Message-Generated Kripke Semantics

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ABSTRACT

We show how to generate multi-agent Kripke models from message exchanges. With these models we can analyze the epistemic consequences of a message exchange. One novelty in this approach is that we include the messages in our logical language. This allows us to model messages that mention other messages and agents that reason about messages. Our framework can be used to model a wide range of different communication scenarios.

Categories and Subject Descriptors

E.4 [**Coding and Information Theory**]: Formal Models of Communication; H.1.2 [**User/Machine Systems**]: Human Information Processing; H.3.4 [**Systems and Software**]: Information Networks; I.2.0 [**Artificial Intelligence**]: Cognitive Simulation

Keywords

Agent communication, message semantics, epistemic Kripke models, dynamic epistemic logic

1. INTRODUCTION

This paper is a proposal to combine the best of history-based message interpretation, as in [4] and [1], and dynamic epistemic semantics, as in [2, 3].

We model communication between agents by means of message sequences. Here a message is assumed to be a formula sent by one agent to a group of other agents. We assume all communication to be truthful, so all formulas that are sent in messages must be true. We also assume that the communication is reliable, so any message that is sent is also received and immediately read.

We define a logical language containing both messages and epistemic operators. This allows us to reason about what knowledge agents have about the messages themselves. Some interesting examples of communication we can model with our framework are:

Send Communication step consisting of a single message m.

Acknowledgement Receipt of a message m can be expressed as (j, m, s_m) where $j \in r_m$.

Reply Reply to sending of m with reply-contents ψ can be expressed as $(j, m \wedge \psi, s_m)$ where $j \in r_m$.

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Forward Forwarding of m can be expressed as (j, m, k) where $j \in r_m$, $k \notin r_m$.

Forward with annotation Forwarding of m with annotation ψ can be expressed as $(j, m \wedge \psi, k)$ where $j \in r_m$, $k \notin r_m$.

Cc There is no distinction between addressee list and cc list. The distinction between addressee and cc belongs to pragmatics: an addressee is supposed to reply to a message while someone on a cc-list incurs no such obligation. We think it is safe to ignore the difference for now.

Bcc A message m with bcc-list $\{j_1, \dots, j_n\}$ can be treated as a sequence of messages m, (s_m, m, j_1), …, (s_m, m, j_n). Each member on the bcc list of m gets a separate message from the sender of m to the effect that message m was sent.

2. FACTUAL COMMUNICATION

Let P be a set of proposition letters. Let N be a finite set of agents.

Definition 1. *Let L_0 be the language given by ψ and let L be the language given by ϕ in the following construct:*

$$\begin{aligned}
\phi &::= \psi \mid \neg\phi \mid \phi \wedge \phi \mid [m]\varphi \mid [\alpha]\phi \\
m &::= (i, \psi, G) \text{ where } i \in G \subseteq N \\
\psi &::= \top \mid p \mid m \mid \neg\psi \mid \psi \wedge \psi \text{ where } p \in P \\
\alpha &::= i \mid ?\phi \mid \alpha;\alpha \mid \alpha \cup \alpha \mid \alpha^* \text{ where } i \in N
\end{aligned}$$

L_0 is propositional logic enriched with factual messages. The formula m expresses that message m was sent at some moment in the past. If $m = (i, \psi, G)$ is a message, we use b_m for its body ψ, s_m for its sender i, and r_m for its recipient set G. The body of a message must be from the basic language L_0, so it cannot contain arbitary L-formulas.

The language L contains an epistemic modality $[\alpha]\phi$ which is standard for epistemic logic: $[i]\phi$ expresses that agent i knows ϕ, $[(\bigcup_{i \in G} i)^*]\phi$ expresses common knowledge in the group G. The message modality $[m]\phi$ expresses that immediately after sending message m, ϕ will hold.

For each formula we define its vocabulary: the set of propositions and messages used in it.

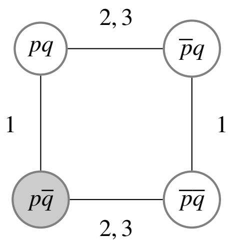
Definition 2 (Vocabulary of ψ).

$$\begin{aligned}
V_p &:= \{p\} \\
V_m &:= \{m\} \cup V_{b_m} \\
V_{\neg\psi} &:= V_\psi \\
V_{\psi_1 \wedge \psi_2} &:= V_{\psi_1} \cup V_{\psi_2}
\end{aligned}$$

We interpret the formulas from L on Kripke models as is standard in epistemic logic. Specifically, $[\alpha]\phi$ holds in a state s of a Kripke model iff for all states t such that there is an α-path from s to t, ϕ holds in t.

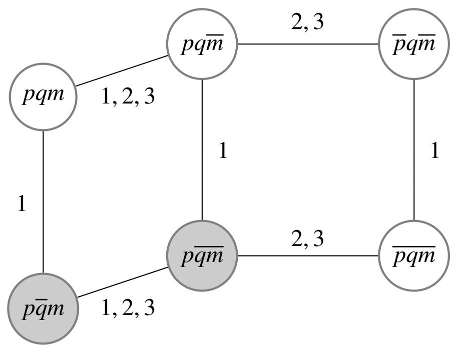
We need to define the interpretation of our new modality $[m]\phi$. For this purpose, we define a 'message update' in the style of [3]. Rather than giving a formal definition, we will give an example to demonstrate our modeling procedure.

Assume 1 knows (only) about p and 2 and 3 have common knowledge about q. Suppose p is true and q false. Given that the initial facts only mention p and q, we can assume that the initial vocabulary is the set $\{p, q\}$. Our initial Kripke model looks like this:



As usual, a link for agent i between two worlds indicates that agent i cannot distinguish the two worlds and does not know which one of them is the case. The grey shading indicates the actual world.

Now message $m@(1, p \vee q, 2)$ gets sent. The first step of processing m is expansion of the model to include m as a new vocabulary element m. Now m can be either true or false at each node (true means the message was sent, false means that it was not). If it was sent, then the sender must know its contents. This rules out situations where m is true and $K_1(p \vee q)$ is false, and it gives the following Kripke model:



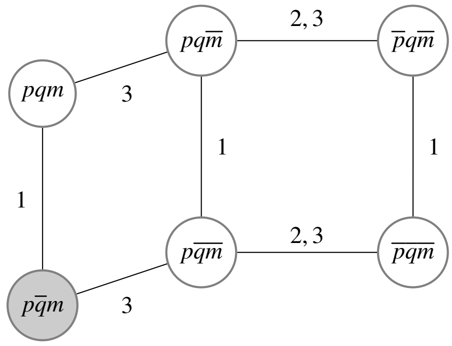
Convention: an i link exists if there is an i path in the picture, so all relations are equivalence relations. Note that the picture represents that no-one knows whether m was actually sent. What we have done is make the awareness of the agents include a new element, the message m. Both of the situations $p\overline{q}m$ and $p\overline{qm}$ could be true, and this is common knowledge at this stage.

Note that none of the agents learns anything new about the facts of the world. All of them become aware of the existence of a certain message that can be sent or not. Since the message can only be sent in worlds where 1 knows $p \vee q$, $\overline{p}qm$ and $\overline{pq}m$ are ruled out from the set of worlds.

Now the epistemic effect of the actual sending of m is three-fold:

- it rules out $p\overline{qm}$ from the set of candidates for the actual world;
- it erases accessibility links for 2 between $p \vee q$ and $\neg(p \vee q)$ worlds, indicating that 2 has learned from m that $p \vee q$ is true.
- it erases accessibility links for 1 and 2 between worlds where the message was sent and worlds where it was not, indicating that 1 and 2 now know whether m was sent, but 3 still does not.

These effects are expressed in the following model, which models the final result of sending m:



Note that $p\overline{qm}$ is no longer a candidate for the actual world. Agent 3 still cannot distinguish situations where m was sent from situations where m was not sent. But as a result of the sending action, 2 now knows everything there is to know about the vocabulary: that p is true, that q is false, and that m was sent.

If we consider the class of models that are generated in such a way from a sequence of sent messages, then the following axiom is sound:

$$m@(i, \psi, G) \to \psi$$

This indicates that we are indeed modeling truthful communication.

3. CURRENT AND FURTHER WORK

We have found a sound and complete axiomatisation of our language using reduction axioms in the style of [3].

We are also considering an extension with messages containing any formula of L, not just L_0. This would allow the agents to send messages containing information like "Alice does not know that Bob sent this message". We are currently working on a sound and complete axiomatization of this language.

Another extension we are investigating is to lift the restriction to truthful communication and consider the effects of lying.

There could also be a great use of our framework in a distributed setting, where every agent has a local Kripke model expressing his knowledge. We are currently investigating this perspective by tracing the logical connections between the distributed and the global views on communication histories.

4. REFERENCES

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