Comment on "Mechanism of Branching in Negative Ionization Fronts"

When the fingers of discharge streamers emerge from a planar ionization front due to a Laplacian instability, their initial spacing is determined by the band of unstable transversal Fourier perturbations and generically dominated by the fastest growing modes. The Letter [1] therefore aims to calculate the temporal growth rate $s(k)$ of modes with wave number k, when the electric field far ahead of the ionization front is E_∞. In earlier work [2–4], $s(k)$ was determined in a pure reaction-drift model for the free electrons, i.e., in the limit of vanishing electron diffusion $D_e = 0$. For negative streamers in pure gases like nitrogen or argon, electron diffusion $D_e > 0$ should be included into the discharge model. This is attempted in [1] in the limit of large field $|E_\infty|$ ahead of the front. A different, extensive analysis with different results can be found in [5]. Below we show that the expansion and calculation in [1] are inconsistent, that the result contradicts a known analytical asymptote, and that it does not fit the cross-checked numerical results presented in [5]. Furthermore, we find in [5] that the most unstable wavelength does not scale as $D_e^{1/3}$ as claimed in [1], but as $D_e^{1/4}$.

In [1], ionization fronts are only considered in the limit $|E_\infty| \gg 1$ ahead of the front which amounts to a saturating impact ionization cross section $\alpha(E) \to 1$. For $|E_\infty| \gg 1$, planar fronts obey [[1], Eq. (7)] after all fields are rescaled with E_∞. For any finite E_∞, a diffusive layer of width $1/\Lambda^* = \sqrt{D_e/[|E_\infty|\alpha(E_\infty)]}$ forms in the leading edge of the front [6]. (We denote the diffusion constant D from [2–6] by D_e to distinguish it from the $D = D_e/|E_\infty|$ in [1].) Following the calculation in [1], Eq. (8) reproduces the diffusive layer for large $|E_\infty|$, but the nonlinear term is incomplete. Then the dispersion relation is calculated by the expansion (11)–(13) about the planar ionization front. Here the expansion of the electron density n_e starts in order δ^2 (where δ is the small expansion parameter), while the expansions of ion density n_p and field E start in order δ. The absence of order δ in the expansion of n_e is unexpected, not explained, and in contradiction with the calculation for $D_e = 0$ in [4].

Jumping to the result of [1], the dispersion relation in Eq. (21) is given as $s = |E_\infty k|/[2(1 + |k|)] - D_e k^2$ in the present notation. The small k limit $s = |E_\infty k|/2 + O(k^2)$ of [[1], Eq. (21)] is consistent neither with the limit $D_e = 0$, where the asymptote $s(k) = |E_\infty k|$ for $|k| \ll \alpha(E_\infty)/2$ was derived in [4], nor with the case $D_e > 0$ where the asymptote $s = c^*|k|$, $c^* = E d_E v^*|_{E_\infty}$, $v^*(E) = |E| + 2\sqrt{D_e|E|\alpha(E)}$ was proposed in [2] and analytically confirmed in [5].

Furthermore, in [5], dispersion curves $s(k)$ for a range of fields E_∞ and diffusion constants D_e are derived as an eigenvalue problem for s; they are plotted in Fig. 1. In one case, the curve is confirmed by numerical solutions of an initial value problem; the curves are also consistent with the analytical small k asymptote. The results for positive s are conveniently fitted as $s(k) = c^*|k|(1 - 4|k|/\Lambda^*)/(1 + a|k|)$ with $a \approx 3/\alpha(E_\infty)$ [5]. Figure 1 also shows the prediction from [1] for $E_\infty = -10$ and $D_e = 0.1$; here the reduced diffusion constant $D_e/|E_\infty|$ is as small as 0.01, and the assumptions $|E_\infty| \gg 1$ and $D_e/|E_\infty| \ll 1$ from [1] hold. However, Fig. 1 shows that the data of [5] and the prediction of [1] clearly differ. Therefore also the scaling prediction [[1], Eq. (23)] for the spacing of emergent streamers does not hold; rather our physical arguments and the numerical data in [5] suggest that the fastest growing mode is $k_{\max} = (\sqrt{1 + a\Lambda^*/4} - 1)/a \propto D_e^{-1/4}$ for $\Lambda^* \gg 1$.

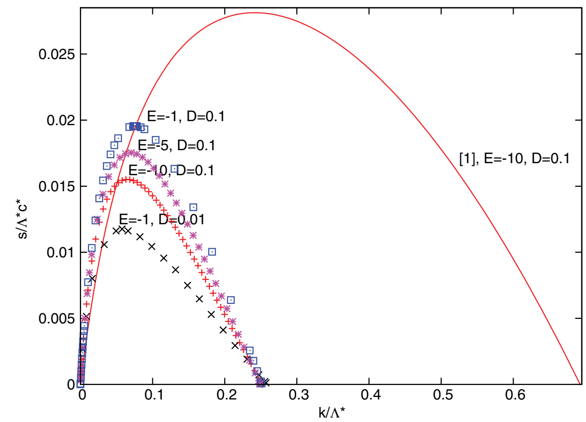

FIG. 1 (color online). Symbols: scaled dispersion relation $s(k)/(c^*\Lambda^*)$ [5] as function of scaled Fourier number k/Λ^* for $E_\infty = -1, -5, -10$ and $D_e = 0.1$ and for $E_\infty = -1$ and $D_e = 0.01$. Line: rescaled prediction [[1], (21)] for $E_\infty = -10$ and $D_e = 0.1$.

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Received 15 June 2007; published 23 September 2008
DOI: 10.1103/PhysRevLett.101.139501
PACS numbers: 52.80.Hc, 05.45.−a, 47.54.−r

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