Chapter 3: Relational Model

- Structure of Relational Databases
- Relational Algebra
- Tuple Relational Calculus
- Domain Relational Calculus
- Extended Relational-Algebra-Operations
- Modification of the Database
- Views

Example of a Relation

account-number	*branch-name*	*balance*
A-101	Downtown	500
A-102	Perryridge	400
A-201	Brighton	900
A-215	Mianus	700
A-217	Brighton	750
A-222	Redwood	700
A-305	Round Hill	350

Basic Structure

- Formally, given sets D_1, D_2, …. D_n a **relation** *r* is a subset of $D_1 \times D_2 \times \ldots \times D_n$
  Thus a relation is a set of n-tuples $(a_1, a_2, \ldots, a_n)$ where each $a_i \in D_i$
- Example: if

  customer-name = {Jones, Smith, Curry, Lindsay}
  customer-street = {Main, North, Park}
  customer-city = {Harrison, Rye, Pittsfield}

  Then *r* = { (Jones, Main, Harrison),
  (Smith, North, Rye),
  (Curry, North, Rye),
  (Lindsay, Park, Pittsfield)}

  is a relation over *customer-name* x *customer-street* x *customer-city*

Attribute Types

- Each attribute of a relation has a name
- The set of allowed values for each attribute is called the **domain** of the attribute
- Attribute values are (normally) required to be **atomic**, that is, indivisible
  - E.g. multivalued attribute values are not atomic
  - E.g. composite attribute values are not atomic
- The special value *null* is a member of every domain
- The null value causes complications in the definition of many operations
  - we shall ignore the effect of null values in our main presentation and consider their effect later

Relation Schema

- $A_1, A_2, \ldots, A_n$ are *attributes*
- $R = (A_1, A_2, \ldots, A_n)$ is a *relation schema*

  E.g. *Customer-schema* =
  (*customer-name, customer-street, customer-city*)
- $r(R)$ is a *relation* on the *relation schema R*

  E.g. *customer (Customer-schema)*

Relation Instance

- The current values (*relation instance*) of a relation are specified by a table
- An element *t* of *r* is a *tuple*, represented by a *row* in a table

attributes (or columns)

customer-name	*customer-street*	customer-city
Jones	Main	Harrison
Smith	North	Rye
Curry	North	Rye
Lindsay	Park	Pittsfield

tuples (or rows)

customer

Relations are Unordered

- Order of tuples is irrelevant (tuples may be stored in an arbitrary order)
- E.g. *account* relation with unordered tuples

account-number	*branch-name*	*balance*
A-101	Downtown	500
A-215	Mianus	700
A-102	Perryridge	400
A-305	Round Hill	350
A-201	Brighton	900
A-222	Redwood	700
A-217	Brighton	750

Database

- A database consists of multiple relations
- Information about an enterprise is broken up into parts, with each relation storing one part of the information

  E.g.: *account* : stores information about accounts
  depositor : stores information about which customer owns which account
  customer : stores information about customers

- Storing all information as a single relation such as
  bank(*account-number, balance, customer-name*, ..)
  results in
  - repetition of information (e.g. two customers own an account)
  - the need for null values (e.g. represent a customer without an account)
- Normalization theory (Chapter 7) deals with how to design relational schemas

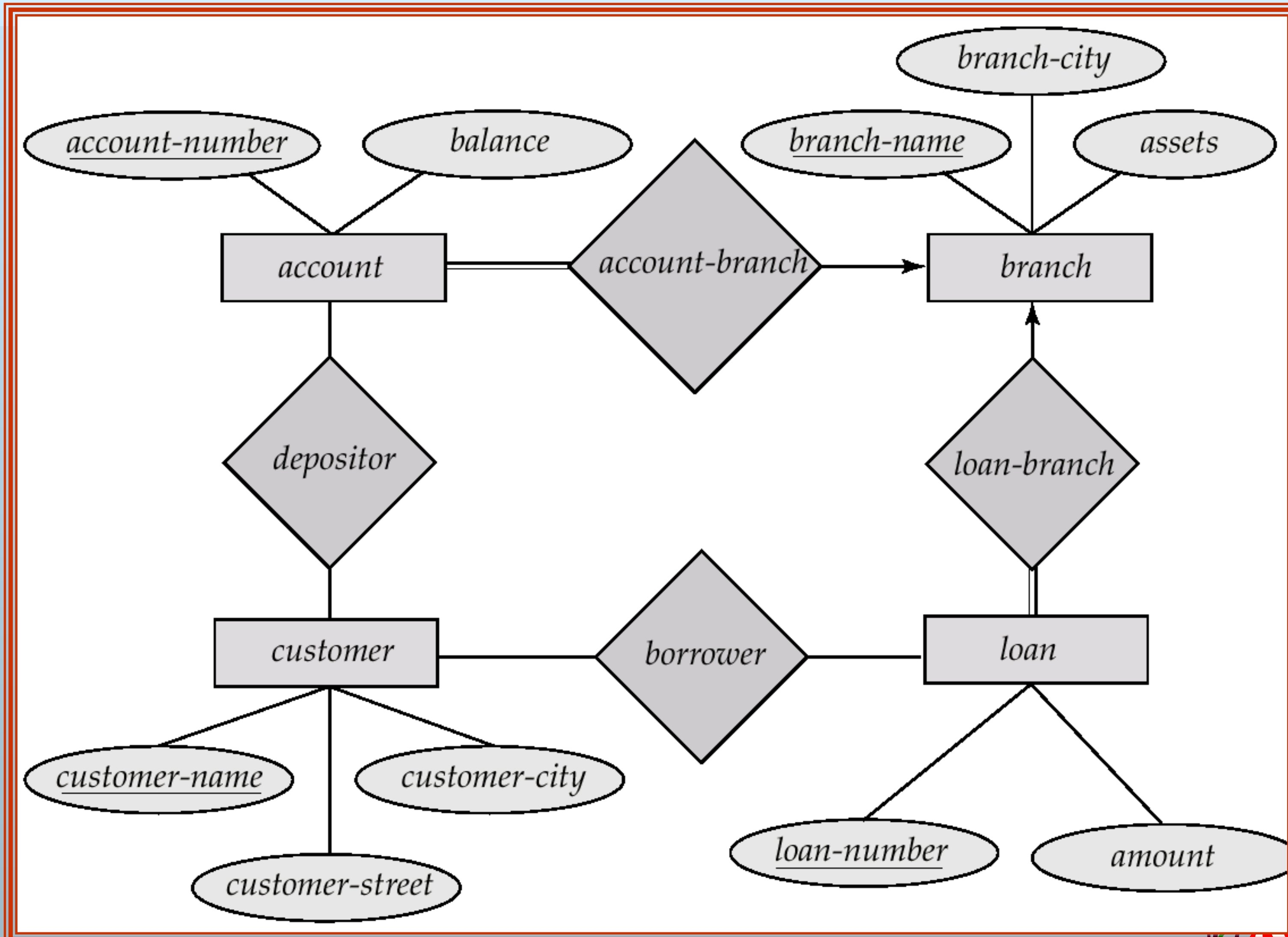
The *customer* Relation

customer-name	*customer-street*	*customer-city*
Adams	Spring	Pittsfield
Brooks	Senator	Brooklyn
Curry	North	Rye
Glenn	Sand Hill	Woodside
Green	Walnut	Stamford
Hayes	Main	Harrison
Johnson	Alma	Palo Alto
Jones	Main	Harrison
Lindsay	Park	Pittsfield
Smith	North	Rye
Turner	Putnam	Stamford
Williams	Nassau	Princeton

The *depositor* Relation

customer-name	*account-number*
Hayes	A-102
Johnson	A-101
Johnson	A-201
Jones	A-217
Lindsay	A-222
Smith	A-215
Turner	A-305

E-R Diagram for the Banking Enterprise



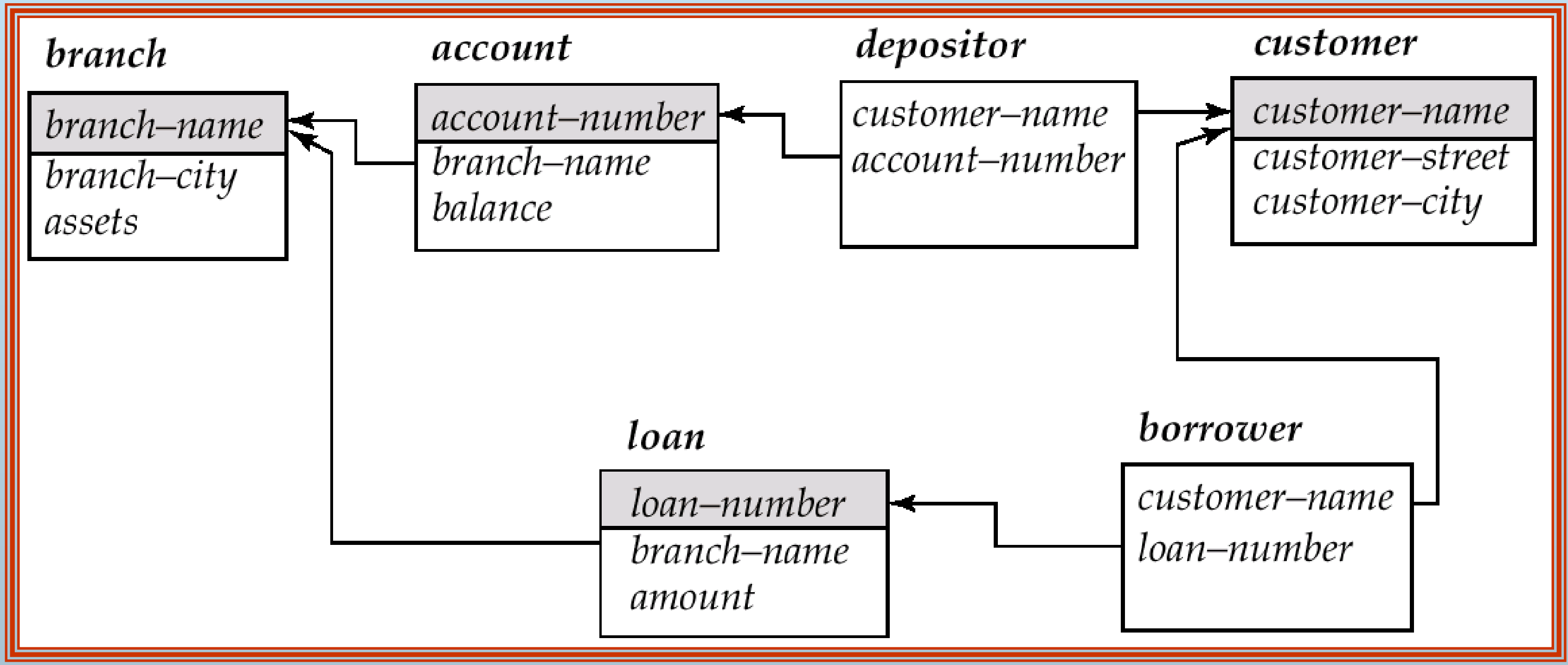
Keys

- Let K $\subseteq$ R
- *K* is a ***superkey*** of *R* if values for *K* are sufficient to identify a unique tuple of each possible relation *r(R)*
  - by "possible *r*" we mean a relation *r* that could exist in the enterprise we are modeling.
  - Example: {*customer-name, customer-street*} and {*customer-name*} are both superkeys of *Customer*, if no two customers can possibly have the same name.
- *K* is a ***candidate key*** if *K* is minimal
  Example: {*customer-name*} is a candidate key for *Customer*, since it is a superkey (assuming no two customers can possibly have the same name), and no subset of it is a superkey.

Determining Keys from E-R Sets

- **Strong entity set**. The primary key of the entity set becomes the primary key of the relation.
- **Weak entity set**. The primary key of the relation consists of the union of the primary key of the strong entity set and the discriminator of the weak entity set.
- **Relationship set**. The union of the primary keys of the related entity sets becomes a super key of the relation.
  - For binary many-to-one relationship sets, the primary key of the "many" entity set becomes the relation's primary key.
  - For one-to-one relationship sets, the relation's primary key can be that of either entity set.
  - For many-to-many relationship sets, the union of the primary keys becomes the relation's primary key

Schema Diagram for the Banking Enterprise



Query Languages

- Language in which user requests information from the database.
- Categories of languages
  - procedural
  - non-procedural
- "Pure" languages:
  - Relational Algebra
  - Tuple Relational Calculus
  - Domain Relational Calculus
- Pure languages form underlying basis of query languages that people use.

Relational Algebra

- Procedural language
- Six basic operators
  - select
  - project
  - union
  - set difference
  - Cartesian product
  - rename
- The operators take two or more relations as inputs and give a new relation as a result.

Select Operation – Example

- Relation *r*

A	*B*	*C*	*D*
α	α	*1*	*7*
α	β	*5*	*7*
β	β	*12*	*3*
β	β	*23*	*10*

- $\sigma_{A=B \land D > 5}(r)$

A	*B*	*C*	*D*
α	α	*1*	*7*
β	β	*23*	*10*

Select Operation

- Notation: $\sigma_p(r)$
- *p* is called the selection predicate
- Defined as:

$$\sigma_p(\mathbf{r}) = \{t \mid t \in r \textbf{ and } p(t)\}$$

  Where *p* is a formula in propositional calculus consisting of terms connected by : ∧ (**and**), ∨ (**or**), ¬ (**not**)
  Each term is one of:

  <attribute> *op* <attribute> or <constant>

  where *op* is one of: =, ≠, >, ≥. <. ≤

- Example of selection:

$$\sigma_{\textit{branch-name}=\text{“Perryridge”}}(\textit{account})$$

Project Operation – Example

- Relation *r*:

A	B	C
α	10	1
α	20	1
β	30	1
β	40	2

- $\Pi_{A,C}(r)$

A	C
α	1
α	1
β	1
β	2

=

A	C
α	1
β	1
β	2

Project Operation

- Notation:

$$\Pi_{A1,\ A2,\ \ldots,\ Ak}\ (r)$$

where A_1, A_2 are attribute names and r is a relation name.

- The result is defined as the relation of k columns obtained by erasing the columns that are not listed
- Duplicate rows removed from result, since relations are sets
- E.g. To eliminate the *branch-name* attribute of *account*

$$\Pi_{account\text{-}number,\ balance}\ (account)$$

Union Operation – Example

- Relations *r, s:*

r

A	B
α	1
α	2
β	1

s

A	B
α	2
β	3

$r \cup s$:

A	B
α	1
α	2
β	1
β	3

Union Operation

- Notation: $r \cup s$
- Defined as:

$$r \cup s = \{t \mid t \in r \text{ or } t \in s\}$$

- For $r \cup s$ to be valid.
  1. *r, s* must have the *same arity* (same number of attributes)
  2. The attribute domains must be *compatible* (e.g., 2nd column of *r* deals with the same type of values as does the 2nd column of *s*)
- E.g. to find all customers with either an account or a loan

$$\Pi_{customer\text{-}name}\,(depositor) \;\cup\; \Pi_{customer\text{-}name}\,(borrower)$$

Set Difference Operation – Example

- Relations *r, s:*

A	B
α	1
α	2
β	1

r

A	B
α	2
β	3

s

r – s:

A	B
α	1
β	1

Set Difference Operation

- Notation $r - s$
- Defined as:

$$r - s = \{t \mid t \in r \textbf{ and } t \notin s\}$$

- Set differences must be taken between *compatible* relations.
  - *r* and *s* must have the *same arity*
  - attribute domains of *r* and *s* must be compatible

Cartesian-Product Operation-Example

Relations *r, s*:

A	B
α	1
β	2

r

C	D	E
α	10	a
β	10	a
β	20	b
γ	10	b

s

r x *s*:

A	B	C	D	E
α	1	α	10	a
α	1	β	10	a
α	1	β	20	b
α	1	γ	10	b
β	2	α	10	a
β	2	β	10	a
β	2	β	20	b
β	2	γ	10	b

Cartesian-Product Operation

- Notation *r* x *s*
- Defined as:

$$r \times s = \{t\, q \mid t \in r \textbf{ and } q \in s\}$$

- Assume that attributes of r(R) and s(S) are disjoint. (That is, $R \cap S = \emptyset$).
- If attributes of *r(R)* and *s(S*) are not disjoint, then renaming must be used.

Composition of Operations

- Can build expressions using multiple operations
- Example: $\sigma_{A=C}(r \times s)$
- $r \times s$

A	B	C	D	E
α	1	α	10	a
α	1	β	10	a
α	1	β	20	b
α	1	γ	10	b
β	2	α	10	a
β	2	β	10	a
β	2	β	20	b
β	2	γ	10	b

- $\sigma_{A=C}(r \times s)$

A	B	C	D	E
α	1	α	10	a
β	2	β	20	a
β	2	β	20	b

Rename Operation

- Allows us to name, and therefore to refer to, the results of relational-algebra expressions.
- Allows us to refer to a relation by more than one name.

Example:

$$\rho_X(E)$$

returns the expression *E* under the name *X*

If a relational-algebra expression *E* has arity *n*, then

$$\rho_{X(A1, A2, \ldots, An)}(E)$$

returns the result of expression *E* under the name *X*, and with the attributes renamed to *A1, A2, …., An*.

Banking Example

branch (branch-name, branch-city, assets)

customer (customer-name, customer-street, customer-only)

account (account-number, branch-name, balance)

loan (loan-number, branch-name, amount)

depositor (customer-name, account-number)

borrower (customer-name, loan-number)

Example Queries

- Find all loans of over $1200

$$\sigma_{amount\ >\ 1200}\ (loan)$$

- Find the loan number for each loan of an amount greater than $1200

$$\Pi_{loan\text{-}number}\ (\sigma_{amount\ >\ 1200}\ (loan))$$

Example Queries

- Find the names of all customers who have a loan, an account, or both, from the bank

$$\Pi_{customer\text{-}name}\,(borrower) \cup \Pi_{customer\text{-}name}\,(depositor)$$

- Find the names of all customers who have a loan and an account at bank.

$$\Pi_{customer\text{-}name}\,(borrower) \cap \Pi_{customer\text{-}name}\,(depositor)$$

Example Queries

- Find the names of all customers who have a loan at the Perryridge branch.

$$\Pi_{customer\text{-}name}\ (\sigma_{branch\text{-}name=\text{“Perryridge”}}\ (\sigma_{borrower.loan\text{-}number\ =\ loan.loan\text{-}number}(borrower\ x\ loan)))$$

- Find the names of all customers who have a loan at the Perryridge branch but do not have an account at any branch of the bank.

$$\Pi_{customer\text{-}name}\ (\sigma_{branch\text{-}name\ =\ \text{“Perryridge”}}\ (\sigma_{borrower.loan\text{-}number\ =\ loan.loan\text{-}number}(\text{borrower x loan})))\ -\ \Pi_{customer\text{-}name}(\text{depositor})$$

Example Queries

- Find the names of all customers who have a loan at the Perryridge branch.
  - Query 1

$$\Pi_{customer\text{-}name}(\sigma_{branch\text{-}name = \text{“Perryridge”}}(\sigma_{borrower.loan\text{-}number = loan.loan\text{-}number}(borrower \times loan)))$$

  - Query 2

$$\Pi_{customer\text{-}name}(\sigma_{loan.loan\text{-}number = borrower.loan\text{-}number}((\sigma_{branch\text{-}name = \text{“Perryridge”}}(loan)) \times borrower))$$

Example Queries

Find the largest account balance

- Rename *account* relation as *d*
- The query is:

$$\Pi_{balance}(account) - \Pi_{account.balance}$$
$$(\sigma_{account.balance < d.balance}(account \times \rho_d(account)))$$

Formal Definition

- A basic expression in the relational algebra consists of either one of the following:
  - A relation in the database
  - A constant relation
- Let E_1 and E_2 be relational-algebra expressions; the following are all relational-algebra expressions:
  - $E_1 \cup E_2$
  - $E_1 - E_2$
  - $E_1 \times E_2$
  - $\sigma_p (E_1)$, P is a predicate on attributes in E_1
  - $\Pi_S(E_1)$, S is a list consisting of some of the attributes in E_1
  - $\rho_x (E_1)$, x is the new name for the result of E_1

Additional Operations

We define additional operations that do not add any power to the relational algebra, but that simplify common queries.

- Set intersection
- Natural join
- Division
- Assignment

Set-Intersection Operation

- Notation: $r \cap s$
- Defined as:
- $r \cap s = \{ t \mid t \in r$ **and** $t \in s \}$
- Assume:
  - *r*, *s* have the *same arity*
  - attributes of r and s are compatible
- Note: $r \cap s = r - (r - s)$

Set-Intersection Operation - Example

- Relation r, s:

A	B
α	1
α	2
β	1

r

A	B
α	2
β	3

s

- r ∩ s

A	B
α	2

Natural-Join Operation

- Notation: r $\bowtie$ s
- Let *r* and *s* be relations on schemas *R* and *S* respectively. Then, r $\bowtie$ s is a relation on schema $R \cup S$ obtained as follows:
  - Consider each pair of tuples t_r from *r* and t_s from *s*.
  - If t_r and t_s have the same value on each of the attributes in $R \cap S$, add a tuple *t* to the result, where
    - *t* has the same value as t_r on *r*
    - *t* has the same value as t_s on *s*
- Example:

  R = (*A, B, C, D*)

  S = (*E, B, D*)

  - Result schema = (*A, B, C, D, E*)
  - *r* $\bowtie$ *s* is defined as:

$$\Pi_{r.A,\ r.B,\ r.C,\ r.D,\ s.E}\ (\sigma_{r.B\ =\ s.B\ \wedge\ r.D\ =\ s.D}\ (r\ \ x\ \ s))$$

Natural Join Operation – Example

- Relations r, s:

A	B	C	D
α	1	α	a
β	2	γ	a
γ	4	β	b
α	1	γ	a
δ	2	β	b

r

B	D	E
1	a	α
3	a	β
1	a	γ
2	b	δ
3	b	$\in$

s

$r \bowtie s$

A	B	C	D	E
α	1	α	a	α
α	1	α	a	γ
α	1	γ	a	α
α	1	γ	a	γ
δ	2	β	b	δ

Division Operation

$$r \div s$$

- Suited to queries that include the phrase "for all".
- Let *r* and *s* be relations on schemas R and S respectively where
  - $R = (A_1, \ldots, A_m, B_1, \ldots, B_n)$
  - $S = (B_1, \ldots, B_n)$

  The result of $r \div s$ is a relation on schema

  $R - S = (A_1, \ldots, A_m)$

$$r \div s = \{ t \mid t \in \Pi_{R-S}(r) \wedge \forall\, u \in s \,(tu \in r) \}$$

Division Operation – Example

Relations *r, s*:

A	B
α	1
α	2
α	3
β	1
γ	1
δ	1
δ	3
δ	4
$\in$	6
$\in$	1
β	2

r

B
1
2

s

$r \div s$:

A
α
β

Another Division Example

Relations *r, s*:

A	*B*	*C*	*D*	*E*
α	a	α	a	1
α	a	γ	a	1
α	a	γ	b	1
β	a	γ	a	1
β	a	γ	b	3
γ	a	γ	a	1
γ	a	γ	b	1
γ	a	β	b	1

r

D	*E*
a	1
b	1

s

$r \div s$:

A	*B*	*C*
α	a	γ
γ	a	γ

Division Operation (Cont.)

- Property
  - Let $q - r \div s$
  - Then q is the largest relation satisfying $q \times s \subseteq r$
- Definition in terms of the basic algebra operation
  Let $r(R)$ and $s(S)$ be relations, and let $S \subseteq R$

$$r \div s = \Pi_{R-S}(r) - \Pi_{R-S}((\Pi_{R-S}(r) \times s) - \Pi_{R-S,S}(r))$$

To see why

- $\Pi_{R-S,S}(r)$ simply reorders attributes of r
- $\Pi_{R-S}(\Pi_{R-S}(r) \times s) - \Pi_{R-S,S}(r))$ gives those tuples t in

  $\Pi_{R-S}(r)$ such that for some tuple $u \in s$, $tu \notin r$.

Assignment Operation

- The assignment operation ($\leftarrow$) provides a convenient way to express complex queries.
  - Write query as a sequential program consisting of
    - a series of assignments
    - followed by an expression whose value is displayed as a result of the query.
  - Assignment must always be made to a temporary relation variable.
- Example: Write $r \div s$ as

$$temp1 \leftarrow \Pi_{R-S}(r)$$

$$temp2 \leftarrow \Pi_{R-S}((temp1 \times s) - \Pi_{R-S,S}(r))$$

$$result = temp1 - temp2$$

  - The result to the right of the $\leftarrow$ is assigned to the relation variable on the left of the $\leftarrow$.
  - May use variable in subsequent expressions.

Example Queries

- Find all customers who have an account from at least the "Downtown" and the Uptown" branches.

Query 1

$$\Pi_{CN}(\sigma_{BN=\text{"Downtown"}}(depositor \bowtie account)) \cap$$

$$\Pi_{CN}(\sigma_{BN=\text{"Uptown"}}(depositor \bowtie account))$$

where *CN* denotes customer-name and *BN* denotes *branch-name*.

Query 2

$$\Pi_{customer\text{-}name,\ branch\text{-}name}\ (depositor \bowtie account)$$
$$\div\ \rho_{temp(branch\text{-}name)}\ (\{(\text{"Downtown"}),\ (\text{"Uptown"})\})$$

Example Queries

- Find all customers who have an account at all branches located in Brooklyn city.

$$\Pi_{customer\text{-}name,\ branch\text{-}name}\ (depositor \bowtie account)$$
$$\div\ \Pi_{branch\text{-}name}\ (\sigma_{branch\text{-}city\ =\ \text{“Brooklyn”}}\ (branch))$$

Extended Relational-Algebra-Operations

- Generalized Projection
- Outer Join
- Aggregate Functions

Generalized Projection

- Extends the projection operation by allowing arithmetic functions to be used in the projection list.

$$\Pi_{F1, F2, \ldots, Fn}(E)$$

- *E* is any relational-algebra expression
- Each of $F_1, F_2, \ldots, F_n$ are are arithmetic expressions involving constants and attributes in the schema of *E*.
- Given relation *credit-info(customer-name, limit, credit-balance),* find how much more each person can spend:

$$\Pi_{customer\text{-}name,\ limit\ -\ credit\text{-}balance}\ (credit\text{-}info)$$

Aggregate Functions and Operations

- **Aggregation function** takes a collection of values and returns a single value as a result.

  avg: average value
  min: minimum value
  max: maximum value
  sum: sum of values
  count: number of values

- **Aggregate operation** in relational algebra

$$_{G1,\ G2,\ \ldots,\ Gn}\ \mathcal{G}\ _{F1(A1),\ F2(A2),\ldots,\ Fn(An)}\ (E)$$

  - E is any relational-algebra expression
  - G_1, G_2 …, G_n is a list of attributes on which to group (can be empty)
  - Each F_i is an aggregate function
  - Each A_i is an attribute name

Aggregate Operation – Example

- Relation *r*:

A	*B*	*C*
α	α	7
α	β	7
β	β	3
β	β	10

$\mathcal{G}_{\mathbf{sum(c)}}(\mathrm{r})$

sum-C
27

Aggregate Operation – Example

- Relation *account* grouped by *branch-name*:

branch-name	*account-number*	*balance*
Perryridge	A-102	400
Perryridge	A-201	900
Brighton	A-217	750
Brighton	A-215	750
Redwood	A-222	700

$$_{branch\text{-}name}\,\mathcal{G}_{\,sum(balance)}\,(account)$$

branch-name	*balance*
Perryridge	1300
Brighton	1500
Redwood	700

Aggregate Functions (Cont.)

- Result of aggregation does not have a name
  - Can use rename operation to give it a name
  - For convenience, we permit renaming as part of aggregate operation

$$_{\textit{branch-name}}\, \mathcal{G}\, _{\textbf{sum}(\textit{balance})\ \textbf{as}\ \textit{sum-balance}}\, (\textit{account})$$

Outer Join

- An extension of the join operation that avoids loss of information.
- Computes the join and then adds tuples form one relation that does not match tuples in the other relation to the result of the join.
- Uses *null* values:
  - *null* signifies that the value is unknown or does not exist
  - All comparisons involving *null* are (roughly speaking) **false** by definition.
    - Will study precise meaning of comparisons with nulls later

Outer Join – Example

- Relation *loan*

loan-number	*branch-name*	*amount*
L-170	Downtown	3000
L-230	Redwood	4000
L-260	Perryridge	1700

- Relation *borrower*

customer-name	*loan-number*
Jones	L-170
Smith	L-230
Hayes	L-155

Outer Join – Example

- **Inner Join**

loan ⋈ *Borrower*

loan-number	*branch-name*	*amount*	*customer-name*
L-170	Downtown	3000	Jones
L-230	Redwood	4000	Smith

- **Left Outer Join**

loan ⟕ *Borrower*

loan-number	*branch-name*	*amount*	*customer-name*
L-170	Downtown	3000	Jones
L-230	Redwood	4000	Smith
L-260	Perryridge	1700	*null*

Outer Join – Example

- **Right Outer Join**

  loan ⟖ *borrower*

loan-number	*branch-name*	*amount*	*customer-name*
L-170	Downtown	3000	Jones
L-230	Redwood	4000	Smith
L-155	*null*	*null*	Hayes

- **Full Outer Join**

  loan ⟗ *borrower*

loan-number	*branch-name*	*amount*	*customer-name*
L-170	Downtown	3000	Jones
L-230	Redwood	4000	Smith
L-260	Perryridge	1700	*null*
L-155	*null*	*null*	Hayes

Null Values

- It is possible for tuples to have a null value, denoted by *null*, for some of their attributes
- *null* signifies an unknown value or that a value does not exist.
- The result of any arithmetic expression involving *null* is *null.*
- Aggregate functions simply ignore null values
  - Is an arbitrary decision. Could have returned null as result instead.
  - We follow the semantics of SQL in its handling of null values
- For duplicate elimination and grouping, null is treated like any other value, and two nulls are assumed to be the same
  - Alternative: assume each null is different from each other
  - Both are arbitrary decisions, so we simply follow SQL

Null Values

- Comparisons with null values return the special truth value *unknown*
  - If *false* was used instead of *unknown*, then *not (A < 5)* would not be equivalent to *A >= 5*
- Three-valued logic using the truth value *unknown*:
  - OR: (*unknown* **or** *true*) = *true*,
    (*unknown* **or** *false*) = *unknown*
    (*unknown* **or** *unknown*) = *unknown*
  - AND: *(true* **and** *unknown)* = *unknown,*
    (false **and** *unknown)* = *false,*
    (unknown **and** *unknown)* = *unknown*
  - NOT*: (***not** *unknown)* = *unknown*
  - In SQL "*P* **is unknown"** evaluates to true if predicate *P* evaluates to *unknown*
- Result of select predicate is treated as *false* if it evaluates to *unknown*

Modification of the Database

- The content of the database may be modified using the following operations:
  - Deletion
  - Insertion
  - Updating
- All these operations are expressed using the assignment operator.

Deletion

- A delete request is expressed similarly to a query, except instead of displaying tuples to the user, the selected tuples are removed from the database.
- Can delete only whole tuples; cannot delete values on only particular attributes
- A deletion is expressed in relational algebra by:

$$r \leftarrow r - E$$

where *r* is a relation and *E* is a relational algebra query.

Deletion Examples

- Delete all account records in the Perryridge branch.

$$account \leftarrow account - \sigma_{\text{branch-name} = \text{“Perryridge”}}(account)$$

- Delete all loan records with amount in the range of 0 to 50

$$loan \leftarrow loan - \sigma_{amount \geq 0 \text{ and } amount \leq 50}(loan)$$

- Delete all accounts at branches located in Needham.

$$r_1 \leftarrow \sigma_{\text{branch-city} = \text{“Needham”}}(account \bowtie branch)$$

$$r_2 \leftarrow \Pi_{\text{branch-name, account-number, balance}}(r_1)$$

$$r_3 \leftarrow \Pi_{\text{customer-name, account-number}}(r_2 \bowtie \text{depositor})$$

$$account \leftarrow \text{account} - r_2$$

$$depositor \leftarrow \text{depositor} - r_3$$

Insertion

- To insert data into a relation, we either:
  - specify a tuple to be inserted
  - write a query whose result is a set of tuples to be inserted
- in relational algebra, an insertion is expressed by:

$$r \leftarrow r \cup E$$

  where *r* is a relation and *E* is a relational algebra expression.
- The insertion of a single tuple is expressed by letting *E* be a constant relation containing one tuple.

Insertion Examples

- Insert information in the database specifying that Smith has $1200 in account A-973 at the Perryridge branch.

  $account \leftarrow account \cup \{(\text{“Perryridge”}, \text{A-973}, 1200)\}$

  $\text{depositor} \leftarrow depositor \cup \{(\text{“Smith”}, \text{A-973})\}$

- Provide as a gift for all loan customers in the Perryridge branch, a $200 savings account. Let the loan number serve as the account number for the new savings account.

  $r_1 \leftarrow (\sigma_{branch\text{-}name\ =\ \text{“}Perryridge\text{”}}(borrower \bowtie \text{loan}))$

  $account \leftarrow account \cup \Pi_{branch\text{-}name,\ account\text{-}number, 200}\ (r_1)$

  $\text{depositor} \leftarrow depositor \cup \Pi_{customer\text{-}name,\ loan\text{-}number}(r_1)$

Updating

- A mechanism to change a value in a tuple without charging *all* values in the tuple
- Use the generalized projection operator to do this task

$$r \leftarrow \Pi_{F1,\ F2,\ \ldots,\ Fl,}\ (r)$$

- Each F_i is either
  - the *i*th attribute of *r*, if the *i*th attribute is not updated, or,
  - if the attribute is to be updated F_i is an expression, involving only constants and the attributes of *r*, which gives the new value for the attribute

Update Examples

- Make interest payments by increasing all balances by 5 percent.

$$account \leftarrow \Pi_{AN,\ BN,\ BAL\ *\ 1.05}\ (account)$$

where *AN*, *BN* and *BAL* stand for *account-number*, *branch-name* and *balance*, respectively.

- Pay all accounts with balances over $10,000 6 percent interest and pay all others 5 percent

$$account \leftarrow \Pi_{AN,\ BN,\ BAL\ *\ 1.06}\ (\sigma_{BAL\ >\ 10000}\ (account)) \cup \Pi_{AN,\ BN,\ BAL\ *\ 1.05}\ (\sigma_{BAL\ \leq\ 10000}\ (account))$$

Views

- In some cases, it is not desirable for all users to see the entire logical model (i.e., all the actual relations stored in the database.)
- Consider a person who needs to know a customer's loan number but has no need to see the loan amount. This person should see a relation described, in the relational algebra, by

$$\Pi_{\textit{customer-name, loan-number}}(\textit{borrower} \bowtie \textit{loan})$$

- Any relation that is not of the conceptual model but is made visible to a user as a "virtual relation" is called a **view**.

View Definition

- A view is defined using the **create view** statement which has the form

  create view *v* **as** <query expression

  where <query expression> is any legal relational algebra query expression. The view name is represented by *v.*
- Once a view is defined, the view name can be used to refer to the virtual relation that the view generates.
- View definition is not the same as creating a new relation by evaluating the query expression
  - Rather, a view definition causes the saving of an expression; the expression is substituted into queries using the view.

View Examples

- Consider the view (named *all-customer*) consisting of branches and their customers.

  create view *all-customer* **as**

  $$\Pi_{branch\text{-}name,\ customer\text{-}name}\ (depositor \bowtie account)$$
  $$\cup\ \Pi_{branch\text{-}name,\ customer\text{-}name}\ (borrower \bowtie loan)$$

- We can find all customers of the Perryridge branch by writing:

  $$\Pi_{branch\text{-}name}$$
  $$(\sigma_{branch\text{-}name\ =\ \text{“Perryridge”}}\ (all\text{-}customer))$$

Updates Through View

- Database modifications expressed as views must be translated to modifications of the actual relations in the database.
- Consider the person who needs to see all loan data in the *loan* relation except *amount.* The view given to the person, *branch-loan,* is defined as:

  create view *branch-loan* **as**

  $$\Pi_{branch\text{-}name,\ loan\text{-}number}\ (loan)$$

- Since we allow a view name to appear wherever a relation name is allowed, the person may write:

$$branch\text{-}loan \leftarrow branch\text{-}loan \cup \{(\text{“Perryridge”}, \text{L-37})\}$$

Updates Through Views (Cont.)

- The previous insertion must be represented by an insertion into the actual relation *loan* from which the view *branch-loan* is constructed.
- An insertion into *loan* requires a value for *amount*. The insertion can be dealt with by either.
  - rejecting the insertion and returning an error message to the user.
  - inserting a tuple (“L-37”, “Perryridge”, *null*) into the *loan* relation
- Some updates through views are impossible to translate into database relation updates
  - create view v as $\sigma_{\textit{branch-name} = \text{“Perryridge”}}$ (*account*))

    $v \leftarrow v \cup$ (L-99, Downtown, 23)
- Others cannot be translated uniquely
  - *all-customer* $\leftarrow$ *all-customer* $\cup$ {(“Perryridge”, “John”)}
    - Have to choose loan or account, and create a new loan/account number!

Views Defined Using Other Views

- One view may be used in the expression defining another view
- A view relation v_1 is said to *depend directly* on a view relation v_2 if v_2 is used in the expression defining v_1
- A view relation v_1 is said to *depend on* view relation v_2 if either v_1 depends directly to v_2 or there is a path of dependencies from v_1 to v_2
- A view relation v is said to be *recursive* if it depends on itself.

View Expansion

- A way to define the meaning of views defined in terms of other views.
- Let view v_1 be defined by an expression e_1 that may itself contain uses of view relations.
- View expansion of an expression repeats the following replacement step:

  repeat
  Find any view relation v_i in e_1
  Replace the view relation v_i by the expression defining v_i
  until no more view relations are present in e_1

- As long as the view definitions are not recursive, this loop will terminate

Tuple Relational Calculus

- A nonprocedural query language, where each query is of the form

$$\{t \mid P(t)\}$$

- It is the set of all tuples *t* such that predicate *P* is true for *t*
- *t* is a *tuple variable*, *t*[*A*] denotes the value of tuple *t* on attribute *A*
- $t \in r$ denotes that tuple *t* is in relation *r*
- *P* is a *formula* similar to that of the predicate calculus

Predicate Calculus Formula

1. Set of attributes and constants
2. Set of comparison operators: (e.g., $<$, $\leq$, $=$, $\neq$, $>$, $\geq$)
3. Set of connectives: and ($\wedge$), or (v), not ($\neg$)
4. Implication ($\Rightarrow$): x $\Rightarrow$ y, if x if true, then y is true

$$x \Rightarrow y \equiv \neg x \vee y$$

5. Set of quantifiers:
   - $\exists\, t \in r\, (Q(t)) \equiv$ "there exists" a tuple in *t* in relation *r* such that predicate *Q(t)* is true
   - $\forall t \in r\, (Q(t)) \equiv Q$ is true "for all" tuples *t* in relation *r*

Banking Example

- *branch (branch-name, branch-city, assets)*
- *customer (customer-name, customer-street, customer-city)*
- *account (account-number, branch-name, balance)*
- *loan (loan-number, branch-name, amount)*
- *depositor (customer-name, account-number)*
- *borrower (customer-name, loan-number)*

Example Queries

- Find the *loan-number, branch-name,* and *amount* for loans of over $1200

$$\{t \mid t \in loan \wedge t\,[amount] > 1200\}$$

- Find the loan number for each loan of an amount greater than $1200

$$\{t \mid \exists\, s \in \text{loan}\ (t[\textit{loan-number}] = s[\textit{loan-number}] \wedge s\,[amount] > 1200)\}$$

Notice that a relation on schema [*loan-number*] is implicitly defined by the query

Example Queries

- Find the names of all customers having a loan, an account, or both at the bank

$$\{t \mid \exists s \in \textit{borrower}(\, t[\textit{customer-name}] = s[\textit{customer-name}])$$
$$\lor\ \exists u \in \textit{depositor}(\, t[\textit{customer-name}] = u[\textit{customer-name}])$$

- Find the names of all customers who have a loan and an account at the bank

$$\{t \mid \exists s \in \textit{borrower}(\, t[\textit{customer-name}] = s[\textit{customer-name}])$$
$$\land\ \exists u \in \textit{depositor}(\, t[\textit{customer-name}] = u[\textit{customer-name}])$$

Example Queries

- Find the names of all customers having a loan at the Perryridge branch

$$\{t \mid \exists s \in borrower(t[customer\text{-}name] = s[customer\text{-}name] \\ \land \exists u \in loan(u[branch\text{-}name] = \text{"Perryridge"} \\ \land\ u[loan\text{-}number] = s[loan\text{-}number]))\}$$

- Find the names of all customers who have a loan at the Perryridge branch, but no account at any branch of the bank

$$\{t \mid \exists s \in borrower(\ t[customer\text{-}name] = s[\text{customer-name}] \\ \land \exists u \in loan(u[branch\text{-}name] = \text{"Perryridge"} \\ \land\ u[loan\text{-}number] = s[\text{loan-}number])) \\ \land \textbf{ not } \exists v \in depositor\ (v[customer\text{-}name] = \\ t[\text{customer-name}])\ \}$$

Example Queries

- Find the names of all customers having a loan from the Perryridge branch, and the cities they live in

$$\{t \mid \exists s \in loan(s[branch\text{-}name] = \text{"Perryridge"} \\ \wedge \exists u \in borrower\ (u[loan\text{-}number] = s[loan\text{-}number] \\ \wedge\ t\,[customer\text{-}name] = u[customer\text{-}name]) \\ \wedge \exists\ v \in customer\ (u[customer\text{-}name] = v[customer\text{-}name] \\ \wedge\ t[customer\text{-}city] = v[customer\text{-}city])))\}$$

Example Queries

- Find the names of all customers who have an account at all branches located in Brooklyn:

$$\{t \mid \exists\, c \in \text{customer } (t[\text{customer.name}] = c[\text{customer-name}]) \wedge$$
$$\forall\, s \in \textit{branch}(s[\textit{branch-city}] = \text{"Brooklyn"} \Rightarrow$$
$$\exists\, u \in \textit{account } (s[\textit{branch-name}] = u[\text{branch-name}]$$
$$\wedge\, \exists\, s \in \textit{depositor } (t[\textit{customer-name}] = s[\text{customer-name}]$$
$$\wedge\; s[\textit{account-number}] = u[\text{account-number}]))\,)\}$$

Safety of Expressions

- It is possible to write tuple calculus expressions that generate infinite relations.
- For example, $\{t \mid \neg\, t \in r\}$ results in an infinite relation if the domain of any attribute of relation *r* is infinite
- To guard against the problem, we restrict the set of allowable expressions to safe expressions.
- An expression $\{t \mid P(t)\}$ in the tuple relational calculus is *safe* if every component of *t* appears in one of the relations, tuples, or constants that appear in *P*
  - NOTE: this is more than just a syntax condition.
    - E.g. $\{ t \mid t[A]=5 \lor$ **true** $\}$ is not safe --- it defines an infinite set with attribute values that do not appear in any relation or tuples or constants in *P*.

Domain Relational Calculus

- A nonprocedural query language equivalent in power to the tuple relational calculus
- Each query is an expression of the form:

$$\{ < x_1, x_2, \ldots, x_n > \mid P(x_1, x_2, \ldots, x_n)\}$$

  - $x_1, x_2, \ldots, x_n$ represent domain variables
  - P represents a formula similar to that of the predicate calculus

Example Queries

- Find the *loan-number, branch-name,* and *amount* for loans of over \$1200

$$\{< l, b, a > \mid < l, b, a > \in loan \wedge a > 1200\}$$

- Find the names of all customers who have a loan of over \$1200

$$\{< c > \mid \exists\ l, b, a\ (< c, l > \in borrower \wedge < l, b, a > \in loan \wedge a > 1200)\}$$

- Find the names of all customers who have a loan from the Perryridge branch and the loan amount:

$$\{< c, a > \mid \exists\ l\ (< c, l > \in borrower \wedge \exists b(< l, b, a > \in loan \wedge b = \text{“Perryridge”}))\}$$

or $\{< c, a > \mid \exists\ l\ (< c, l > \in borrower \wedge < l, \text{“Perryridge”}, a > \in loan)\}$

Example Queries

- Find the names of all customers having a loan, an account, or both at the Perryridge branch:

$$\{< c > \mid \exists\, l\ (\{< c, l > \in \mathit{borrower}$$
$$\wedge\ \exists\, b,a(< l, b, a > \in \mathit{loan} \wedge b = \text{“Perryridge”}))$$
$$\vee\ \exists\, a(< c, a > \in \mathit{depositor}$$
$$\wedge\ \exists\, b,n(< a, b, n > \in \mathit{account} \wedge b = \text{“Perryridge”}))\}$$

- Find the names of all customers who have an account at all branches located in Brooklyn:

$$\{< c > \mid \exists\, \text{s}, n\ (< c, s, n > \in \text{customer}) \wedge$$
$$\forall\, x,y,z(< x, y, z > \in \mathit{branch} \wedge y = \text{“Brooklyn”}) \Rightarrow$$
$$\exists\, a,b(< x, y, z > \in \mathit{account} \wedge < c,a > \in \mathit{depositor})\}$$

Safety of Expressions

$$\{ < x_1, x_2, \ldots, x_n > \mid P(x_1, x_2, \ldots, x_n)\}$$

is safe if all of the following hold:

1. All values that appear in tuples of the expression are values from *dom*(P) (that is, the values appear either in P or in a tuple of a relation mentioned in P).
2. For every "there exists" subformula of the form $\exists\ x\ (P_1(x))$, the subformula is true if and only if there is a value of x in *dom*(P_1) such that $P_1(x)$ is true.
3. For every "for all" subformula of the form $\forall_x\ (P_1\ (x))$, the subformula is true if and only if $P_1(x)$ is true for all values x from *dom* (P_1).

End of Chapter 3

Result of $\sigma_{\text{branch-name = “Perryridge”}}$ (*loan*)

loan-number	*branch-name*	*amount*
L-15	Perryridge	1500
L-16	Perryridge	1300

Loan Number and the Amount of the Loan

loan-number	*amount*
L-11	900
L-14	1500
L-15	1500
L-16	1300
L-17	1000
L-23	2000
L-93	500

Names of All Customers Who Have Either a Loan or an Account

customer-name
Adams
Curry
Hayes
Jackson
Jones
Smith
Williams
Lindsay
Johnson
Turner

Customers With An Account But No Loan

customer-name
Johnson
Lindsay
Turner

Result of *borrower* × *loan*

customer-name	*borrower. loan-number*	*loan. loan-number*	*branch-name*	*amount*
Adams	L-16	L-11	Round Hill	900
Adams	L-16	L-14	Downtown	1500
Adams	L-16	L-15	Perryridge	1500
Adams	L-16	L-16	Perryridge	1300
Adams	L-16	L-17	Downtown	1000
Adams	L-16	L-23	Redwood	2000
Adams	L-16	L-93	Mianus	500
Curry	L-93	L-11	Round Hill	900
Curry	L-93	L-14	Downtown	1500
Curry	L-93	L-15	Perryridge	1500
Curry	L-93	L-16	Perryridge	1300
Curry	L-93	L-17	Downtown	1000
Curry	L-93	L-23	Redwood	2000
Curry	L-93	L-93	Mianus	500
Hayes	L-15	L-11		900
Hayes	L-15	L-14		1500
Hayes	L-15	L-15		1500
Hayes	L-15	L-16		1300
Hayes	L-15	L-17		1000
Hayes	L-15	L-23		2000
Hayes	L-15	L-93		500
...	...	...	...	...
...	...	...	...	...
...	...	...	...	...
Smith	L-23	L-11	Round Hill	900
Smith	L-23	L-14	Downtown	1500
Smith	L-23	L-15	Perryridge	1500
Smith	L-23	L-16	Perryridge	1300
Smith	L-23	L-17	Downtown	1000
Smith	L-23	L-23	Redwood	2000
Smith	L-23	L-93	Mianus	500
Williams	L-17	L-11	Round Hill	900
Williams	L-17	L-14	Downtown	1500
Williams	L-17	L-15	Perryridge	1500
Williams	L-17	L-16	Perryridge	1300
Williams	L-17	L-17	Downtown	1000
Williams	L-17	L-23	Redwood	2000
Williams	L-17	L-93	Mianus	500

Result of $\sigma_{\text{branch-name} = \text{"Perryridge"}}$ (*borrower* × *loan*)

customer-name	*borrower. loan-number*	*loan. loan-number*	*branch-name*	*amount*
Adams	L-16	L-15	Perryridge	1500
Adams	L-16	L-16	Perryridge	1300
Curry	L-93	L-15	Perryridge	1500
Curry	L-93	L-16	Perryridge	1300
Hayes	L-15	L-15	Perryridge	1500
Hayes	L-15	L-16	Perryridge	1300
Jackson	L-14	L-15	Perryridge	1500
Jackson	L-14	L-16	Perryridge	1300
Jones	L-17	L-15	Perryridge	1500
Jones	L-17	L-16	Perryridge	1300
Smith	L-11	L-15	Perryridge	1500
Smith	L-11	L-16	Perryridge	1300
Smith	L-23	L-15	Perryridge	1500
Smith	L-23	L-16	Perryridge	1300
Williams	L-17	L-15	Perryridge	1500
Williams	L-17	L-16	Perryridge	1300

Result of $\Pi_{customer\text{-}name}$

customer-name
Adams
Hayes

Result of the Subexpression

balance
500
400
700
750
350

Largest Account Balance in the Bank

balance
900

Customers Who Live on the Same Street and In the Same City as Smith

customer-name
Curry
Smith

Customers With Both an Account and a Loan at the Bank

customer-name
Hayes
Jones
Smith

Result of $\Pi_{\textit{customer-name, loan-number, amount}}$ (*borrower* $\bowtie$ *loan*)

customer-name	*loan-number*	*amount*
Adams	L-16	1300
Curry	L-93	500
Hayes	L-15	1500
Jackson	L-14	1500
Jones	L-17	1000
Smith	L-23	2000
Smith	L-11	900
Williams	L-17	1000

Result of $\Pi_{branch\text{-}name}(\sigma_{customer\text{-}city = \text{“Harrison”}}(customer \bowtie account \bowtie depositor))$

branch-name
Brighton
Perryridge

Result of $\Pi_{branch\text{-}name}(\sigma_{branch\text{-}city = \text{“Brooklyn”}}(branch))$

branch-name
Brighton
Downtown

Result of $\Pi_{customer\text{-}name,\ branch\text{-}name}(depositor \bowtie account)$

customer-name	*branch-name*
Hayes	Perryridge
Johnson	Downtown
Johnson	Brighton
Jones	Brighton
Lindsay	Redwood
Smith	Mianus
Turner	Round Hill

The *credit-info* Relation

customer-name	*branch-name*
Hayes	Perryridge
Johnson	Downtown
Johnson	Brighton
Jones	Brighton
Lindsay	Redwood
Smith	Mianus
Turner	Round Hill

Result of $\Pi_{customer\text{-}name,\ (limit\ -\ credit\text{-}balance)\ as\ credit\text{-}available}(credit\text{-}info)$.

customer-name	*credit-available*
Curry	250
Jones	5300
Smith	1600
Hayes	0

The *pt-works* Relation

employee-name	*branch-name*	*salary*
Adams	Perryridge	1500
Brown	Perryridge	1300
Gopal	Perryridge	5300
Johnson	Downtown	1500
Loreena	Downtown	1300
Peterson	Downtown	2500
Rao	Austin	1500
Sato	Austin	1600

The *pt-works* Relation After Grouping

employee-name	*branch-name*	*salary*
Rao	Austin	1500
Sato	Austin	1600
Johnson	Downtown	1500
Loreena	Downtown	1300
Peterson	Downtown	2500
Adams	Perryridge	1500
Brown	Perryridge	1300
Gopal	Perryridge	5300

Result of $_{branch\text{-}name}\mathcal{G}_{sum(salary)}$ (pt-works)

branch-name	*sum of salary*
Austin	3100
Downtown	5300
Perryridge	8100

Result of $_{branch\text{-}name}\mathcal{G}_{\text{sum } salary,\ \text{max}(salary) \text{ as } max\text{-}salary}$ (*pt-works*)

branch-name	*sum-salary*	*max-salary*
Austin	3100	1600
Downtown	5300	2500
Perryridge	8100	5300

The *employee* and *ft-works* Relations

employee-name	*street*	*city*
Coyote	Toon	Hollywood
Rabbit	Tunnel	Carrotville
Smith	Revolver	Death Valley
Williams	Seaview	Seattle

employee-name	*branch-name*	*salary*
Coyote	Mesa	1500
Rabbit	Mesa	1300
Gates	Redmond	5300
Williams	Redmond	1500

The Result of *employee* $\bowtie$ *ft-works*

employee-name	*street*	*city*	*branch-name*	*salary*
Coyote	Toon	Hollywood	Mesa	1500
Rabbit	Tunnel	Carrotville	Mesa	1300
Williams	Seaview	Seattle	Redmond	1500

The Result of *employee* ⟕ *ft-works*

employee-name	*street*	*city*	*branch-name*	*salary*
Coyote	Toon	Hollywood	Mesa	1500
Rabbit	Tunnel	Carrotville	Mesa	1300
Williams	Seaview	Seattle	Redmond	1500
Smith	Revolver	Death Valley	*null*	*null*

Result of *employee* ⟖ *ft-works*

employee-name	*street*	*city*	*branch-name*	*salary*
Coyote	Toon	Hollywood	Mesa	1500
Rabbit	Tunnel	Carrotville	Mesa	1300
Williams	Seaview	Seattle	Redmond	1500
Gates	*null*	*null*	Redmond	5300

Result of *employee* ⟗ *ft-works*

employee-name	*street*	*city*	*branch-name*	*salary*
Coyote	Toon	Hollywood	Mesa	1500
Rabbit	Tunnel	Carrotville	Mesa	1300
Williams	Seaview	Seattle	Redmond	1500
Smith	Revolver	Death Valley	*null*	*null*
Gates	*null*	*null*	Redmond	5300

Tuples Inserted Into *loan* and *borrower*

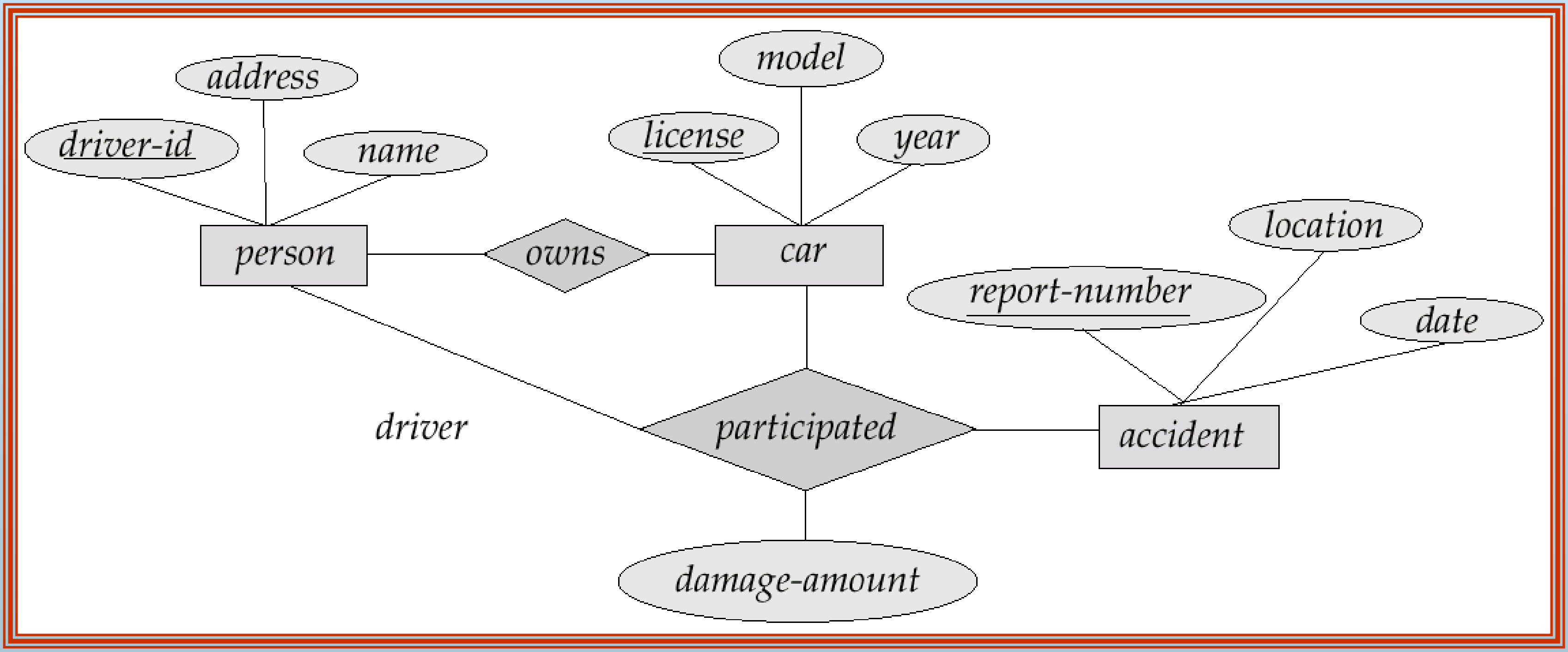
loan-number	*branch-name*	*amount*
L-11	Round Hill	900
L-14	Downtown	1500
L-15	Perryridge	1500
L-16	Perryridge	1300
L-17	Downtown	1000
L-23	Redwood	2000
L-93	Mianus	500
null	*null*	1900

customer-name	*loan-number*
Adams	L-16
Curry	L-93
Hayes	L-15
Jackson	L-14
Jones	L-17
Smith	L-11
Smith	L-23
Williams	L-17
Johnson	*null*

Names of All Customers Who Have a Loan at the Perryridge Branch

customer-name
Adams
Hayes

E-R Diagram


address
driver-id
name
model
license
year
person
owns
car
location
report-number
date
driver
participated
accident
damage-amount

The *branch* Relation

branch-name	*branch-city*	*assets*
Brighton	Brooklyn	7100000
Downtown	Brooklyn	9000000
Mianus	Horseneck	400000
North Town	Rye	3700000
Perryridge	Horseneck	1700000
Pownal	Bennington	300000
Redwood	Palo Alto	2100000
Round Hill	Horseneck	8000000

The *loan* Relation

loan-number	*branch-name*	*amount*
L-11	Round Hill	900
L-14	Downtown	1500
L-15	Perryridge	1500
L-16	Perryridge	1300
L-17	Downtown	1000
L-23	Redwood	2000
L-93	Mianus	500

The *borrower* Relation

customer-name	*loan-number*
Adams	L-16
Curry	L-93
Hayes	L-15
Jackson	L-14
Jones	L-17
Smith	L-11
Smith	L-23
Williams	L-17