



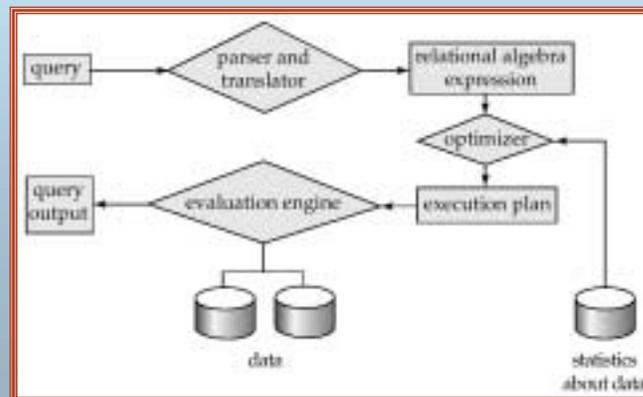
Chapter 13: Query Processing

- Overview
- Measures of Query Cost
- Selection Operation
- Sorting
- Join Operation
- Other Operations
- Evaluation of Expressions



Basic Steps in Query Processing

1. Parsing and translation
2. Optimization
3. Evaluation





Basic Steps in Query Processing (Cont.)

- Parsing and translation
 - ★ translate the query into its internal form. This is then translated into relational algebra.
 - ★ Parser checks syntax, verifies relations
- Evaluation
 - ★ The query-execution engine takes a query-evaluation plan, executes that plan, and returns the answers to the query.



Basic Steps in Query Processing : Optimization

- A relational algebra expression may have many equivalent expressions
 - ★ E.g., $\sigma_{balance < 2500}(\Pi_{balance}(account))$ is equivalent to $\Pi_{balance}(\sigma_{balance < 2500}(account))$
- Each relational algebra operation can be evaluated using one of several different algorithms
 - ★ Correspondingly, a relational-algebra expression can be evaluated in many ways.
- Annotated expression specifying detailed evaluation strategy is called an **evaluation-plan**.
 - ★ E.g., can use an index on *balance* to find accounts with $balance < 2500$,
 - ★ or can perform complete relation scan and discard accounts with $balance \geq 2500$





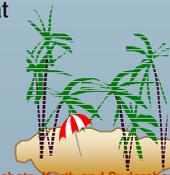
Basic Steps: Optimization (Cont.)

- **Query Optimization:** Amongst all equivalent evaluation plans choose the one with lowest cost.
 - ★ Cost is estimated using statistical information from the database catalog
 - e.g. number of tuples in each relation, size of tuples, etc.
- In this chapter we study
 - ★ How to measure query costs
 - ★ Algorithms for evaluating relational algebra operations
 - ★ How to combine algorithms for individual operations in order to evaluate a complete expression
- In Chapter 14
 - ★ We study how to optimize queries, that is, how to find an evaluation plan with lowest estimated cost



Measures of Query Cost

- Cost is generally measured as total elapsed time for answering query
 - ★ Many factors contribute to time cost
 - *disk accesses, CPU, or even network communication*
- Typically disk access is the predominant cost, and is also relatively easy to estimate. Measured by taking into account
 - ★ Number of seeks * average-seek-cost
 - ★ Number of blocks read * average-block-read-cost
 - ★ Number of blocks written * average-block-write-cost
 - Cost to write a block is greater than cost to read a block
 - data is read back after being written to ensure that the write was successful





Measures of Query Cost (Cont.)

- For simplicity we just use *number of block transfers from disk* as the cost measure
 - ★ We ignore the difference in cost between *sequential* and *random* I/O for simplicity
 - ★ We also ignore CPU costs for simplicity
- Costs depends on the size of the buffer in main memory
 - ★ Having more memory reduces need for disk access
 - ★ Amount of real memory available to buffer depends on other concurrent OS processes, and hard to determine ahead of actual execution
 - ★ We often use worst case estimates, assuming only the minimum amount of memory needed for the operation is available
- Real systems take CPU cost into account, differentiate between sequential and random I/O, and take buffer size into account
- We do not include cost to writing output to disk in our cost formulae



Selection Operation

- **File scan** – search algorithms that locate and retrieve records that fulfill a selection condition.
- Algorithm **A1** (*linear search*). Scan each file block and test all records to see whether they satisfy the selection condition.
 - ★ Cost estimate (number of disk blocks scanned) = b_r
 - b_r denotes number of blocks containing records from relation r
 - ★ If selection is on a key attribute, cost = $(b_r/2)$
 - stop on finding record
 - ★ Linear search can be applied regardless of
 - selection condition or
 - ordering of records in the file, or
 - availability of indices





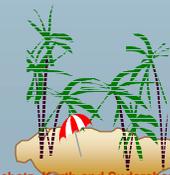
Selection Operation (Cont.)

- **A2** (*binary search*). Applicable if selection is an equality comparison on the attribute on which file is ordered.
 - ★ Assume that the blocks of a relation are stored contiguously
 - ★ Cost estimate (number of disk blocks to be scanned):
 - $\lceil \log_2(b_r) \rceil$ — cost of locating the first tuple by a binary search on the blocks
 - Plus number of blocks containing records that satisfy selection condition
 - Will see how to estimate this cost in Chapter 14



Selections Using Indices

- **Index scan** – search algorithms that use an index
 - ★ selection condition must be on search-key of index.
- **A3** (*primary index on candidate key, equality*). Retrieve a single record that satisfies the corresponding equality condition
 - ★ $Cost = HT_i + 1$
- **A4** (*primary index on nonkey, equality*) Retrieve multiple records.
 - ★ Records will be on consecutive blocks
 - ★ $Cost = HT_i + \text{number of blocks containing retrieved records}$
- **A5** (*equality on search-key of secondary index*).
 - ★ Retrieve a single record if the search-key is a candidate key
 - $Cost = HT_i + 1$
 - ★ Retrieve multiple records if search-key is not a candidate key
 - $Cost = HT_i + \text{number of records retrieved}$
 - Can be very expensive!
 - each record may be on a different block
 - one block access for each retrieved record





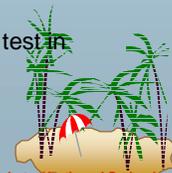
Selections Involving Comparisons

- Can implement selections of the form $\sigma_{A \leq v}(r)$ or $\sigma_{A \geq v}(r)$ by using
 - ★ a linear file scan or binary search,
 - ★ or by using indices in the following ways:
- **A6 (primary index, comparison).** (Relation is sorted on A)
 - For $\sigma_{A \geq v}(r)$ use index to find first tuple $\geq v$ and scan relation sequentially from there
 - For $\sigma_{A \leq v}(r)$ just scan relation sequentially till first tuple $> v$; do not use index
- **A7 (secondary index, comparison).**
 - For $\sigma_{A \geq v}(r)$ use index to find first index entry $\geq v$ and scan index sequentially from there, to find pointers to records.
 - For $\sigma_{A \leq v}(r)$ just scan leaf pages of index finding pointers to records, till first entry $> v$
 - In either case, retrieve records that are pointed to
 - requires an I/O for each record
 - Linear file scan may be cheaper if many records are to be fetched!



Implementation of Complex Selections

- **Conjunction:** $\sigma_{\theta_1 \wedge \theta_2 \wedge \dots \wedge \theta_n}(r)$
- **A8 (conjunctive selection using one index).**
 - ★ Select a combination of θ_i and algorithms A1 through A7 that results in the least cost for $\sigma_{\theta_i}(r)$.
 - ★ Test other conditions on tuple after fetching it into memory buffer.
- **A9 (conjunctive selection using multiple-key index).**
 - ★ Use appropriate composite (multiple-key) index if available.
- **A10 (conjunctive selection by intersection of identifiers).**
 - ★ Requires indices with record pointers.
 - ★ Use corresponding index for each condition, and take intersection of all the obtained sets of record pointers.
 - ★ Then fetch records from file
 - ★ If some conditions do not have appropriate indices, apply test in memory.





Algorithms for Complex Selections

- **Disjunction:** $\sigma_{\theta_1 \vee \theta_2 \vee \dots \vee \theta_n}(r)$.
- **A11** (*disjunctive selection by union of identifiers*).
 - ★ Applicable if *all* conditions have available indices.
 - Otherwise use linear scan.
 - ★ Use corresponding index for each condition, and take union of all the obtained sets of record pointers.
 - ★ Then fetch records from file
- **Negation:** $\sigma_{\neg\theta}(r)$
 - ★ Use linear scan on file
 - ★ If very few records satisfy $\neg\theta$, and an index is applicable to θ
 - Find satisfying records using index and fetch from file



Sorting

- We may build an index on the relation, and then use the index to read the relation in sorted order. May lead to one disk block access for each tuple.
- For relations that fit in memory, techniques like quicksort can be used. For relations that don't fit in memory, **external sort-merge** is a good choice.





External Sort-Merge

Let M denote memory size (in pages).

1. Create sorted runs. Let i be 0 initially.

Repeatedly do the following till the end of the relation:

- (a) Read M blocks of relation into memory
- (b) Sort the in-memory blocks
- (c) Write sorted data to run R_i ; increment i .

Let the final value of i be N

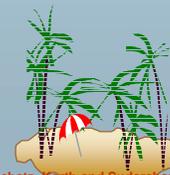
2. Merge the runs (N-way merge). We assume (for now) that $N < M$.

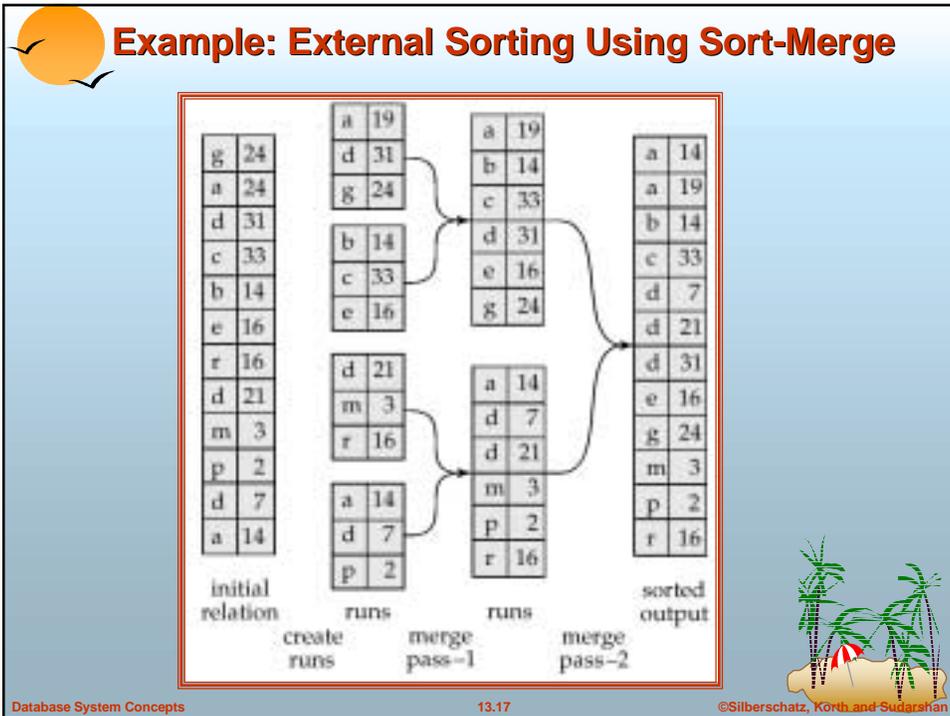
1. Use N blocks of memory to buffer input runs, and 1 block to buffer output. Read the first block of each run into its buffer page
2. **repeat**
 1. Select the first record (in sort order) among all buffer pages
 2. Write the record to the output buffer. If the output buffer is full write it to disk.
 3. Delete the record from its input buffer page.
If the buffer page becomes empty then
read the next block (if any) of the run into the buffer.
3. **until** all input buffer pages are empty:



External Sort-Merge (Cont.)

- If $i \geq M$, several merge passes are required.
 - ★ In each pass, contiguous groups of $M - 1$ runs are merged.
 - ★ A pass reduces the number of runs by a factor of $M - 1$, and creates runs longer by the same factor.
 - E.g. If $M=11$, and there are 90 runs, one pass reduces the number of runs to 9, each 10 times the size of the initial runs
 - ★ Repeated passes are performed till all runs have been merged into one.





External Merge Sort (Cont.)

- Cost analysis:
 - ★ Total number of merge passes required: $\lceil \log_{M-1}(b_r/M) \rceil$.
 - ★ Disk accesses for initial run creation as well as in each pass is $2b_r$
 - for final pass, we don't count write cost
 - we ignore final write cost for all operations since the output of an operation may be sent to the parent operation without being written to disk

Thus total number of disk accesses for external sorting:

$$b_r (2 \lceil \log_{M-1}(b_r/M) \rceil + 1)$$

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Join Operation

- Several different algorithms to implement joins
 - ★ Nested-loop join
 - ★ Block nested-loop join
 - ★ Indexed nested-loop join
 - ★ Merge-join
 - ★ Hash-join
- Choice based on cost estimate
- Examples use the following information
 - ★ Number of records of *customer*: 10,000 *depositor*: 5000
 - ★ Number of blocks of *customer*: 400 *depositor*: 100



Nested-Loop Join

- To compute the theta join $r \bowtie_{\theta} s$
 - for each tuple t_r in r do begin**
 - for each tuple t_s in s do begin**
 - test pair (t_r, t_s) to see if they satisfy the join condition θ
 - if they do, add $t_r \cdot t_s$ to the result.
 - end**
 - end**
- r is called the **outer relation** and s the **inner relation** of the join.
- Requires no indices and can be used with any kind of join condition.
- Expensive since it examines every pair of tuples in the two relations.





Nested-Loop Join (Cont.)

- In the worst case, if there is enough memory only to hold one block of each relation, the estimated cost is $n_r * b_s + b_r$ disk accesses.
- If the smaller relation fits entirely in memory, use that as the inner relation. Reduces cost to $b_r + b_s$ disk accesses.
- Assuming worst case memory availability cost estimate is
 - ★ $5000 * 400 + 100 = 2,000,100$ disk accesses with *depositor* as outer relation, and
 - ★ $1000 * 100 + 400 = 1,000,400$ disk accesses with *customer* as the outer relation.
- If smaller relation (*depositor*) fits entirely in memory, the cost estimate will be 500 disk accesses.
- Block nested-loops algorithm (next slide) is preferable.



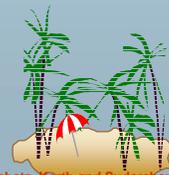
Block Nested-Loop Join

- Variant of nested-loop join in which every block of inner relation is paired with every block of outer relation.

```

for each block  $B_r$  of  $r$  do begin
  for each block  $B_s$  of  $s$  do begin
    for each tuple  $t_r$  in  $B_r$  do begin
      for each tuple  $t_s$  in  $B_s$  do begin
        Check if  $(t_r, t_s)$  satisfy the join condition
        if they do, add  $t_r \cdot t_s$  to the result.
      end
    end
  end
end

```





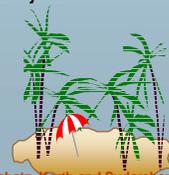
Block Nested-Loop Join (Cont.)

- Worst case estimate: $b_r * b_s + b_r$ block accesses.
 - ★ Each block in the inner relation s is read once for each *block* in the outer relation (instead of once for each tuple in the outer relation)
- Best case: $b_r + b_s$ block accesses.
- Improvements to nested loop and block nested loop algorithms:
 - ★ In block nested-loop, use $M - 2$ disk blocks as blocking unit for outer relations, where M = memory size in blocks; use remaining two blocks to buffer inner relation and output
 - Cost = $\lceil b_r / (M-2) \rceil * b_s + b_r$
 - ★ If equi-join attribute forms a key or inner relation, stop inner loop on first match
 - ★ Scan inner loop forward and backward alternately, to make use of the blocks remaining in buffer (with LRU replacement)
 - ★ Use index on inner relation if available (next slide)



Indexed Nested-Loop Join

- Index lookups can replace file scans if
 - ★ join is an equi-join or natural join and
 - ★ an index is available on the inner relation's join attribute
 - Can construct an index just to compute a join.
- For each tuple t_r in the outer relation r , use the index to look up tuples in s that satisfy the join condition with tuple t_r .
- Worst case: buffer has space for only one page of r , and, for each tuple in r , we perform an index lookup on s .
- Cost of the join: $b_r + n_r * c$
 - ★ Where c is the cost of traversing index and fetching all matching s tuples for one tuple or r
 - ★ c can be estimated as cost of a single selection on s using the join condition.
- If indices are available on join attributes of both r and s , use the relation with fewer tuples as the outer relation.





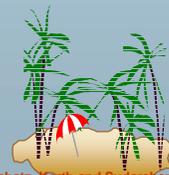
Merge-Join (Cont.)

- Can be used only for equi-joins and natural joins
- Each block needs to be read only once (assuming all tuples for any given value of the join attributes fit in memory)
- Thus number of block accesses for merge-join is $b_r + b_s$ + the cost of sorting if relations are unsorted.
- **hybrid merge-join:** If one relation is sorted, and the other has a secondary B⁺-tree index on the join attribute
 - ★ Merge the sorted relation with the leaf entries of the B⁺-tree .
 - ★ Sort the result on the addresses of the unsorted relation's tuples
 - ★ Scan the unsorted relation in physical address order and merge with previous result, to replace addresses by the actual tuples
 - Sequential scan more efficient than random lookup



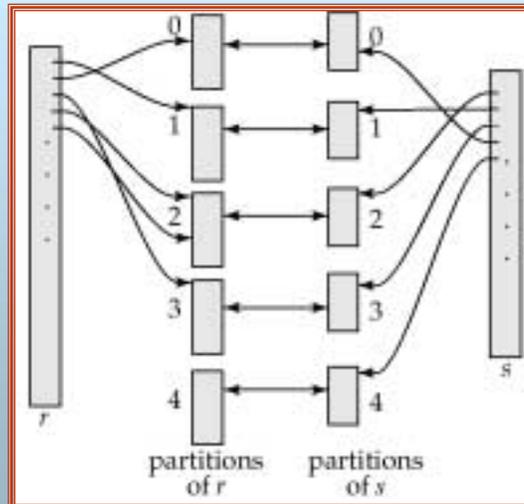
Hash-Join

- Applicable for equi-joins and natural joins.
- A hash function h is used to partition tuples of both relations
- h maps *JoinAttrs* values to $\{0, 1, \dots, n\}$, where *JoinAttrs* denotes the common attributes of r and s used in the natural join.
 - ★ r_0, r_1, \dots, r_n denote partitions of r tuples
 - Each tuple $t_r \in r$ is put in partition r_i where $i = h(t_r[\text{JoinAttrs}])$.
 - ★ s_0, s_1, \dots, s_n denotes partitions of s tuples
 - Each tuple $t_s \in s$ is put in partition s_i where $i = h(t_s[\text{JoinAttrs}])$.
- *Note:* In book, r_i is denoted as H_{r_i} , s_i is denoted as H_{s_i} and n is denoted as n_h .





Hash-Join (Cont.)



Hash-Join (Cont.)

- r tuples in r_i need only to be compared with s tuples in s_i
Need not be compared with s tuples in any other partition, since:
 - ★ an r tuple and an s tuple that satisfy the join condition will have the same value for the join attributes.
 - ★ If that value is hashed to some value i , the r tuple has to be in r_i and the s tuple in s_i .





Hash-Join Algorithm

The hash-join of r and s is computed as follows.

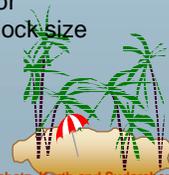
1. Partition the relation s using hashing function h . When partitioning a relation, one block of memory is reserved as the output buffer for each partition.
2. Partition r similarly.
3. For each i :
 - (a) Load s_i into memory and build an in-memory hash index on it using the join attribute. This hash index uses a different hash function than the earlier one h .
 - (b) Read the tuples in r_i from the disk one by one. For each tuple t_r , locate each matching tuple t_s in s_i using the in-memory hash index. Output the concatenation of their attributes.

Relation s is called the **build input** and r is called the **probe input**.



Hash-Join algorithm (Cont.)

- The value n and the hash function h is chosen such that each s_i should fit in memory.
 - ★ Typically n is chosen as $\lceil b_s/M \rceil * f$ where f is a “fudge factor”, typically around 1.2
 - ★ The probe relation partitions s_i need not fit in memory
- **Recursive partitioning** required if number of partitions n is greater than number of pages M of memory.
 - ★ instead of partitioning n ways, use $M - 1$ partitions for s
 - ★ Further partition the $M - 1$ partitions using a different hash function
 - ★ Use same partitioning method on r
 - ★ Rarely required: e.g., recursive partitioning not needed for relations of 1GB or less with memory size of 2MB, with block size of 4KB.





Handling of Overflows

- **Hash-table overflow** occurs in partition s_j if s_j does not fit in memory. Reasons could be
 - ★ Many tuples in s with same value for join attributes
 - ★ Bad hash function
- Partitioning is said to be **skewed** if some partitions have significantly more tuples than some others
- Overflow resolution can be done in build phase
 - ★ Partition s_j is further partitioned using different hash function.
 - ★ Partition r_j must be similarly partitioned.
- Overflow avoidance performs partitioning carefully to avoid overflows during build phase
 - ★ E.g. partition build relation into many partitions, then combine them
- Both approaches fail with large numbers of duplicates
 - ★ Fallback option: use block nested loops join on overflowed partitions



Cost of Hash-Join

- If recursive partitioning is not required: cost of hash join is $3(b_r + b_s) + 2 * n_h$
- If recursive partitioning required, number of passes required for partitioning s is $\lceil \log_{M-1}(b_s) - 1 \rceil$. This is because each final partition of s should fit in memory.
- The number of partitions of probe relation r is the same as that for build relation s ; the number of passes for partitioning of r is also the same as for s .
- Therefore it is best to choose the smaller relation as the build relation.
- Total cost estimate is:
$$2(b_r + b_s \lceil \log_{M-1}(b_s) - 1 \rceil) + b_r + b_s$$
- If the entire build input can be kept in main memory, n can be set to 0 and the algorithm does not partition the relations into temporary files. Cost estimate goes down to $b_r + b_s$.





Example of Cost of Hash-Join

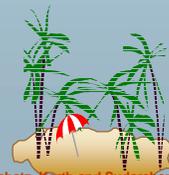
customer ⋈ *depositor*

- Assume that memory size is 20 blocks
- $b_{depositor} = 100$ and $b_{customer} = 400$.
- *depositor* is to be used as build input. Partition it into five partitions, each of size 20 blocks. This partitioning can be done in one pass.
- Similarly, partition *customer* into five partitions, each of size 80. This is also done in one pass.
- Therefore total cost: $3(100 + 400) = 1500$ block transfers
 - ★ ignores cost of writing partially filled blocks



Hybrid Hash-Join

- Useful when memory sized are relatively large, and the build input is bigger than memory.
- **Main feature of hybrid hash join:**
 - Keep the first partition of the build relation in memory.
- E.g. With memory size of 25 blocks, *depositor* can be partitioned into five partitions, each of size 20 blocks.
- Division of memory:
 - ★ The first partition occupies 20 blocks of memory
 - ★ 1 block is used for input, and 1 block each for buffering the other 4 partitions.
- *customer* is similarly partitioned into five partitions each of size 80; the first is used right away for probing, instead of being written out and read back.
- Cost of $3(80 + 320) + 20 + 80 = 1300$ block transfers for hybrid hash join, instead of 1500 with plain hash-join.
- Hybrid hash-join most useful if $M \gg \sqrt{b_s}$





Other Operations : Aggregation

- **Aggregation** can be implemented in a manner similar to duplicate elimination.
 - ★ Sorting or hashing can be used to bring tuples in the same group together, and then the aggregate functions can be applied on each group.
 - ★ *Optimization*: combine tuples in the same group during run generation and intermediate merges, by computing partial aggregate values
 - For count, min, max, sum: keep aggregate values on tuples found so far in the group.
 - When combining partial aggregate for count, add up the aggregates
 - For avg, keep sum and count, and divide sum by count at the end



Other Operations : Set Operations

- **Set operations** (\cup , \cap and $-$): can either use variant of merge-join after sorting, or variant of hash-join.
- E.g., Set operations using hashing:
 1. Partition both relations using the same hash function, thereby creating r_1, \dots, r_n and s_1, s_2, \dots, s_n
 2. Process each partition i as follows. Using a different hashing function, build an in-memory hash index on r_i after it is brought into memory.
 - 3. – $r \cup s$: Add tuples in s_i to the hash index if they are not already in it. At end of s_i add the tuples in the hash index to the result.
 - $r \cap s$: output tuples in s_i to the result if they are already there in the hash index.
 - $r - s$: for each tuple in s_i , if it is there in the hash index, delete it from the index. At end of s_i add remaining tuples in the hash index to the result.





Other Operations : Outer Join

- **Outer join** can be computed either as
 - ★ A join followed by addition of null-padded non-participating tuples.
 - ★ by modifying the join algorithms.
- Modifying merge join to compute $r \sqsupset \bowtie s$
 - ★ In $r \sqsupset \bowtie s$, non participating tuples are those in $r - \Pi_R(r \bowtie s)$
 - ★ Modify merge-join to compute $r \sqsupset \bowtie s$: During merging, for every tuple t_r from r that do not match any tuple in s , output t_r padded with nulls.
 - ★ Right outer-join and full outer-join can be computed similarly.
- Modifying hash join to compute $r \sqsupset \bowtie s$
 - ★ If r is probe relation, output non-matching r tuples padded with nulls
 - ★ If r is build relation, when probing keep track of which r tuples matched s tuples. At end of s_i output non-matched r tuples padded with nulls



Evaluation of Expressions

- So far: we have seen algorithms for individual operations
- Alternatives for evaluating an entire expression tree
 - ★ **Materialization**: generate results of an expression whose inputs are relations or are already computed, **materialize** (store) it on disk. Repeat.
 - ★ **Pipelining**: pass on tuples to parent operations even as an operation is being executed
- We study above alternatives in more detail



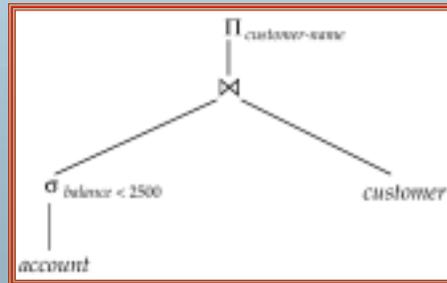


Materialization

- **Materialized evaluation:** evaluate one operation at a time, starting at the lowest-level. Use intermediate results materialized into temporary relations to evaluate next-level operations.
- E.g., in figure below, compute and store

$$\sigma_{balance < 2500}(account)$$

then compute the store its join with *customer*, and finally compute the projections on *customer-name*.



Materialization (Cont.)

- Materialized evaluation is always applicable
- Cost of writing results to disk and reading them back can be quite high
 - ★ Our cost formulas for operations ignore cost of writing results to disk, so
 - Overall cost = Sum of costs of individual operations + cost of writing intermediate results to disk
- **Double buffering:** use two output buffers for each operation, when one is full write it to disk while the other is getting filled
 - ★ Allows overlap of disk writes with computation and reduces execution time





Pipelining

- **Pipelined evaluation** : evaluate several operations simultaneously, passing the results of one operation on to the next.
- E.g., in previous expression tree, don't store result of

$$\sigma_{balance < 2500}(account)$$

- ★ instead, pass tuples directly to the join.. Similarly, don't store result of join, pass tuples directly to projection.
- Much cheaper than materialization: no need to store a temporary relation to disk.
- Pipelining may not always be possible – e.g., sort, hash-join.
- For pipelining to be effective, use evaluation algorithms that generate output tuples even as tuples are received for inputs to the operation.
- Pipelines can be executed in two ways: **demand driven** and **producer driven**



Pipelining (Cont.)

- In **demand driven** or **lazy** evaluation
 - ★ system repeatedly requests next tuple from top level operation
 - ★ Each operation requests next tuple from children operations as required, in order to output its next tuple
 - ★ In between calls, operation has to maintain "**state**" so it knows what to return next
 - ★ Each operation is implemented as an **iterator** implementing the following operations
 - open()
 - E.g. file scan: initialize file scan, store pointer to beginning of file as state
 - E.g. merge join: sort relations and store pointers to beginning of sorted relations as state
 - next()
 - E.g. for file scan: Output next tuple, and advance and store file pointer
 - E.g. for merge join: continue with merge from earlier state till next output tuple is found. Save pointers as iterator state.
 - close()





Pipelining (Cont.)

- In produce-driven or **eager** pipelining
 - ★ Operators produce tuples eagerly and pass them up to their parents
 - Buffer maintained between operators, child puts tuples in buffer, parent removes tuples from buffer
 - if buffer is full, child waits till there is space in the buffer, and then generates more tuples
 - ★ System schedules operations that have space in output buffer and can process more input tuples



Evaluation Algorithms for Pipelining

- Some algorithms are not able to output results even as they get input tuples
 - ★ E.g. merge join, or hash join
 - ★ These result in intermediate results being written to disk and then read back always
- Algorithm variants are possible to generate (at least some) results on the fly, as input tuples are read in
 - ★ E.g. hybrid hash join generates output tuples even as probe relation tuples in the in-memory partition (partition 0) are read in
 - ★ **Pipelined join technique**: Hybrid hash join, modified to buffer partition 0 tuples of both relations in-memory, reading them as they become available, and output results of any matches between partition 0 tuples
 - When a new r_0 tuple is found, match it with existing s_0 tuples, output matches, and save it in r_0
 - Symmetrically for s_0 tuples





Complex Joins

- Join involving three relations: $loan \bowtie depositor \bowtie customer$
- **Strategy 1.** Compute $depositor \bowtie customer$; use result to compute $loan \bowtie (depositor \bowtie customer)$
- **Strategy 2.** Compute $loan \bowtie depositor$ first, and then join the result with $customer$.
- **Strategy 3.** Perform the pair of joins at once. Build and index on $loan$ for $loan\text{-}number$, and on $customer$ for $customer\text{-}name$.
 - ★ For each tuple t in $depositor$, look up the corresponding tuples in $customer$ and the corresponding tuples in $loan$.
 - ★ Each tuple of $depositor$ is examined exactly once.
- Strategy 3 combines two operations into one special-purpose operation that is more efficient than implementing two joins of two relations.

