \mathbf{v}(\mathbf{E})$ where $\mathbf{E}$ is the local field, and the exterior is non-ionized and charge-free, so $\nabla^2\Phi = 0$. The field $\mathbf{E} = -\nabla\Phi$ far from the streamer is specified; we presently consider the case where this far field is constant.
The problem of a moving boundary of arbitrary shape in 2 dimensions can be reduced to a one-dimensional problem by conformal mapping methods. Such methods have been developed previously for the study of interfacial instabilities in two-fluid-flow, the so-called Saffman-Taylor problem. With the last simplification of $\mathbf{v}(\mathbf{E}) \propto \mathbf{E}$, the complex dynamics of the ionization boundary in some cases can be solved even fully analytically. We present the solutions recently found by B. Meulenbroek, A. Rocco and U. Ebert. They reproduce the transients and the branching instability as observed in the numerical solutions.

4. References

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