Structure formation in a DC-driven "barrier" discharge: stability analysis and numerical solutions

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A DC-driven "barrier" discharge is a gas discharge layer and a high-Ohmic semiconductor layer sandwiched between planar electrodes to which a DC voltage is applied. The system resembles a dielectric barrier discharge, but is even simpler, as the external boundary conditions allow for a completely homogeneous and stationary state. Whole phase transition diagrams to purely oscillating states or to states with spatio-temporal structures were determined in experiments that actually explored the regime between Townsend and glow discharge. We discuss the experimental conditions and our present theoretical understanding, that is based on stability analysis and numerical solutions of a fluid model for the gas discharge with space charge effects and a linear model for the semiconductor.

1. Experimental observations

A dielectric barrier discharge is used frequently for gas treatment; it consists of a layered structure of at least one dielectric layer and a gas discharge layer placed between planar electrodes to which an AC voltage is applied [1, 2]. Here an even simpler physical system is studied: a system with essentially the same set-up, but with a DC voltage supply, i.e., a stationary drive. The system is illustrated in Fig. 1. To allow current to flow under DC conditions, the dielectric layer is replaced by a high-Ohmic semiconductor.

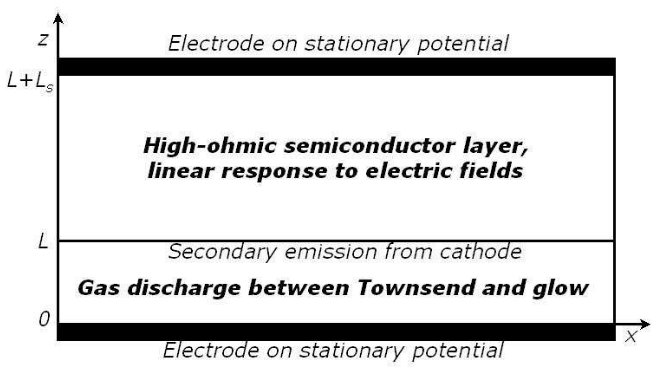

Figure 1: Scheme of semiconductor and gas discharge sandwiched between planar electrodes to which a DC voltage is applied.

Independently of the technical applications, AC and DC system both have attracted much attention in the pattern formation community in the past two decades, as they are easy to operate, have convenient length and time scales and a wealth of spontaneously created spatio-temporal patterns that can be observed from below through a transparent ITO layer acting as electrode. When the transversal extension of the layers is large enough, experiments show homogeneous stationary and oscillating modes, and patterns with spatial and spatio-temporal structures like stripes, spots, and spirals [3-18]. The aspect ratio of a thin layer with wide lateral extension and the observation from above are reminiscent of Rayleigh-Bénard convection as the classical pattern forming system in hydrodynamics.

The experiments on DC driven "barrier" discharges in [5-9] describe not only phenomena at one particular set of parameters, but Refs. [6, 9] also explore parameter space and draw phase diagrams for the transition between different states; therefore we concentrate on those — we are not aware of other experimental investigations of such phase diagrams. These experiments are performed in nitrogen at 40 mbar in a gap of 1 mm. The semiconductor layer consists of 1.5 mm of GaAs. The applied voltages are in the range of 580 V to 740 V. Through photosensitive doping, the conductivity of the semiconductor layer can be increased by an order of magnitude; the dark conductivity is $\sigma_s = 3.2 \times 10^{-8}/(\Omega\,\mathrm{cm})$. These parameters imply that the product of pressure and distance pd of the gas discharge is short, but still sufficiently far on the right hand side of the Paschen curve that the transition from Townsend to glow discharge is subcritical [19, 20], i.e., that the voltage at stationary opera-

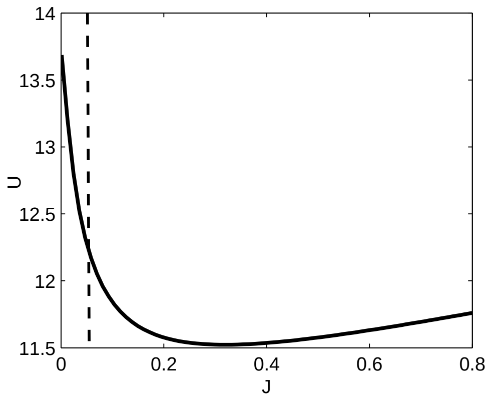

Figure 2: Solid line: Typical current voltage characteristics of the gas discharge in non-dimensional units showing the transition from Townsend to glow discharge. Dashed line: load line of the semiconductor in the case when it is furthest from the Townsend regime $J \to 0$. The intersection of the lines indicates the stationary solution of the system.

tion drops when the current rises, as shown in Fig. 2 as a solid line. The resistance of the semiconductor together with the applied voltage constrain the operation to the transition regime from Townsend to glow discharge. The load line depicted as dashed line in Fig. 2 is the one that is furthest from the Townsend regime $J \to 0$ that is relevant for these experiments. The intersection of discharge characteristics and load line defines the homogeneous stationary state of the system.

However, the system can approach two other states [6, 9]: one where it stays spatially perfectly homogeneous but oscillates in time, and one where structures in space and time are formed. Can these states be predicted by a simple fluid model for the discharge coupled to a linearly approximated semiconductor?

2. The fluid model

The gas-discharge is modeled as a fluid of electrons and positive ions with particle densities $n_{\rm e}$ and n_+:

$$\partial_t n_{\rm e} - \nabla \cdot \left(\mu_{\rm e} n_{\rm e}\, \mathbf{E}\right) = source, \quad (1)$$

$$\partial_t n_+ + \nabla \cdot \left(\mu_+ n_+\, \mathbf{E}\right) = source, \quad (2)$$

that are coupled to Poisson's equation

$$\nabla \cdot \mathbf{E} = \frac{e}{\varepsilon_0}\left(n_+ - n_{\rm e}\right), \quad \mathbf{E} = -\nabla\Phi. \quad (3)$$

Here, Φ is the electric potential, $\mathbf{E}$ is the electric field, e is the elementary charge, ε_0 is the dielectric constant, and $\mu_{{\rm e},+}$ are the mobilities of electrons and ions. The source term accounts for impact ionization by accelerated electrons within the gas

$$source = |\mu_{\rm e} n_{\rm e}\, \mathbf{E}|\, \alpha\left(|\mathbf{E}|\right), \quad (4)$$

furthermore secondary electron emission by ion impact onto the cathode is included through

$$\left|\mu_{\rm e} n_{\rm e}\, \mathbf{E}\right| = \gamma \left|\mu_+ n_+\, \mathbf{E}\right| \quad (5)$$

everywhere on the cathode. The cathode of the discharge is the semiconductor surface, cf. Fig. 1.

The semiconductor is characterized by its extensions, conductivity σ_s and dielectric constant ε_s, and a fixed voltage U is applied to the system.

On the boundary between gas layer and semiconductor, accumulated surface charges Σ create a discontinuity of the normal electric field

$$\Sigma = \left(\varepsilon_s \varepsilon_0 \mathbf{E}_s - \varepsilon_0 \mathbf{E}_g\right) \cdot \hat{\mathbf{n}}, \quad (6)$$

where $\mathbf{E}_{s,g}$ are the fields on the semiconductor or the gas discharge side of the interface, and $\hat{\mathbf{n}}$ is the normal vector on the boundary directed into the semiconductor.

3. Predictions

Predictions on oscillating states were made previously in [21, 22] where we resolved only the dynamics in the direction normal to the layers and found good agreement with experiments after adjusting only one parameter, namely the secondary emission constant γ. Here we report results when the transversal directions are included, a first paper on the subject is presently under review [23].
Basically, there are three complementary theoretical approaches to the problem:

3.1. Effective reaction-diffusion systems: The similarity with other pattern forming systems has motivated many attempts to derive an effective reaction-diffusion model only in the two transversal directions where the "reactants" are the local current or the electric potential on the gas semiconductor interface [3, 24, 25]. These models are appealing, however, they never have been derived from the full three-dimensional physics in an interesting parameter regime, as discussed in [21].

3.2. 2D and 3D simulations: One can solve the dynamics numerically in 2 or 3 spatial dimensions. 3D simulations of the AC system were reported only last year [26]. We will present 2D [23] as well as 3D solutions of our DC system. The drawback of simulations is that only finite time spans can be investigated for a limited set of system parameters and

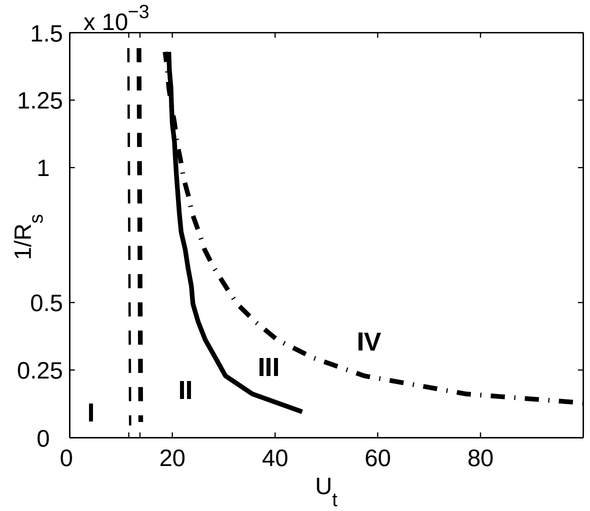

Figure 3: Calculated bifurcation diagram as a function of applied voltage $\mathcal{U}_t$ and conductivity $1/\mathcal{R}_s$ of the semiconductor. Dashed lines: minimal voltage $\mathcal{U}_{\rm glow}$ (thin) in the glow regime and Townsend voltage $\mathcal{U}_{\rm Town}$ (thick) of the gas discharge. Solid line: Re $\lambda(k^*)$=0. Dash-dotted line: $k^* = 0$. In region I, a discharge cannot form. In region II, the homogeneous stationary discharge is linearly stable, in region III, it is destabilized by traveling wave packages, and in region IV, it is destabilized by purely temporal modes.

initial conditions.

3.3. Stability analysis: One can perform a linear stability analysis of the homogeneous stationary state [23]. Our result is shown in Figure 3. We will discuss the relation of this prediction with simulations and experiment.

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