Ebert and Hundsdorfer Reply: In our paper [1] we presented numerical evidence that negative streamers in homogeneous fields can branch spontaneously if the field is sufficiently high, we gave a physical explanation and presented a stability analysis. Our aim was to study generic features of electric breakdown in ionizable matter within the minimal discharge model. We search for understanding by a combination of analysis and simulations, since simulations can always be subject to errors and are constrained to a limited part of phase space.

Kulikovsky's Comment [2] addresses four questions: (i) details of our simulational techniques, (ii) the inclusion of additional ionization processes in the model, in particular, for positive streamers, (iii) the Laplacian instability, the Firsov limit and regularizing length scales, and (iv) the value of effectively 2D simulations in (r, z) versus truly 3D simulations in (x, y, z).

(i) The numerical results in [1] were derived on a somewhat coarse numerical grid, and we meanwhile have published results on a finer numerical grid and with a no flow boundary condition on the cathode in [3]. We published the earlier results in [1] together with arguments that the configuration immediately before the numerically observed branching (up to $t = 365$ in Figs. 1 and 2 in [1]) indeed should be physically unstable. We learned from Pasko that his group with their numerical code could reproduce the qualitative features and, in particular, the branching observed in our simulations.

Kulikovsky wonders whether our streamer branching is physical or due to a numerical artifact. In particular, he wonders whether the total current within our numerical scheme is conserved on finite grids. Indeed, it is: we use a finite-volume scheme based on local mass balances. The densities and potential are calculated at the cell centers and the fluxes and electric field at the cell boundaries. This seems to correspond with Kulikovsky's notion of a "staggered grid." The mass conservation of the finite-volume scheme implies charge conservation $\partial_t q + \nabla \cdot \mathbf{j} = 0$. The electric field $\mathbf{E} = -\nabla\Phi$ is calculated at each time step from the Poisson equation $q = -\nabla^2\Phi$ and the boundary conditions. Therefore the total current is conserved on finite grids. More details of the numerical scheme will be given elsewhere.

(ii) Our work deals with negative discharges that are prevailing in natural processes like sparks or the recently observed sprite discharges [4]. They can be described by our minimal model consisting of impact ionization, particle drift, and the Poisson equation. Various additions have been suggested for particular systems: photoionization for positive streamers in air or other complex gases under lab conditions, background ionization for sprite discharges at high atmospheric altitudes, or tunneling ionization for ultrafast impact ionization fronts in doped semiconductors [5]. Such additional mechanisms [6] are not the subject of our present studies [1,3,7,8].

(iii) Indeed, the lowest order approximation of a Laplacian instability like the Lozansky-Firsov limit discussed in [1] has no characteristic length scale. In [7,8], we therefore have suggested that a length scale (comparable to the curvature correction to Laplacian growth in viscous fingering cf. references in [1,7,8]) could be provided by the inverse cross section of impact ionization, and we have presented an analytical confirmation in [1]. This shows that even without photoionization, the problem has an intrinsic length scale that will correct the lowest order approximation of the Lozansky-Firsov limit. In [1], we also discuss the role of the diffusion length. Of course, in a streamer model extended with photoionization, the inverse cross section of photoionization will introduce a third length scale. Kulikovsky has focused in a recent paper [9] on this length scale.

(iv) The idea that streamer branching could only be verified by truly 3D simulations, is quite common but not true. We repeat [1,3]: Our effectively 2D simulation in coordinates (r, z) uses the radial symmetry of the initial streamer. The evolving instability modes typically will break this symmetry. The constraint of radial symmetry suppresses all instability modes that are not radially symmetric. If the pattern becomes unstable even towards radially symmetric instability modes, then for sure it will become unstable without symmetry constraint. Therefore our effectively 2D simulations with radial symmetry "prove" the streamer instability.

Ute Ebert and Willem Hundsdorfer
Centrum voor Wiskunde en Informatica
P.O. Box 94079, 1090 GB Amsterdam, The Netherlands

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