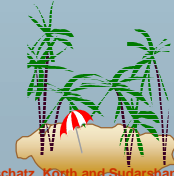




Appendix C: Advanced Normalization Theory

- Reasoning with MVDs
- Higher normal forms
 - Join dependencies and PJNF
 - DKNF



Theory of Multivalued Dependencies

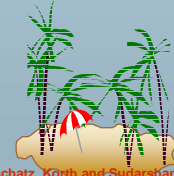
- Let D denote a set of functional and multivalued dependencies. The closure D^+ of D is the set of all functional and multivalued dependencies logically implied by D .
- Sound and complete inference rules for functional and multivalued dependencies:
 1. **Reflexivity rule.** If α is a set of attributes and $\beta \subseteq \alpha$, then $\alpha \twoheadrightarrow \beta$ holds.
 2. **Augmentation rule.** If $\alpha \twoheadrightarrow \beta$ holds and γ is a set of attributes, then $\gamma \alpha \twoheadrightarrow \gamma \beta$ holds.
 3. **Transitivity rule.** If $\alpha \twoheadrightarrow \beta$ holds and $\gamma \alpha \twoheadrightarrow \gamma \beta$ holds, then $\alpha \twoheadrightarrow \gamma$ holds.





Theory of Multivalued Dependencies (Cont.)

4. **Complementation rule.** If $\alpha \twoheadrightarrow \beta$ holds, then $\alpha \twoheadrightarrow R - \beta - \alpha$ holds.
5. **Multivalued augmentation rule.** If $\alpha \twoheadrightarrow \beta$ holds and $\gamma \subseteq R$ and $\delta \subseteq \gamma$, then $\gamma \alpha \twoheadrightarrow \delta \beta$ holds.
6. **Multivalued transitivity rule.** If $\alpha \twoheadrightarrow \beta$ holds and $\beta \twoheadrightarrow \gamma$ holds, then $\alpha \twoheadrightarrow \gamma - \beta$ holds.
7. **Replication rule.** If $\alpha \rightarrow \beta$ holds, then $\alpha \twoheadrightarrow \beta$.
8. **Coalescence rule.** If $\alpha \twoheadrightarrow \beta$ holds and $\gamma \subseteq \beta$ and there is a δ such that $\delta \subseteq R$ and $\delta \cap \beta = \emptyset$ and $\delta \rightarrow \gamma$, then $\alpha \rightarrow \gamma$ holds.



Simplification of the Computation of D^+

- We can simplify the computation of the closure of D by using the following rules (proved using rules 1-8).
 - **Multivalued union rule.** If $\alpha \twoheadrightarrow \beta$ holds and $\alpha \twoheadrightarrow \gamma$ holds, then $\alpha \twoheadrightarrow \beta\gamma$ holds.
 - **Intersection rule.** If $\alpha \twoheadrightarrow \beta$ holds and $\alpha \twoheadrightarrow \gamma$ holds, then $\alpha \twoheadrightarrow \beta \cap \gamma$ holds.
 - **Difference rule.** If $\alpha \twoheadrightarrow \beta$ holds and $\alpha \twoheadrightarrow \gamma$ holds, then $\alpha \twoheadrightarrow \beta - \gamma$ holds and $\alpha \twoheadrightarrow \gamma - \beta$ holds.





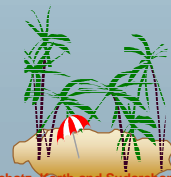
Example

- $R = (A, B, C, G, H, I)$
 $D = \{A \twoheadrightarrow B$
 $B \twoheadrightarrow HI$
 $CG \twoheadrightarrow H\}$
- Some members of D^+ :
 - $A \twoheadrightarrow CGHI$.
 Since $A \twoheadrightarrow B$, the complementation rule (4) implies that
 $A \twoheadrightarrow R - B - A$.
 Since $R - B - A = CGHI$, so $A \twoheadrightarrow CGHI$.
 - $A \twoheadrightarrow HI$.
 Since $A \twoheadrightarrow B$ and $B \twoheadrightarrow HI$, the multivalued transitivity
 rule (6) implies that $A \twoheadrightarrow HI - B$.
 Since $HI - B = HI$, $A \twoheadrightarrow HI$.



Example (Cont.)

- Some members of D^+ (cont.):
 - $B \twoheadrightarrow H$.
 Apply the coalescence rule (8); $B \twoheadrightarrow HI$ holds.
 Since $H \subseteq HI$ and $CG \twoheadrightarrow H$ and $CG \cap HI = \emptyset$, the
 coalescence rule is satisfied with α being B , β being HI , δ being CG ,
 and γ being H . We conclude that $B \twoheadrightarrow H$.
 - $A \twoheadrightarrow CG$.
 $A \twoheadrightarrow CGHI$ and $A \twoheadrightarrow HI$.
 By the difference rule, $A \twoheadrightarrow CGHI - HI$.
 Since $CGHI - HI = CG$, $A \twoheadrightarrow CG$.



Normalization Using Join Dependencies

- Join dependencies constrain the set of legal relations over a schema R to those relations for which a given decomposition is a lossless-join decomposition.
- Let R be a relation schema and R_1, R_2, \dots, R_n be a decomposition of R . If $R = R_1 \cup R_2 \cup \dots \cup R_n$, we say that a relation $r(R)$ satisfies the *join dependency* $*(R_1, R_2, \dots, R_n)$ if:

$$r = \Pi_{R_1}(r) \Join \Pi_{R_2}(r) \Join \dots \Join \Pi_{R_n}(r)$$

A join dependency is *trivial* if one of the R_i is R itself.

- A join dependency $*(R_1, R_2)$ is equivalent to the multivalued dependency $R_1 \cap R_2 \twoheadrightarrow R_2$. Conversely, $\alpha \twoheadrightarrow \beta$ is equivalent to $*(\alpha \cup (R - \beta), \alpha \cup \beta)$
- However, there are join dependencies that are not equivalent to any multivalued dependency.



Project-Join Normal Form (PJNF)

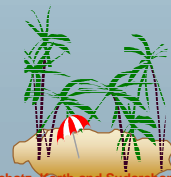
- A relation schema R is in PJNF with respect to a set D of functional, multivalued, and join dependencies if for all join dependencies in D^+ of the form

$$*(R_1, R_2, \dots, R_n) \text{ where each } R_i \subseteq R$$

$$\text{and } R = R_1 \cup R_2 \cup \dots \cup R_n$$

at least one of the following holds:

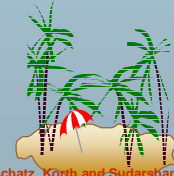
- $*(R_1, R_2, \dots, R_n)$ is a trivial join dependency.
 - Every R_i is a superkey for R .
- Since every multivalued dependency is also a join dependency, every PJNF schema is also in 4NF.





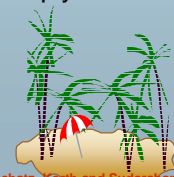
Example

- Consider *Loan-info-schema* = (*branch-name*, *customer-name*, *loan-number*, *amount*).
- Each loan has one or more customers, is in one or more branches and has a loan amount; these relationships are independent, hence we have the join dependency
- $\star(=(\textit{loan-number}, \textit{branch-name}), (\textit{loan-number}, \textit{customer-name}), (\textit{loan-number}, \textit{amount}))$
- *Loan-info-schema* is not in PJNF with respect to the set of dependencies containing the above join dependency. To put *Loan-info-schema* into PJNF, we must decompose it into the three schemas specified by the join dependency:
 - (*loan-number*, *branch-name*)
 - (*loan-number*, *customer-name*)
 - (*loan-number*, *amount*)



Domain-Key Normal Form (DKNF)

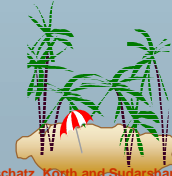
- **Domain declaration.** Let A be an attribute, and let **dom** be a set of values. The domain declaration $A \subseteq \mathbf{dom}$ requires that the A value of all tuples be values in **dom**.
- **Key declaration.** Let R be a relation schema with $K \subseteq R$. The key declaration **key** (K) requires that K be a superkey for schema R ($K \rightarrow R$). All key declarations are functional dependencies but not all functional dependencies are key declarations.
- **General constraint.** A general constraint is a predicate on the set of all relations on a given schema.
- Let **D** be a set of domain constraints and let **K** be a set of key constraints for a relation schema R . Let **G** denote the general constraints for R . Schema R is in DKNF if $\mathbf{D} \cup \mathbf{K}$ logically imply **G**.





Example

- Accounts whose *account-number* begins with the digit 9 are special high-interest accounts with a minimum balance of 2500.
- General constraint: "If the first digit of $t[\textit{account-number}]$ is 9, then $t[\textit{balance}] \geq 2500$."
- DKNF design:
 - Regular-acct-schema* = (*branch-name*, *account-number*, *balance*)
 - Special-acct-schema* = (*branch-name*, *account-number*, *balance*)
- Domain constraints for $\{\textit{Special-acct-schema}\}$ require that for each account:
 - The account number begins with 9.
 - The balance is greater than 2500.



DKNF rephrasing of PJNF Definition

- Let $R = (A_1, A_2, \dots, A_n)$ be a relation schema. Let $\text{dom}(A_i)$ denote the domain of attribute A_i , and let all these domains be infinite. Then all domain constraints \mathbf{D} are of the form $A_i \subseteq \text{dom}(A_i)$.
- Let the general constraints be a set \mathbf{G} of functional, multivalued, or join dependencies. If F is the set of functional dependencies in \mathbf{G} , let the set \mathbf{K} of key constraints be those nontrivial functional dependencies in F^+ of the form $\alpha \rightarrow R$.
- Schema R is in PJNF if and only if it is in DKNF with respect to \mathbf{D} , \mathbf{K} , and \mathbf{G} .

