PDL as a Multi-Agent Strategy Logic

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Overview

Games and Strategies

Extending PDL to a Strategic Game Language

Epistemic Strategy Logic

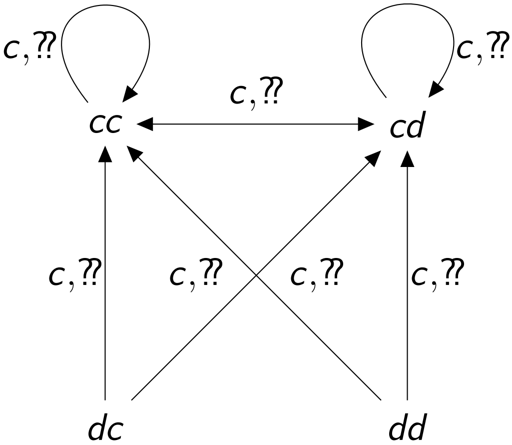
The PD Game

	cooperate	defect
cooperate	c, c	c, d
defect	d, c	d, d

	cooperate	defect
cooperate	2, 2	0, 3
defect	3, 0	1, 1

Output function: map from strategy pairs to outcomes.

Cooperation Strategy for Player 1 in PD



MASL Language

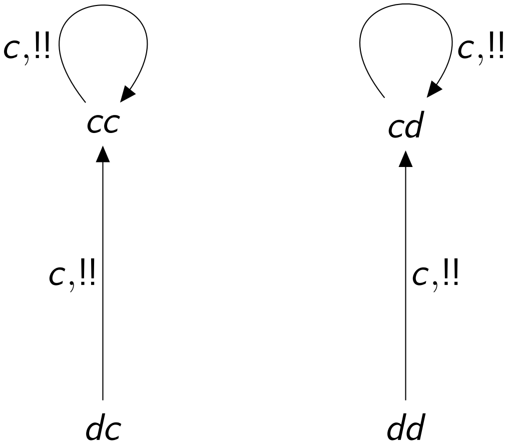
$$
\begin{aligned}
t_i &::= a \mid ⁇ \mid !! \\
\mathbf{c} &::= (t_1 \ldots, t_n) \\
\phi &::= \top \mid \mathbf{c} \mid p \mid \neg\phi \mid \phi_1 \wedge \phi_2 \mid [\gamma]\phi \\
\gamma &::= \mathbf{c} \mid ?\phi \mid \gamma_1 ; \gamma_2 \mid \gamma_1 \cup \gamma_2 \mid \gamma^*
\end{aligned}
$$

Term and Vector evaluation

$$
\begin{aligned}
[\![a]\!]^{S_i,s,i} &= \{a\} \\
[\![??]\!]^{S_i,s,i} &= S_i \\
[\![!!]\!]^{S_i,s,i} &= \{s[i]\}
\end{aligned}
$$

$$[\![(t_1 .. t_n)]\!]^{S,s} = [\![t_1]\!]^{S_1,s,1} \times \cdots \times [\![t_n]\!]^{S_n,s,n}$$

Interpretation of $(c, !!)$ in PD



Truth for MASL

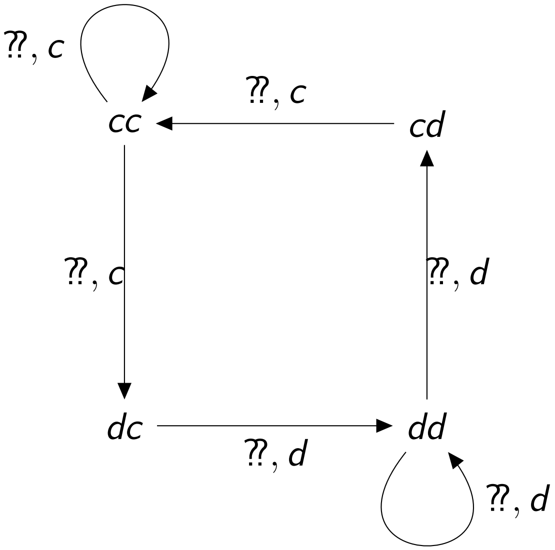
$M = (N, S, o)$ with $o : S \to P$.

$$
\begin{array}{rcl}
M, s \models \top & & \text{always} \\
M, s \models \mathbf{c} & \text{iff} & s \in [\![\mathbf{c}]\!]^{S,s} \\
M, s \models p & \text{iff} & s \in o^{-1}(p) \\
M, s \models \neg\phi & \text{iff} & M, s \not\models \phi \\
M, s \models \phi_1 \wedge \phi_2 & \text{iff} & \cdots \\
M, s \models [\gamma]\phi & \text{iff} & \cdots \\
[\![\mathbf{c}]\!]^M & = & \{(s, t) \mid t \in [\![\mathbf{c}]\!]^{S,s}\} \\
[\![?\phi]\!]^M & = & \{(s, s) \mid M, s \models \phi\} \\
[\![\gamma_1 ; \gamma_2]\!]^M & = & [\![\gamma_1]\!]^M \circ [\![\gamma_2]\!]^M \\
[\![\gamma_1 \cup \gamma_2]\!]^M & = & [\![\gamma_1]\!]^M \cup [\![\gamma_2]\!]^M \\
[\![\gamma^*]\!]^M & = & ([\![\gamma]\!]^M)^*.
\end{array}
$$

Formula for 'Game is Nash'

$$\langle(\overline{??})\rangle \bigwedge_{i \in N} \bigvee_{v \in U} (u_i \geq v \wedge [(i, \overline{!!})]\neg u_i > v).$$

Meta-strategy: Tit-for-Tat



$$(?(c, ⁇); (⁇, c) \ \cup \ ?(d, ⁇); (⁇, d))^*$$

Axioms

$[\mathbf{c}]\mathbf{c}$.
$\langle \mathbf{c} \rangle \top$.
$\langle \mathbf{c} \rangle \phi \rightarrow [\mathbf{c}]\phi$ (if $\mathbf{c}$ determined).
$[\mathbf{c}]\phi \leftrightarrow \bigwedge_{a \in S_i} [\mathbf{c}^i_a]\phi$.
$(i_a, \overline{!!}) \rightarrow (\mathbf{c} \leftrightarrow \mathbf{c}^i_a)$.

plus PDL axioms . . .

MASL is complete

Canonical model construction

MASL and coalition logic

$$\begin{aligned}
\mathrm{Tr}(p) &:= p \\
\mathrm{Tr}(\neg\phi) &:= \neg\mathrm{Tr}(\phi) \\
\mathrm{Tr}(\phi_1 \wedge \phi_2) &:= \mathrm{Tr}(\phi_1) \wedge \mathrm{Tr}(\phi_2) \\
\mathrm{Tr}([C]\phi) &:= \bigvee_{\mathbf{c} \in \dot{C}} [\mathbf{c}]\mathrm{Tr}(\phi).
\end{aligned}$$

$\dot{C}$ defined by

$$\left\{ (t_1, \ldots, t_n) \mid \begin{array}{ll} t_i \in S_i & \text{if } i \in C, \\ t_i = ?? & \text{otherwise} \end{array} \right\}.$$

$M, s \models_{CL} \phi$ iff $M, s \models_{MASL} \mathrm{Tr}(\phi)$.

Epistemic MASL

$$\phi \quad ::= \quad \cdots \mid [\gamma]\phi \mid [\alpha]\phi$$

$$\gamma \quad ::= \quad \cdots$$

$$\alpha \quad ::= \quad i \mid i\check{} \mid ?\phi \mid \alpha_1 ; \alpha_2 \mid \alpha_1 \cup \alpha_2 \mid \alpha^*$$

Define $\mathbf{i}$ as $(i \cup i\check{})^*$.
Then $\mathbf{i}$ is a reflexive, symmetric and transitive knowledge operator.

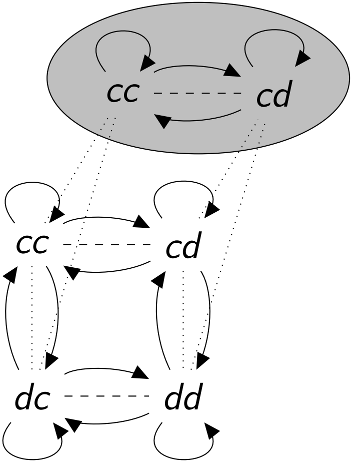
Intensional game forms

$$(N, W, R_1, \ldots, R_n)$$

where

- W is a set of pairs (G, s) where $G = (N, S)$ is a game form with $s \in S$,
- each R_i is a binary relation on W.

Restriction in Epistemic PD



Knowing Dictatorship

A knowing dictator is a player who *knows* that he is always able to get the best deal:

$$[\mathbf{i}][(\overline{??})] \bigvee_{v \in U} \bigwedge_{j \in N - \{i\}} (\neg u_j > v \wedge \langle (i, !!) \rangle u_i \geq v).$$

Player 2 in restricted PD is a dictator, but not a knowing dictator. For all he knows, he could end up in state *dd*.